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Brief paper

State-space H_∞ controller design for descriptor systems[☆]

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ABSTRACT

This paper proposes a new linear matrix inequality (LMI) method to design state-space H_∞ controllers for linear time-invariant descriptor systems. Unlike preceding studies, where descriptor-type controllers are first computed and then numerically transformed to state-space controllers, the proposed method carries out the transformation analytically in the parameter domain. We derive a necessary and sufficient LMI condition for the existence of a state-space controller with the same dynamic order of the descriptor system to be controlled, which makes the closed-loop system regular, impulse-free, stable, and guarantees the H_∞ norm bound imposed on the closed-loop transfer function. Furthermore, we present parameterization of all such state-space controllers by variables satisfying the LMI condition and an arbitrary nonsingular matrix. The LMIs utilized in this paper are strict ones, that is, those containing no equality, while LMIs with equality constraints have been extensively used in the analysis and design for descriptor systems. The strict LMIs play key roles in deriving the results of this paper.

1. Introduction

This paper considers H_∞ control of general linear time-invariant descriptor systems including irregular or impulsive ones. There have been a number of preceding studies using linear matrix inequalities (LMIs), which deal with descriptor-type controllers of the same size as the systems to be controlled. Necessary and sufficient conditions have been proposed for the existence of such H_∞ controllers, and coefficients of controllers are given by the solutions of LMIs (see, e.g., Masubuchi, Kamitane, Ohara, & Suda, 1997, Rehm & Allgöwer, 2001, Uezato & Ikeda, 1999 and Xu & Lam, 2006). Theoretically, these results are satisfactory.

However, it is not easy to compute the control inputs from the measured outputs by descriptor-type controllers, because we do not have an efficient way of solving descriptor-type equations, that is, differential equations under algebraic constraints. Therefore, we usually transform the descriptor-type controllers to input–output equivalent state-space controllers or transfer functions. The transformations are carried out in the numerical domain. This idea would be fine in practical control.

In this paper, we take a different approach, the original idea of which the authors adopted in deriving state-space stabilizing controllers for descriptor systems (Inoue, Wada, Ikeda, & Uezato, 2012). We obtain state-space controllers for a descriptor system without computing descriptor-type controllers numerically. The state-space controllers are realized by treating descriptor-type controllers in the parameter domain, where the coefficients of the descriptor-type controllers are expressed by variables satisfying LMIs, which describe a necessary and sufficient condition for the existence of a descriptor-type H_∞ controller, and arbitrary parameters. We analytically transform the descriptor-type controllers to input–output equivalent state-space controllers whose dimension is the same as the dynamic order (the rank of the coefficient matrix for the time-derivative of the descriptor variable) of the descriptor-type controller under a necessary and sufficient condition for the equivalent transformation. In this way, we can derive all parameterized state-space H_∞ controllers for a given system, which make

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the closed-loop systems regular, impulse-free, stable, and guarantee the specified H_∞ norm bound on the closed-loop transfer functions.

The coefficient matrices of the state-space H_∞ controllers are expressed in terms of the solutions of the LMIs and an arbitrary nonsingular matrix. It is shown that the nonsingular matrix plays the role of the equivalent transformation of the state space and thus does not affect the input–output property of the controllers. This finding is a contribution of the present paper.

The LMIs which we utilize in this paper are strict ones (Uezato & Ikeda, 1999), namely, those not containing any equality, while LMIs with equality constraints are extensively used in analysis and design for descriptor systems (see, e.g., Masubuchi et al., 1997 and Rehm & Allgöwer, 2001). The strict LMIs play key roles in obtaining the results of this paper.

Direct design of state-space controllers, that is, design not through descriptor-type controllers, was studied. Rehm and Allgöwer (1998) proposed two conditions for the existence of state-space H_∞ controllers for descriptor systems. One is expressed by bi-affine matrix inequalities, which is a necessary and sufficient condition. The other is expressed by LMIs, but is only a necessary condition.

The authors of this paper also proposed direct design of strictly proper state-space H_∞ controllers for a regular and impulse-free descriptor system via an LMI approach and gave an existence condition in Inoue, Wada, Ikeda, and Uezato (2011). The present paper extends that result to general descriptor systems including those being irregular or impulsive and provides a parameterized form of all proper state-space H_∞ controllers. The approach here comes essentially from the same idea as Inoue et al. (2011), but makes the process of deriving the state-space controller more understandable by using a parameterized descriptor-type controller.

2. System and controller

Let us deal with a linear time-invariant descriptor system

$$\begin{cases} E\dot{x} = Ax + B_1 w + B_2 u, \\ z = C_1 x, \\ y = C_2 x, \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$ is the descriptor variable, $w \in \mathbb{R}^p$ is the disturbance input, $u \in \mathbb{R}^m$ is the control input, $z \in \mathbb{R}^q$ is the controlled output, $y \in \mathbb{R}^\ell$ is the measured output, and $E, A \in \mathbb{R}^{n\times n}$, $B_1 \in \mathbb{R}^{n\times p}$, $B_2 \in \mathbb{R}^{n\times m}$, $C_1 \in \mathbb{R}^{q\times n}$, $C_2 \in \mathbb{R}^{\ell\times n}$ are constant coefficient matrices. The matrix E may be singular and we denote rank E by r $(\le n)$. Then, only an r-dimensional component of the descriptor variable x contributes the dynamics of the system (1). For this reason, we called rank E the dynamic order (e.g., Inoue et al., 2012) of the descriptor system in Introduction. We note that although the direct transmission paths from w and u to z and y are not seen explicitly in (1), such paths can be included by augmenting the descriptor variable if necessary (e.g., Masubuchi et al., 1997). In this paper, we treat general descriptor systems including those being irregular or impulsive. We assume that the triple (E, A, B_2) is stabilizable and controllable at infinity, and (C_2, E, A) is detectable and observable at infinity (Verghese, Levy, & Kailath, 1981).

