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Brief paper

Observers for invariant systems on Lie groups with biased input measurements and homogeneous outputs[☆]

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ABSTRACT

This paper provides a new observer design methodology for invariant systems whose state evolves on a Lie group with outputs in a collection of related homogeneous spaces and where the measurement of system input is corrupted by an unknown constant bias. The key contribution of the paper is to study the combined state and input bias estimation problem in the general setting of Lie groups, a question for which only case studies of specific Lie groups are currently available. We show that any candidate observer (with the same state space dimension as the observed system) results in non-autonomous error dynamics, except in the trivial case where the Lie-group is Abelian. This precludes the application of the standard non-linear observer design methodologies available in the literature and leads us to propose a new design methodology based on employing invariant cost functions and general gain mappings. We provide a rigorous and general stability analysis for the case where the underlying Lie group allows a faithful matrix representation. We demonstrate our theory in the example of rigid body pose estimation and show that the proposed approach unifies two competing pose observers published in prior literature.

1. Introduction

The study of dynamical systems on Lie groups has been an active research area for the past decade. Work in this area is motivated by applications in analytical mechanics, robotics and geometric control for mechanical systems (Agrachev & Sachkov, 2004; Bloch, 2003; Bullo, 2005; Jurdjevic, 1997). Many mechanical systems carry a natural symmetry or invariance structure expressed as invariance properties of their dynamical models under transformation by a symmetry group. For totally symmetric kinematic systems, the system can be lifted to an invariant system on the symmetry group (Mahony, Trumpf, & Hamel, 2013). In most practical situations, obtaining a reliable measurement of the internal states of such physical systems directly is not possible and it is necessary to use a state observer.

Systematic observer design methodologies for invariant systems on Lie groups have been proposed that lead to strong stability and robustness properties. Specifically, Bonnabel, Martin, and Rouchon (2008a,b, 2009) consider observers which consist of a copy of the system and a correction term, along with a constructive method to find suitable symmetry-preserving correction terms. The construction utilizes the invariance of the system and the moving frame method, leading to local convergence properties of the observers. The authors propose methods in Lageman, Trumpf, and Mahony (2009, 2010, 2008) to achieve almost globally convergent observers. A key aspect of the design approach proposed in Lageman et al. (2009, 2010, 2008) is the use of the invariance properties of the system to ensure that the error dynamics are globally defined and are autonomous. This leads to a straight forward stability analysis and excellent performance in practice. More recent extensions to early work in this area was the consideration of output measurements where a partial state measurement is generated by an action of the Lie group on a homogeneous output space (Bonnabel et al., 2008a,b, 2009; Khosravian, Trumpf, Mahony, & Lageman, 2013; Lageman et al., 2009, 2008; Mahony et al., 2013). Design methodologies exploiting symmetries and invariance of the system can be applied to many real world scenarios such as attitude estimator design on the Lie group $\mathrm{SO}(3)$ (Bonnabel et al.,

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2009; Brás, Cunha, Vasconcelos, Silvestre, & Oliveira, 2011; Grip, Fossen, Johansen, & Saberi, 2012; Khosravian & Namvar, 2010; Mahony, Hamel, & Pflimlin, 2008; Vasconcelos, Silvestre, & Oliveira, 2008), pose estimation on the Lie group $\mathrm{SE}(3)$ (Baldwin, Mahony, Trumpf, Hamel, & Cheviron, 2007; Hua, Zamani, Trumpf, Mahony, & Hamel, 2011; Rehbinder & Ghosh, 2003; Vasconcelos, Cunha, Silvestre, & Oliveira, 2010), homography estimation on the Lie group $\mathrm{SL}(3)$ (Hamel, Mahony, Trumpf, Morin, & Hua, 2011), and motion estimation of chained systems on nilpotent Lie groups (Leonard & Krishnaprasad, 1995) (e.g. front-wheel drive cars or kinematic cars with k trailers).

All asymptotically stable observer designs for kinematic systems on Lie groups depend on a measurement of system input. In practice, measurements of system input are often corrupted by an unknown bias that must be estimated and compensated to achieve good observer error performance. The specific cases of attitude estimation on $\mathrm{SO}(3)$ and pose estimation on $\mathrm{SE}(3)$ have been studied independently, and methods have been proposed for the concurrent estimation of state and input measurement bias (Mahony et al., 2008; Vasconcelos et al., 2010, 2008). These methods strongly depend on particular properties of the specific Lie groups $\mathrm{SO}(3)$ or $\mathrm{SE}(3)$ and do not directly generalize to general Lie groups. To the authors' knowledge, there is no existing work on combined state and input bias estimation for general classes of invariant systems.

In this paper, we tackle the problem of observer design for general invariant systems on Lie groups with homogeneous outputs when the measurement of system input is corrupted by an unknown constant bias. The observer is required to be implementable based on available sensor measurements; the system input in the Lie algebra, corrupted by an unknown bias, along with a collection of partial state measurements (i.e. outputs) that ensure observability of the state. For bias free input measurements, it is always possible to obtain autonomous dynamics for the standard error (Lageman et al., 2009, 2010, 2008), and previous observer design methodologies for systems on Lie groups rely on the autonomy of the resulting error dynamics. However, for concurrent state and input measurement bias estimation, we show that *any* implementable candidate observer (with the same state space dimension as the observed system) yields non-autonomous error dynamics unless the Lie group is Abelian (Theorem 4.1). This result explains why the previous general observer design methodologies for the bias-free case do not apply and why the special cases considered in prior works (Hua et al., 2011; Vasconcelos et al., 2010) do not naturally lead to a general theory.

