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Online fault diagnosis for nonlinear power systems[☆]

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ABSTRACT

This paper considers the problem of automatic fault diagnosis for transmission lines in large scale power networks. Since faults in transmission lines threatens stability of the entire power network, fast and reliable fault diagnosis is an important problem in transmission line protection. This work is the first paper exploiting sparse signal recovery for the fault-diagnosis problem in power networks with nonlinear swing-type dynamics. It presents a novel and scalable technique to detect, isolate and identify transmission faults using a relatively small number of observations by exploiting the sparse nature of the faults. Buses in power networks are typically described by second-order nonlinear swing equations. Based on this description, the problem of fault diagnosis for transmission lines is formulated as a compressive sensing or sparse signal recovery problem, which is then solved using a sparse Bayesian formulation. An iterative reweighted ℓ_1-minimisation algorithm based on the sparse Bayesian learning update is then derived to solve the fault diagnosis problem efficiently. With the proposed framework, a real-time fault monitoring scheme can be built using only measurements of phase angles at the buses.

1. Introduction

Power networks are large-scale spatially distributed systems. Being critical infrastructures, they possess strict safety and reliability constraints. The design of monitoring schemes to diagnose anomalies caused by unpredicted or sudden faults on power networks is thus of great importance (Shahidehpour, Tinney, & Fu, 2005). To be consistent with the international definition of the fault diagnosis problem, the recommendations of the IFAC Technical Committee *SAFEPROCESS* is accordingly employed in what follows. Namely, this work proposes a method to: (1) decide whether there is an occurrence of a fault and the time of this occurrence (*i.e. detection*), (2) establish the location of the detected fault (*i.e. isolation*), and (3) determine the size and time-varying behaviour of the detected fault (*i.e. identification*).

Since power networks are typically large-scale and have nonlinear dynamics, fault diagnosis over transmission lines can be a very challenging problem. This paper draws inspiration from the fields of signal processing and machine learning to combine compressive sensing and variational Bayesian inference techniques so as to offer an efficient method for fault diagnosis.

Most of the literature available on fault diagnosis focuses on systems approximated by linear dynamics (Ding, 2008), with applications in networked system (Dong, Wang, & Gao, 2012), modern complex processes (Yin, Ding, Haghani, Hao, & Zhang, 2012), etc. Beyond linear systems descriptions, the dynamics of buses in power networks can be described by the so-called swing equations where the active power flows are nonlinear functions of the phase angles. Works that have considered fault detection and isolation in power networks include (Mohajerin Esfahani, Vrakopoulou, Andersson, & Lygeros, 2012; Shames, Teixeira, Sandberg, & Johansson, 2011; Zhang, Zhang, Polycarpou, & Parisini, 2014). Shames et al. (2011) focuses on distributed fault detection and isolation using linearised swing dynamics and the faults are considered to be additive. The method developed in Zhang et al. (2014) is used to detect sensor faults assuming that such faults appear as biased faults added to the measurement equation. In Mohajerin

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Esfahani et al. (2012), a fault detection and isolation residual generator is presented for nonlinear systems with additive faults. The nonlinearities in Mohajerin Esfahani et al. (2012) are not imposed *a priori* on the model structure but treated as disturbances with some known patterns.

To summarise, the works (Ding, 2008; Dong et al., 2012; Shames et al., 2011; Yin et al., 2012) use linear systems to characterise the dynamics of power networks and the faults are assumed to be additive. Though the system dynamics are nonlinear in Mohajerin Esfahani et al. (2012) and Zhang et al. (2014), the faults are still assumed to be additive. The methods developed on the basis of these conservative assumptions yield several problems. Firstly, the linear approximation to nonlinear swing equations can only be used when the phase angles are close to each other. However, when the system is strained and faults appear, phase angles can often be far apart. Therefore, a linear approximation is inappropriate in strained power network situations. Secondly, it is well-known that a large portion of power system faults occurring in transmission lines do not involve additive faults, e.g. a short-circuit fault occurring on the transmission lines between generators would correspond to some changes in the parameters of the nonlinear terms appearing in the swing equation (Kundur, Balu, & Lauby, 1994). Furthermore, the inevitable and frequent introduction of new components in a power network contributes to the vulnerability of transmission lines, which, if not appropriately controlled, can lead to cascading failures (Hines, Balasubramaniam, & Sanchez, 2009; Jiang, Yang, Lin, Liu, & Ma, 2000). Such cascading failures cannot be captured by additive faults. Finally, the methods mentioned above only address fault detection and isolation rather than identification, which is crucial to take appropriate actions when faults occur on transmission lines.

Contributions. The power networks considered in this paper are described by the *nonlinear swing equations* with additive process noise. The faults are assumed to occur on the transmission lines of the power network. The problem of fault diagnosis, i.e. detection, isolation and identification, of such nonlinear power networks is formulated as a compressive sensing or sparse signal recovery problem. To solve this problem we consider a sparse Bayesian formulation of the fault identification problem, which is then casted as a nonconvex optimisation problem. Finally, the problem is relaxed into a convex problem and solved efficiently using an iterative reweighted ℓ_1-minimisation algorithm. The resulting efficiency of the proposed method enables real-time detection of faults in large-scale networks.

Outline. The outline of the paper is as follows. Section 2 introduces the nonlinear model of power networks considered in this paper. Section 3 formulates the fault diagnosis problem as a compressive sensing or sparse signal recovery problem. Section 4 shows how the resulting nonconvex optimisation problem can be relaxed into a convex optimisation problem and solved efficiently using an iterative reweighted ℓ_1-minimisation algorithm. Section 5 applies the method to a power network with 20 buses and 80 transmission lines and, finally, Section 6 concludes and discusses several future problems.