We consider a dynamic controller of the form

$$\Sigma_C(\hat{E}, \hat{A}, \hat{B}, \hat{C}, \hat{D}) : \begin{cases} \hat{E}\dot{\xi} = \hat{A}\xi + \hat{B}y, \\ u = \hat{C}\xi + \hat{D}y, \end{cases} \tag{2}$$

where $\xi \in \mathbb{R}^k$ is the descriptor variable of the controller and $\hat{E}$, $\hat{A} \in \mathbb{R}^{k\times k}$, $\hat{B} \in \mathbb{R}^{k\times\ell}$, $\hat{C} \in \mathbb{R}^{m\times k}$, $\hat{D} \in \mathbb{R}^{m\times\ell}$ are constant matrices. In this paper, we treat only the following two cases.

$$\text{(a) } \hat{E} = E, \quad k = n, \qquad \text{(b) } \hat{E} = I_r, \quad k = r. \tag{3}$$

In the case (a), the controller (2) is the descriptor-type considered extensively in preceding studies, e.g., Masubuchi et al. (1997), Rehm and Allgöwer (2001), Uezato and Ikeda (1999), Xu and Lam (2006), and Zhang, Huang, and Lam (2003). In the case (b), it is a state-space controller with the dimension of the dynamic order of the descriptor system (1). Although it might be interesting to consider other $\hat{E}$ matrices, the authors of the present paper believe that it is good enough to treat only these two cases for the H_∞ control problem.

The closed-loop system composed of the system (1) and the controller (2) is written using the combined descriptor variable $x_c = [x^T \quad \xi^T]^T$ as

$$\begin{cases} \hat{E}_c\dot{x}_c = A_c x_c + B_c w, \\ z = C_c x_c, \end{cases} \tag{4}$$

where

$$\hat{E}_c = \begin{bmatrix} E & 0 \\ 0 & \hat{E} \end{bmatrix}, \qquad A_c = \begin{bmatrix} A + B_2\hat{D}C_2 & B_2\hat{C} \\ \hat{B}C_2 & \hat{A} \end{bmatrix},$$

$$B_c = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}, \qquad C_c = \begin{bmatrix} C_1 & 0 \end{bmatrix}. \tag{5}$$

The descriptor system (4) is said to be *regular* if $\det(s\hat{E}_c - A_c) \not\equiv 0$. In addition, the system is said to be *impulse-free* if $\deg\det(s\hat{E}_c - A_c) = \operatorname{rank}\hat{E}_c$. When (4) is regular and impulse-free, it has a proper transfer function

$$G_{zw(4)}(s) = C_c(s\hat{E}_c - A_c)^{-1}B_c, \tag{6}$$

and there exists a unique and continuous solution $x_c(t)$, $t > 0$ for any initial value $x_c(0)$ and any input $w(t)$ which is continuous at almost every t. The system is said to be *stable* if it is regular, impulse-free, and all roots of the polynomial $\det(s\hat{E}_c - A_c)$ have negative real parts. This paper considers descriptor-type and state-space controllers (2) which make the closed-loop system (4) stable and the H_∞ norm $\|G_{zw(4)}\|_\infty$ of the transfer function $G_{zw(4)}(s)$ less than a specified value.

3. State-space controllers

In this section, we present a necessary and sufficient condition for the existence of state-space H_∞ controllers for descriptor systems, and give their coefficient matrices. For this, we use the following matrices (Uezato & Ikeda, 1999). Matrices $E_L, E_R \in \mathbb{R}^{n\times r}$ are of full column rank and satisfy $E = E_L E_R^T$. Matrices $U, V \in \mathbb{R}^{n\times(n-r)}$ are of full column rank and their column vectors are composed of bases of KerE^T and Ker$E$, respectively. From these definitions, we see that

$$E^T U = 0, \qquad EV = 0, \qquad E_L^T U = 0, \qquad E_R^T V = 0 \tag{7}$$

and the identities

$$I_n = E_L(E_L^T E_L)^{-1}E_L^T + U(U^T U)^{-1}U^T,$$

$$I_n = E_R(E_R^T E_R)^{-1}E_R^T + V(V^T V)^{-1}V^T \tag{8}$$

hold. We note that although the matrices E_L, E_R, U, and V are not unique, all the discussions and results in this paper do not depend on their choices, because images of the matrices, $\operatorname{Im} E_L = \operatorname{Im} E$, $\operatorname{Im} E_R = \operatorname{Im} E^T$, $\operatorname{Im} U = \operatorname{Ker} E^T$, $\operatorname{Im} V = \operatorname{Ker} E$ are invariant.

We introduce LMIs and a matrix to express existence conditions of H_∞ controllers and their coefficient matrices. We use matrix variables $F \in \mathbb{R}^{n\times n}$, $G \in \mathbb{R}^{n\times\ell}$, $H \in \mathbb{R}^{m\times n}$, $J \in \mathbb{R}^{m\times\ell}$, P,