Automatica 55 (2015) 95–107

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Model order reduction of random parameter-dependent linear systems ☆

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ARTICLE INFO

Article history:
Received 8 May 2013
Received in revised form
4 February 2015
Accepted 4 February 2015
Available online 22 March 2015

Keywords:
Model reduction
Uncertain systems
Generalized polynomial chaos
Random parameters
Balanced transformation
Controllability and observability Gramians
Hankel singular values

ABSTRACT

This paper proposes a new method to perform model reduction of linear time invariant (LTI) systems where parameters are random variables governed by probabilistic laws. It combines the well-known truncation balanced realization (TBR) technique together with the generalized polynomial chaos (GPC) formalism, a powerful tool for uncertainty propagation. GPC formalism is used to represent and compute a random parameter-dependent balancing transformation (RPD-BT) which puts the random LTI system in a balanced form almost surely within the probabilistic range of the uncertain parameters. Model reduction is then performed by truncating almost surely weakly controllable and observable states, yielding a random parameter dependent truncated balanced realization (RPD-TBR). The truncation error's moments are shown to be bounded by Hankel singular values' moments, which are also estimated using GPC formalism. As an illustrative example, the proposed method is applied to a simple mechanical model of a two-degrees of freedom mass–spring system with uncertain stiffness and damping.

1. Introduction

Model reduction of dynamical systems is an important practical and theoretical issue in dynamical system's modeling, simulation and control design, and therefore also a major issue in control theory. Different methodologies have been proposed to construct, starting from a complex model, a simpler description which retains the most important properties of the original one.

For systems without uncertainties, there exist numerous methods for model reduction, such as the Truncated Balanced Realization (TBR) (Fujimoto & Tsubakino, 2008; Hahn & Edgar, 2002; Moore, 1981; Scherpen, 1993; Wood, Goddard, & Glover, 1996), Krylov Subspace Methods (also known as Moment Matching, Antoulas, 2001, 2005; Grimme, Sorensen, & Van Dooren, 1995), Modal Reduction Methods (Davison, 1968; Varga, 1995), Singular Perturbations based methods (Djennoune & Bettayeb, 2003; Kokotovic & Sanutti, 1968) or Proper Orthogonal Decomposition (POD) methods (Berkooz, Holmes, & Lumley, 1993). However, when one or several system parameters are uncertain, model reduction becomes a lot more complicated, the main difficulty being to preserve the effects of parameter variations on the system's behavior.

Methods which have been proposed to perform model reduction for uncertain models are for the most part extensions of deterministic techniques. For example, the balanced truncation method, in which state variables are ordered according to their degrees of controllability and observability (measured by Hankel singular values), has been extended to systems with structured uncertainty modeled by linear fractional transformation (Beck, Doyle, & Glover, 1996). This extension relies on a state transformation which renders the solutions of appropriate linear matrix inequalities (LMIs) equal and diagonal under some constraints on minimal eigenvalues. Truncation of the states having small singular values is then performed to obtain a reduced order model. The method requires the system to be represented as a linear fractional transformation (LFT). This condition is restrictive since the LFT representation is not easy to establish, particularly for high dimension systems. Sun and Hahn have extended balancing and proper orthogonal decomposition (POD) techniques to systems with uncertain parameters

☆ This work has been supported by French Agence Nationale de la Recherche (ANR) through project CHAPERSOA ANR-09-Blan-0162-01. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Fabrizio Dabbene under the direction of Editor Richard Middleton.

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http://dx.doi.org/10.1016/j.automatica.2015.02.027
0005-1098/

(Sun & Hahn, 2006). An important advantage of their method is that it preserves the influence of uncertain parameters by including parametric uncertainty into the procedure used for computing the state transformation in the balancing and POD techniques. The idea is to lump together the inputs and the uncertain parameters in the same vector of extended inputs. Empirical Gramians for balancing and the correlation matrix for POD are then computed for the system with the new input vector.

Another technique which has been extended to LTI systems with uncertain parameters is the moment matching approach. Weile and co-authors considered the case where the state matrix depends linearly on one parameter (Weile, Michielssen, Grimme, & Gallivan, 1999). They calculate a projection matrix matching the first moments of the transfer function $G(s, p)$ to the uncertain parameter p. This result extends the procedure which in the deterministic case enables to derive a projection matrix matching these first moments to the Laplace variable s. A generalization of this result to LTI systems with multiple uncertain parameters is given in Daniel, Siong, Lee, and White (2004). However, these methods require an affine dependence of the state matrix to the system parameters. Sufficiently smooth nonlinearities are also allowed.

Trofino and Coutinho have used the LMI approach to solve a robust order reduction problem (Trofino & Coutinho, 2004). Their method is based on the solution of an LMI optimization problem, in which an upper bound on the H_2 or H_∞ norm of the approximation error is minimized. A major drawback of LMIs based techniques for model reduction is their infeasibility even for medium size problems. Otherwise and for models with interval description of parameters uncertainty, a combination of the Routh–Padé approximation method with interval arithmetic can be considered to perform model reduction (Bandyopadhyay, Upadhye, & Ismail, 1997). The main drawback of this technique is the possible instability of the resulting reduced models. In this context, other studies have proposed algorithms to reduce the likelihood of losing stability (Dolgin & Zeheb, 2003; Wang, Li, & Wang, 2012).

