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Brief paper

Stabilization of networked control systems with both network-induced delay and packet dropout[☆]

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ABSTRACT

This paper is concerned with the mean-square stabilization problem for discrete-time networked control systems (NCSs). It is assumed that control signal is sent to plant over a lossy communication channel, where network-induced delay and packet dropout occur simultaneously. A necessary and sufficient stabilizing condition is developed in terms of the unique positive-definite solutions to some coupled algebraic Riccati equations (CAREs). The contributions of this paper are twofold. First, an existence theorem of the maximum packet dropout rate is proposed. Second, for one-dimensional single-input system and the decoupled multi-input system, it is shown that the NCS is stabilizable iff the network-induced delay and the packet dropout rate satisfy some simple algebraic inequalities. If the network-induced delay is known a priori, the maximum packet dropout rate is given explicitly in terms of network-induced delay and unstable eigenvalues of the system matrix. If the packet dropout rate is known a priori, the maximum allowable delay bound is also given explicitly.

1. Introduction

Recently, networked control systems (NCSs) have attracted considerable interest due to their wide applications in different areas, such as automated highway systems, unmanned aerial vehicles and aircrafts (see Hespanha, Naghshtabrizi, & Xu, 2007; Schenato, Sinopoli, Franceschetti, Poolla, & Sastry, 2007; Zhang, Branicky, & Phillips, 2001, and the references therein). In contrast to classical feedback control systems, NCSs have various advantages including low cost, high flexibility, ease of installation and maintenance. However, the limited communication resources and unreliability of communication channel introduce some challenging problems, such as packet dropout (Elia, 2005; Hu & Yan, 2007; Xiong & Lam, 2007), transmission delay (Gao, Meng, & Chen, 2008; Khalil & Wang, 2011), quantization (Fu & Xie, 2005), etc. For instance, network-induced delay often occurs while exchanging data among devices connected to the shared medium. This delay can degrade the performance of NCSs and even destabilize the whole system. Besides, when control signal transmits from controller to plant through a lossy communication channel, data packet dropout may occur. As a consequence, it is of great significance to study the NCSs with both network-induced delay and packet dropout.

Since the NCSs with network-induced delay and packet dropout can be treated as stochastic systems with input delay and multiplicative noises, a great deal of available results can be extended to study the NCSs. For stochastic delay-free system, some necessary and sufficient stabilizing conditions have been developed in terms of generalized algebraic Riccati equation (GARE) (Huang, Zhang, & Zhang, 2008), or linear matrix inequalities (LMIs) (Ait Rami & Zhou, 2000). Moreover, Xiao, Xie, and Qiu (2012) addressed the stabilization problem for discrete-time NCSs over fading channels and showed that under the channel/controller co-design framework, the NCS can be stabilized by the state feedback control iff the overall mean-square channel capacity is greater than the topological entropy. On the other hand, the packet dropout problem has been an active area of research. Sinopoli et al. (2004) investigated the effect of packet dropout on the convergence property of the state estimation error covariance and proved the existence of the critical packet dropout rate, beyond which the error covariance would diverge. Besides, Hu and Yan (2007) introduced an optimization algorithm for computing the value of maximum packet dropout rate under the constraint that the NCS has prescribed nominal closed-loop poles.

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However, the aforementioned literatures have not taken network-induced delays into account. As cited in Zhang and Yu (2008), due to the simultaneous appearance of time delay and packet dropout, the analysis and synthesis for such NCS is much challenging. In recent years, some studies have focused on the stabilization problem in the presence of both data packet dropout and network-induced delay. The efficiency of various methods, including the switched approach (Zhang & Yu, 2008), Lyapunov–Krasovskii functional approach (Sun & Jiang, 2013), and predictive control method (Liu, 2010), is mostly on the LMIs and only sufficient conditions are available. Meanwhile, for the NCS with both network-induced delay and packet dropout, how to compute the maximum packet dropout rate is also a challenging problem in a long-term time. Therefore, the stabilization problem for such NCSs is very involved. Recently, Zhang, Li, and Xu (2014) investigated the LQ optimal control problem for multiplicative-noise stochastic systems with input delay, where the optimal control is the feedback of the conditional expectation of the state. Based on the CAREs, the necessary and sufficient stabilizing condition was developed.

Motivated by Zhang, Li, and Xu's work (2014), in this paper, we are concerned with discrete-time NCS with both network-induced delay and packet dropout. The necessary and sufficient stabilizing condition is developed in terms of the unique positive-definite solutions to some newly derived CAREs. Second, under some basic assumptions, it is shown that there exists a unique maximum packet dropout rate. Specifically, for one-dimensional single-input system, the NCS is stabilization iff the network-induced delay and the packet dropout rate satisfy a simple inequality, which can be used to derive the maximum packet dropout rate and the maximum allowable delay bound. If the network-induced delay is known a priori, the maximum packet dropout rate can be derived explicitly, which is uniquely determined by the network-induced delay and the unstable eigenvalue of the system matrix. If the packet dropout rate is known a priori, the maximum allowable delay bound can be derived explicitly in terms of the unstable eigenvalues and the packet dropout rate. Similar results can be derived for the decoupled multi-input system.

The rest of this paper is organized as follows. In Section 2, the NCS under consideration is described and the problems to be solved are formulated. In Section 3, the existence theorem of the maximum packet dropout rate is developed and some explicit computation formula for the maximum packet dropout rate and the maximum allowable delay bound are derived for one-dimensional single-input system and the decoupled multi-input system. Concluding remarks are given in Section 4.