Notation. The notation in this paper is standard. Bold symbols are used to denote vectors and matrices. For a matrix $\mathbf{A} \in \mathbb{R}^{M\times N}$, $\mathbf{A}_{i,j} \in \mathbb{R}$ denotes the element in the ith row and jth column, $\mathbf{A}_{i,:} \in \mathbb{R}^{1\times N}$ denotes its ith row, $\mathbf{A}_{:,j} \in \mathbb{R}^{M\times 1}$ denotes its jth column. For a column vector $\boldsymbol{\alpha} \in \mathbb{R}^{N\times 1}$, α_i denotes its ith element. In particular, $\mathbf{I}_l$ denotes the identity matrix of size $l \times l$. We simply use $\mathbf{I}$ when the dimension is obvious from context. $\|\mathbf{w}\|_1$ and $\|\mathbf{w}\|_2$ denote the ℓ_1 and ℓ_2 norms of the vector $\mathbf{w}$, respectively. $\|\mathbf{w}\|_0$ denotes the ℓ_0 "norm" of the vector $\mathbf{w}$, which counts the number of nonzero elements in the vector $\mathbf{w}$. diag $[\gamma_1, \ldots, \gamma_N]$ denotes a diagonal matrix with principal diagonal elements being $\gamma_1, \ldots, \gamma_N$. $\mathbb{E}(\boldsymbol{\alpha})$ stands for the expectation of stochastic variable $\boldsymbol{\alpha}$.

2. Model formulation

Power systems are examples of complex systems in which generators and loads are dynamically interconnected. Hence, they can be seen as networked systems, where each bus is a node in the network. We assume that all the buses in the network are connected to synchronous machines (motors or generators). The nonlinear model for the active power flow in a transmission line connected between bus i and bus j is given as follows. For $i = 1, \ldots, n$, the behaviour of bus/node i can be represented by the swing equation (Kundur et al., 1994; Shames et al., 2011; Zhang et al., 2014)

$$m_i\ddot{\delta}_i(t) + d_i\dot{\delta}_i(t) - P_{mi}(t) = -\sum_{j\in N_i} P_{ij}(t), \tag{1}$$

where δ_i is the phase angle of bus i, m_i and d_i are the inertia and damping coefficients of the motors and generators, respectively, P_{mi} is the mechanical input power, P_{ij} is the active power flow from bus i to j, and N_i is the neighbourhood set of bus i where bus j and i share a transmission line or communication link.

Considering that there are no power losses nor ground admittances, and letting $V_i = |V_i|e^{\tilde{j}\delta_i}$ be the complex voltage of bus i where $\tilde{j}$ represents the imaginary unit, the active power flow between bus i and bus j, P_{ij}, is given by:

$$P_{ij}(t) = w_{ij}^{(1)} \cos(\delta_i(t) - \delta_j(t)) + w_{ij}^{(2)} \sin(\delta_i(t) - \delta_j(t)), \tag{2}$$

where $w_{ij}^{(1)} = |V_i|\,|V_j|G_{ij}$ and G_{ij} is the branch conductance between bus i and bus j; and $w_{ij}^{(2)} = |V_i|\,|V_j|B_{ij}$ and B_{ij} is the branch susceptance between bus i and bus j.

If we let $\xi_i(t) = \delta_i(t)$ and $\zeta_i(t) = \dot{\delta}_i(t)$, each bus can be assumed to have double integrator dynamics. The dynamics of bus i can thus be written:

$$\dot{\xi}_i(t) = \zeta_i(t), \tag{3}$$

$$\dot{\zeta}_i(t) = u_i(t) + v_i(t), \tag{4}$$

where ξ_i, ζ_i are scalar states, $v_i(t)$ is a known scalar external input, and u_i is the power flow

$$v_i(t) = \frac{P_{mi}(t)}{m_i} \tag{5}$$

$$\begin{aligned} u_i(t) \;=\; & -\frac{d_i}{m_i}\zeta_i(t) - \frac{1}{m_i}\sum_{j\in N_i}[w_{ij}^{(1)} \cos(\xi_i(t) - \xi_j(t)) \\ & + w_{ij}^{(2)} \sin(\xi_i(t) - \xi_j(t))]. \end{aligned} \tag{6}$$

The variables ξ_i and ζ_i can be interpreted as phase and frequency in the context of power networks.

In Shames et al. (2011), the $\cos(\cdot)$ terms are neglected (no branch conductance between buses) and it is assumed that phase angles are close to each other. The dynamics in (1) are then linearised to yield

$$m_i\ddot{\delta}_i(t) + d_i\dot{\delta}_i(t) - P_{mi}(t) = -\sum_{j\in N_i} w_{ij}^{(2)}(\delta_i(t) - \delta_j(t)). \tag{7}$$

Each bus i is assumed to have double integrator dynamics as described in (3) and (4). $u_i(t)$ in (6) becomes a linear equation

$$u_i(t) = -\frac{d_i}{m_i}\xi_i(t) - \frac{1}{m_i}\sum_{j\in N_i} w_{ij}^{(2)}(\xi_i(t) - \xi_j(t)). \tag{8}$$

For the linearised system (8), a bus k is faulty if for some functions $f_{\xi k}(t)$ and $f_{\zeta k}(t)$ not identical to zero either $\dot{\xi}_i(t) = \zeta_i(t) + f_{\xi k}(t)$, or $\dot{\zeta}_i(t) = u_i(t) + v_i(t) + f_{\zeta k}(t)$. The functions $f_{\xi k}(t)$ and $f_{\zeta k}(t)$ are referred to as fault signals. Model-based or observer-based fault