We go on to show that, despite the nonlinear and non-autonomous nature of the error dynamics, there is a natural choice of observer for which we can prove exponential stability of the error dynamics (Theorems 5.1 and 5.2). The approach taken employs a general gain mapping applied to the differential of a cost function rather than the more restrictive gradient-like innovations used in prior work (Khosravian et al., 2013; Lageman et al., 2009, 2010, 2008). We also propose a systematic method for construction of invariant cost functions based on lifting costs defined on the homogeneous output spaces (Proposition 6.1). To demonstrate the generality of the proposed approach we consider the problem of rigid body pose estimation using landmark measurements when the measurements of linear and angular velocity are corrupted by constant unknown biases. We show that for specific choices of gain mappings the resulting observer specializes to either the gradient-like observer of Hua et al. (2011) or the non-gradient pose estimator proposed in Vasconcelos et al. (2010), unifying these two state-of-the-art application papers in a single framework that applies to any invariant kinematic system on a Lie-group. Stability of estimation error is proved for the case where the Lie group allows a faithful matrix representation.

The paper is organized as follows. After briefly clarifying our notation in Section 2, we formulate the problem in Section 3. A standard estimation error is defined and autonomy of the resulting error dynamics is investigated in Section 4. We introduce the proposed observer in Section 5 and investigate the stability of observer error dynamics. Section 6 is devoted to the systematic construction of invariant cost functions. A detailed example in Section 7 and brief conclusions in Section 8 complete the paper. A preliminary version of this work was presented at the CDC 2013 (Khosravian et al., 2013).

2. Notations and definitions

Let G be a finite-dimensional real connected Lie group with associated Lie algebra $\mathfrak{g}$. Denote the identity element of G by I. Left (resp. right) multiplication of $X \in G$ by $S \in G$ is denoted by $L_S X = SX$ (resp. $R_S X = XS$). The Lie algebra $\mathfrak{g}$ can be identified with the tangent space at the identity element of the Lie group, i.e. $\mathfrak{g} \cong T_I G$. For any $u \in \mathfrak{g}$, one can obtain a tangent vector at $S \in G$ by left (resp. right) translation of u denoted by $S[u] := T_I L_S[u] \in T_S G$ (resp. $[u]S := T_I R_S[u] \in T_S G$). The element inside the brackets $[\cdot]$ denotes the vector on which a linear mapping (here the tangent map $T_I L_S \colon \mathfrak{g} \to T_S G$ or $T_I R_S \colon \mathfrak{g} \to T_S G$) acts. The adjoint map at the point $S \in G$ is denoted by $\mathrm{Ad}_S \colon \mathfrak{g} \to \mathfrak{g}$ and is defined by $\mathrm{Ad}_S[u] := S[u]S^{-1} = T_S R_{S^{-1}}[T_I L_S[u]] = T_S R_{S^{-1}} \circ T_I L_S[u]$ where $\circ$ denotes the composition of two maps. For a finite-dimensional vector space V, we denote its corresponding dual and bidual vector spaces by V^* and V^{**} respectively. A linear map $F \colon V^* \to V$ is called positive definite if $v^*[F[v^*]] > 0$ for all $0 \neq v^* \in V^*$. The dual of F is denoted by $F^* \colon V^* \to V^{**}$ and is defined by $F^*[v^*] = v^* \circ F$. The linear map F is called symmetric (resp. anti-symmetric) if $v^*[F[w^*]] = w^*[F[v^*]]$ (resp. $v^*[F[w^*]] = -w^*[F[v^*]]$) for all $v^*, w^* \in V^*$, and it is called symmetric positive definite if it is symmetric and positive definite. We can extend the above notion of symmetry and positiveness to linear maps $H \colon W \to W^*$ as well. Defining $V := W^*$, H is called positive definite if $H^* \colon V^* \to V$ is positive definite and it is called symmetric if H^* is symmetric. Positive definite cost functions on manifolds are also used in the paper and should not be mistaken with positive definite linear maps.

3. Problem formulation

We consider a class of left invariant systems on G given by

$$\dot{X}(t) = X(t)u(t), \qquad X(t_0) = X_0, \tag{1}$$

where $u \in \mathfrak{g}$ is the system input and $X \in G$ is the state. Although the ideas presented in this paper are based on the above left invariant dynamics, they can easily be modified for right invariant systems as was done for instance in Lageman et al. (2010). We assume that $u \colon \mathbb{R}^+ \to \mathfrak{g}$ is continuous and hence a unique solution for (1) exists for all $t \geq t_0$ (Jurdjevic & Sussmann, 1972). In most kinematic mechanical systems, u models the velocity of physical objects. Hence, it is reasonable to assume that u is bounded and continuous.

Let M_i, $i = 1, \ldots, n$ denote a collection of n homogeneous spaces of G, termed *output spaces*. Denote the outputs of system (1) by $y_i \in M_i$. Suppose each output provides a partial measurement of X via

$$y_i = h_i(X, \mathring{y}_i) \tag{2}$$

where $\mathring{y}_i \in M_i$ is the constant (with respect to time) reference output associated with y_i and h_i is a right action of G on M_i, i.e. $h_i(I, y_i) = y_i$ and $h_i(XS, y_i) = h_i(S, h_i(X, y_i))$ for all $y_i \in M_i$ and all $X, S \in G$. To simplify the notation, we define the combined output $y := (y_1, \ldots, y_n)$, the combined reference output $\mathring{y} := (\mathring{y}_1, \ldots, \mathring{y}_n)$, and the combined right action $h(X, \mathring{y}) :=$