More recently, Panzer and co-authors have proposed a novel methodology for model reduction of parameter-dependent linear dynamic systems (Panzer, 2010). The main idea is first to calculate reduced order models for several values of parameters by using suitable projection matrices. The reduced parametric model is then generated by interpolating the state-space representations of those reduced-order models. A similar idea was developed in Amsallem and Farhat (2011). In a previous work, Baur and Benner have combined the balanced truncation method with interpolation algorithms to generate parametric reduced order models (Baur & Benner, 2009). The key idea in this procedure is to compute local reduced order models at several points in the parameters space and then to interpolate them. Key steps in the algorithm are the choice of points in the parameter space where local models are to be calculated and reduced, then the choice of weighting functions. The first problem is dealt with a sparse grid technique, while weighting functions are chosen so as to minimize the interpolation error. The critical issue with Panzer and Amsallem's methods is the choice of an error estimation criterion encapsulating the quality of reduced order models. Baur and Benner's method offers a rigorous error bound for the reduced model within the parameter intervals; this allows computing an error estimate. On the other hand, no state-space realization exists for this method. Other methods have also been developed for particular classes of systems such as polytopic uncertain linear systems (Fen, 1996) and discrete linear systems described by polygons (Dolgin & Zeheb, 2004, 2005).

All the methods discussed so far did not take into account the statistics of the parameter uncertainty. Yet in many cases of practical interest, some information on those distributions can be derived from a combination of modeling priors and experimental/simulation data in the form of probability distributions. This additional information on the model's uncertain parameters can be exploited to develop more efficient model reduction methods. This is the main goal of the present paper, which is devoted to model order reduction of random parameter-dependent (RPD) LTI systems. Our strategy is to feed this parameter probability distribution into the reduction algorithm using the polynomial chaos (PC) formalism. This theory, initially suggested by Wiener (1938), has been pioneered by Ghanem and Spanos (1991) and subsequently extended into the so-called generalized polynomial chaos (GPC) formalism (Xiu & Karniadakis, 2003). It enables to express any second order stochastic process into a series of weighted polynomials orthogonalized with respect to an appropriate probability measure. The tricky part of GPC is the calculation of the 'stochastic modes', i.e. the weighting coefficients in the series, for which so-called intrusive or non-intrusive schemes have been developed Babuska, Tempone, and Zouraris (2004), Babuska, Nobile, and Tempone (2007) and Crestaux, Le Maitre, and Martinez (2009). In recent years, this powerful and versatile tool has been applied to a wide range of control problems. These include stability analysis and prediction of dynamic behaviors of uncertain linear and nonlinear systems (Fisher & Bhattacharya, 2009; Nechak, Berger, & Aubry, 2011, 2012a, 2013), sensitivity analysis (Crestaux et al., 2009; Sudret, 2007), parameter estimation and state observer synthesis (Blanchard, Sandu, & Sandu, 2010; Li & Xiu, 2009; Smith, Monti, & Ponci, 2007) or controller design (Fisher & Bhattacharya, 2009; Hover & Triantafyllou, 2006).

The goal of the present paper which extends preliminary results in Nechak, Raynaud, and Kulcsar (2013) is to combine the GPC formalism with the TBR method to propose a random parameter-dependent model-reduction technique. The final output will be a random parameter dependent truncated balanced realization (RPD-TBR). Applying Monte Carlo (MC) simulation techniques to this reduced-order stochastic model then enables to approximate the statistical moments of various characteristics of the original uncertain systems (e.g. settling time, overshoot or final value, singular values' degrees of controllability/observability, H_2/H_∞ norm). A similar approach was proposed in a recent contribution to perform model reduction of finite-element models of electromagnetic devices exhibiting statistical variability in their parameters (Sumant, Wu, & Cangellaris, 2012). In this approach, the reduced order system's matrices were represented using polynomial chaos expansions, which in turn were computed from a set of reduced order finite element models obtained for specific values of the uncertain parameters (taken from a multidimensional sparse grid according to the Smolyak algorithm). As a consequence, the model reduction procedure needed to be applied to a set of samples of the original uncertain full-order state-space representation. This is clearly a potentially serious drawback, especially when the original system exhibits large dimensions.

The approach to GPC-based model reduction proposed in the present paper – which can be applied to any almost-surely stable RPD continuous-time LTI system in standard state-space form depending on uncertain parameters described as random variables with known probability density function – avoids the need to perform model reduction on a sample set of the original system. The key idea is to apply the GPC formalism to directly compute a RPD transformation which puts the original stochastic model almost surely in balanced form within the full range of the system parameters' distribution. The final RPD-TBR is then generated by deleting states that are almost-surely weakly controllable and observable.

Controllability and observability degrees of state variables are determined by computing RPD-Hankel singular values. More generally, the original system's singular values can also be fully characterized in a probabilistic way (mean value, standard deviation and density function) by using the GPC approach instead of possibly prohibitive MC simulations involving repeated Lyapunov-type