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Teaching multiloop control of nonlinear system: three tanks case study

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Abstract: The paper focuses on teaching advanced MIMO control topics using hydraulic system model. Basic steps of nonlinear MIMO system control design are shown on three tank plant model. After modelling part, comprising nonlinear model based on first principles and its linearization about equilibrium point, two approaches to MIMO system control are demonstrated. The first approach designs a full state feedback PI controller in state space, the second one uses multi-loop approach with individual loop design, overall stability condition is used to detune loop PI controllers for the overall system issues. The results are analysed and compared, important teaching issues are indicated.

Keywords: modelling, multi-input multi-output system, multi-loop control, PI controller, nonlinear system, robust stability

1. INTRODUCTION

Teaching advanced control systems belongs to important topics in mechatronics study. To connect theoretical concepts with applications, modelling and simulation of real plants contribute to increase interest and understanding of control design fundaments, preparing students to real laboratory work and practice, Oggunaike et al. (1994). Hydraulic systems belong to basic laboratory plants, used for simple as well as complex control task demonstration, Rosinová and Kozáková, (2009). Many control issues, including nonlinearities and multi-input multi-output system control can be taught and illustrated on various types of hydraulic plants, as four tank system introduced by Johansson et al. (1999), Johansson (2000).

In this paper, we consider a three tank system model corresponding to the laboratory plant delivered by Inteco company. Series of control design tasks necessary for nonlinear MIMO system controller design is presented. The nonlinear system control based on linearization around working point is adopted. Though this approach works only within limited operation region, its simplicity and applicability in many real plants make it attractive and frequently used in nonlinear systems control. Modelling part comprises nonlinear model and its linearization around equilibrium point. (The results are slightly different from those presented in plant user manual.) The main focus is on controller design and comparison of different control design approaches. State feedback control with full size gain matrices is confronted with a multi-loop controller (called also decentralized controller, according to some sources). Multi-loop control reduces multivariable control problem complexity, and enables to employ decentralized control structure with subsystems having their local control loops. Compared with centralized MIMO controller systems, multi-loop control structure generally yields certain performance deterioration, however, weighted against by important benefits, such as design simplicity, hardware, operation and reliability improvement. Robustness is one of attractive qualities of a multi-loop control scheme, since such control structure can be inherently resistant to a wide range of uncertainties both in subsystems and interconnections.

We study the basic steps of nonlinear MIMO system control design on three tank process case study with 4 inputs and 3 outputs. After the modelling phase, control structure is selected; a full size state feedback control is designed using pole placement technique in the state space. Presented alternative – multi-loop control, requires a choice of appropriate input-output pairing. Further step is independent single loops design so that it guarantees stability of the overall system including interactions. Two alternatives of stability condition for decentralized control structure are used: one based on small gain theorem for complementary sensitivity function and one for systems with no RHP (right half plane) zeros. The obtained results are analysed and compared.

2. MODELLING AND CONTROL OF NONLINEAR THREE TANK SYSTEM

In this section, key points of teaching basics of nonlinear modeling and control, using three tank model, are summarized. The aim of this part of advanced control course is to cover basic steps necessary to design appropriate control for a nonlinear MIMO system, realized and verified on a simulation model of a three tank system.

Modelling part includes:

- analysis of nonlinear three tank mathematical model, input and output variables and their ranges,

- determination of working area – steady state of nonlinear system and control aim

- derivation of linearized model using standard Taylor series approach

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- verification of linearized model: comparison of nonlinear and linear model using the respective Simulink schemes; importance of settling the system into the working point

Controller design part consists of:

- determination of control structure (full size controller versus multi-loop – decentralized controller)

- choosing appropriate input-output pairing for multi-loop controller

- loop controller design (PI controllers)

- testing (robust) stability condition under decentralized controller design

- verifying the designed controllers by simulation in Simulink, comparison of linearized model and nonlinear model results.

Presented material can be preferrably used for project oriented teaching, when the students have to complete tasks and write a report, summarizing all procedures and results. The important part of the project is analysis of the obtained results and comparison of the designed controllers and closed loop responses both for original nonlinear and linearized model.