Notation: A' denotes the transpose of the matrix A and I the identity matrix. $A \geq 0 (>0)$ means that A is a positive-semidefinite (positive-definite) matrix and $A \geq B$ $(>B)$ means that $A - B \geq 0$ (>0). $\mathbb{R}^n$ denotes the n-dimensional real Euclidean space and δ_{ks} the Kronecker function, i.e., $\delta_{ks} = 1$ if $k = s$ and $\delta_{ks} = 0$ if $k \neq s$. $\{w_k, k \in \mathbb{N}\}$ denotes a sequence of real random variables defined on the filtered probability space $(\Omega, \mathcal{F}, \mathcal{P}; \mathcal{F}_k)$ with $\mathcal{F}_0 = \{\emptyset, \Omega\}$ and $\mathcal{F}_k = \sigma\{w_j | j = 0, 1, 2, \ldots, k\}$. Moreover, $\mathcal{P}(A)$ denotes the probability if the event A occurs, and $\hat{x}_{j|k} = E[x_j | \mathcal{F}_k]$ denotes the conditional expectation of x_j w.r.t. $\mathcal{F}_k$.

2. Problem formulation

The NCS under consideration of this paper is depicted in Fig. 1, which is composed of a plant, a controller and a lossy communication channel. It is assumed that the controller is collocated with the plant, and can access the information of the state x_k at any time k. Meanwhile, the designed control signal u_k is sent to the plant through a lossy communication channel where packet dropout and network-induced delay occur simultaneously.

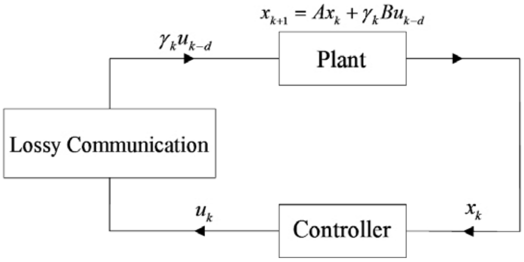

Fig. 1. Networked control system over lossy communication.

To sum up, the input control of Fig. 1 can be modeled by $\gamma_k u_{k-d}$. Here, $\gamma_k = 1$ denotes that the data packet has been successfully delivered to the plant, and $\gamma_k = 0$ signifies the dropout of the data packet. Without loss of generality, the random process $\{\gamma_k\}_{k\geq 0}$ is modeled as an i.i.d. Bernoulli process with probability distribution $\mathcal{P}(\gamma_k = 0) = p$ and $\mathcal{P}(\gamma_k = 1) = 1 - p$, where $p \in (0, 1)$ is named as the packet dropout rate.

The overall NCS in Fig. 1 can be described as

$$x_{k+1} = Ax_k + \gamma_k Bu_{k-d}, \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the state and $u_{k-d} \in \mathbb{R}^p$ is the input control with a constant time delay $d > 0$. x_0, u_i $(i = -1, \ldots, -d)$ are the known initial values, and A, B are constant matrices with compatible dimensions. In this paper, we assume

(H1) A is unstable and B has full-column rank.

Denote $\omega_k = \gamma_k - E[\gamma_k] = \gamma_k - (1 - p)$. Then system (1) can be rewritten as

$$x_{k+1} = Ax_k + (1 - p)Bu_{k-d} + w_k Bu_{k-d}. \tag{2}$$

In this case, $\{w_k, k \in \mathbb{N}\}$ is a sequence of random variables defined on the filtered probability space $(\Omega, \mathcal{F}, \mathcal{P}; \mathcal{F}_k)$ with $E[w_k] = 0$ and $E[w_k w_s] = p(1 - p)\delta_{ks}$. For convenience, the NCS with both network-induced delay and packet dropout will be described as $[A, B, p|d]$ hereinafter.

Remark 1. In this paper, since the network-induced delay is assumed to be constant, at any time k, the control signal arriving at the plant is either u_{k-d} or zero. The problem of packet disorder would be avoided. However, if the network-induced delay is assumed to be random or time-vary, the packet disorder would occur and the stabilization problem is much challenging. In addition, the proposed results of this paper can be extended to study the general NCS, where the lossy communication channel exists in both the sensor-to-controller channel (SCC) and controller-to-actuator channel (CAC) leading to network-induced delay d_{scc} of state signal x_k, and the simultaneous network-induced delay d_{cac} and packet dropout of control signal u_k.

This paper is concerned with the mean-square stabilization problem for the NCSs with both network-induced delay and packet dropout. The main objective is to derive the maximum packet dropout rate and the maximum allowable delay bound. Under some basic assumptions, the existence of maximum packet dropout rate is to be derived. In addition, the necessary and sufficient stabilizing condition is to be derived.

Remark 2. For the NCSs with both network-induced delay and packet dropout, the previous work only provided some sufficient stabilizing conditions based on some LMIs, such as Zhang and Yu (2008), Sun and Jiang (2013), and Liu (2010). It is remarkable that if the stabilizing control is designed to be a feedback of the state, owing to the information shortage, it is difficult to obtain the necessary and sufficient condition. To tackle this difficulty, in Zhang et al. (2014), the stabilizing control was designed to be the feedback of the conditional expectation of the state, and the necessary