3. THREE TANK SYSTEM MODELLING

The model of multi tank system, corresponding to the one, produced by Inteco Co is considered, Fig.1. It comprises three separated tanks placed above each other and interconnected with drain valves (Inteco Co., 2006). The upper tank has a constant cross section, while the others have a variable cross section: trapezoidal middle one and round lower one. These variable cross sections add further nonlinearities in the system, besides physical laws of flow. The water is pumped into the upper tank from the supply tank by a pump driven by a DC motor. The water outflows from the tanks due to gravity through orifices placed in the bottom of tanks. The tank valves act as flow resistors. The area ratio of the valves is controlled and can be used to vary the outflow characteristic. Each tank is equipped with a level sensor based on hydraulic pressure measurement.

The standard procedure of the model development is adopted, consisting of the following steps: developing the mathematical model based on the basic physical laws; determination of equilibrium respective to the working point and the respective model linearization; building the simulation model and its verification.

The nonlinear model is developed in the state space, where liquid levels H_1, H_2, H_3 in the tanks are measurable state variables of the system. The controlled inputs are: liquid flow rate q and valves settings C_1, C_2, C_3. Output variables are the same as state ones, which later simplifies state space controller design. In Fig. 1, the multi tank system components interconnection is shown (Inteco, 2006). In this paper the control strategy based on pump and valves control system is investigated. The nonlinearities caused by shapes of tanks and those introduced by valve flow dynamics cause difficulties in achieving the control aim - a high accuracy control of the tank levels.

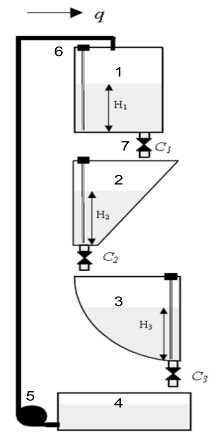

Fig. 1 Physical model of multitank system

3.1 Nonlinear model of three tank system

The mathematical model of multi tank system can be derived from the general model of n-tanks. The model of the process dynamics can be obtained by the mass balance equation

$$\begin{aligned}
&\frac{dV_1}{dH_1}\frac{dH_1}{dt} = q - C_1 H_1^{\alpha_1} - w_{l1} H_1^{\alpha_1} \\
&\frac{dV_2}{dH_2}\frac{dH_2}{dt} = C_1 H_1^{\alpha_1} - C_2 H_2^{\alpha_2} + w_{l1} H_1^{\alpha_1} - w_{l2} H_2^{\alpha_2} \\
&\dots \\
&\frac{dV_n}{dH_n}\frac{dH_n}{dt} = C_{n-1} H_{n-1}^{\alpha_{n-1}} - C_n H_n^{\alpha_n} + w_{l(n-1)} H_{n-1}^{\alpha_{n-1}} - w_{\ln} H_n^{\alpha_n}
\end{aligned} \tag{1}$$

where V_i is the fluid volume of the i-th tank, C_i is the resistance of the i-th output orifice, w_{li} is the resistance of the i-th disturbance orifice and H_i is the level in the i-th tank.

For the laminar flows, the outflow rate from a tank can be described by the Bernoulli law with $\alpha_i = 1/2$. For real configuration, a more general coefficients α_i are applied. From (1), the nonlinear multi tank system model is obtained

$$\frac{dH_1}{dt} = \frac{1}{\beta_1(H_1)} q - \frac{1}{\beta_1(H_1)} C_1 H_1^{\alpha_1} - \frac{1}{\beta_1(H_1)} w_{l1} H_1^{\alpha_1}, \tag{2}$$

$$\frac{dH_2}{dt} = \frac{1}{\beta_2(H_2)} C_1 H_1^{\alpha_1} - \frac{1}{\beta_2(H_2)} C_2 H_2^{\alpha_2} + \frac{1}{\beta_2(H_2)} w_{l1} H_1^{\alpha_1}, \tag{3}$$

$$\begin{aligned}
\frac{dH_3}{dt} = &\frac{1}{\beta_3(H_3)} C_2 H_2^{\alpha_2} - \frac{1}{\beta_3(H_3)} C_3 H_3^{\alpha_3} + \\
&+ \frac{1}{\beta_3(H_3)} w_{l2} H_2^{\alpha_2} - \frac{1}{\beta_3(H_3)} w_{l3} H_3^{\alpha_3}
\end{aligned}, \tag{4}$$

$$\begin{aligned}
&\beta_1(H_1) = aw \\
&\beta_2(H_2) = cw + \frac{H_2}{H_{2\max}} bw \\
&\beta_3(H_3) = w\sqrt{R^2 - (R - H_3)^2}
\end{aligned} \tag{5}$$

where $\beta_i(H_i)$ is the cross sectional area of i-th tank at the level H_i.