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Brief paper

Observer design for a class of uncertain nonlinear systems with sampled outputs—Application to the estimation of kinetic rates in bioreactors[☆]

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ABSTRACT

A continuous–discrete time observer is proposed for a class of uncertain nonlinear systems where the output is available only at non uniformly spaced sampling instants. The underlying correction term depends on the output observation error and is updated in a mixed continuous-discrete fashion. The proposed observer is first introduced under a set of differential equations with instantaneous state impulses corresponding to the measured samples and their estimates. Two features of the proposed observer are worth to be pointed out. The first one consists in the simplicity of its calibration while the second one lies in its comprehensive convergence analysis. More specifically, it is shown that in the case of noise-free sampled outputs, the observation error lies in a ball centered at the origin and its radius is proportional to the bounds of the uncertainties and the sampling partition diameter. Moreover, in the free uncertainties case, the exponential convergence to zero of the observation error is established under a well-defined condition on the maximum value of the sampling partition diameter. The ability of the proposed observer to perform a suitable estimation of the reactions rates in biochemical reactors is highlighted through a simulation study dealing with an ethanolic fermentation.

1. Introduction

Although a considerable research activity has been devoted to the observer design for nonlinear systems over the last decades, the available contributions deal mainly with the continuous-time measurements case (Andrieu & Praly, 2006; Farza, M'Saad, Triki, & Maatoug, 2011; Gauthier & Kupka, 2001; Kazantzis & Kravaris, 1998; Krener & Isidori, 1983; Rajamani, 1998). In order to handle the non continuous availability of the output measurements, many works have focused on the redesign of the continuous time state observers that have been proposed for the underlying systems. An early contribution was made using a high gain observer for a class of nonlinear systems that are observable for any input (Deza, Busvelle, Gauthier, & Rakotopara, 1992). The design was firstly carried out assuming continuous-time output measurements before being appropriately modified to deal with the case where these measurements are only available at sampling instants. Based on the aforementioned contribution, many other observers have been proposed for specific classes of continuous time systems with sampled output measurements (Hammouri, Nadri, & Mota, 2006; Nadri, Hammouri, & Grajales, 2013). In all these contributions, the continuous–discrete time observer operates as follows: a dynamical system, which is similar to the underlying system, is used to provide a state prediction over the sampling intervals. At the sampling instants, the output measurements are used to update the state prediction provided by the dynamical system. An output predictor approach has been proposed in Karafyllis and Kravaris (2009) to cope with the non-availability of the output measurements between the sampling instants. It consists in a continuous time observer involving a suitable predictor of the output over the

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sampling intervals. More specifically, the output prediction is provided by the solution of an ordinary differential equation between two successive sampling instants with the value of the measured output sample as initial condition. The underlying observer is a hybrid system which is able to recover the continuous time observer properties for relatively fast sampling. Another approach was described in Raff, Kögel, and Allgöwer (2008) where the authors proposed an impulsive continuous–discrete time observer for a class of uniformly observable systems with sampled outputs. The correction term of the proposed observer is the product of a constant gain by the difference between the estimated and measured values of the last output sample. The determination of the observer gain is carried out through the resolution of an appropriate LMI.

In this paper, our objective consists in synthesizing a continuous–discrete time observer for a class of uncertain nonlinear systems where the output measurements are available only at non uniformly spaced sampling instants. In the absence of uncertainties, the considered class of nonlinear systems is observable for any input and has been considered in Farza, M'Saad, and Rossignol (2004) for observer design purposes in the case where the output measurements are available in a continuous manner. The proposed continuous–discrete time observer is issued from a redesigned version of the high gain continuous time observer proposed in Farza et al. (2004). This observer shares the impulsive nature of the observer given in Raff et al. (2008) up to an adequate modification of the observer gain matrix. Indeed, unlike in Raff et al. (2008), the gain of the proposed observer does not necessitate the resolution of any system and its expression is given. This gain is time-varying and depends on the sampling periods. There are two main features of the proposed continuous–discrete time observer that are worth to be emphasized with respect to the existing ones. The first one concerns the ease with which the observer gain is updated at sampling instants together with the simplicity of its implementation. The second feature is related to the convergence analysis simplicity and its ability to provide precise expressions of the upper bounds of the sampling partition diameter as well as the rate of the observation error convergence. More specifically, it is shown that the observation error lies in a ball centered at the origin with a radius proportional to the magnitude of the bounds of the uncertainties, the noise measurements and the maximum sampling partition diameter. Moreover, it is shown that in the noise-free outputs case, the ultimate bound of the observation error can be made arbitrarily small, as in the continuous-time output case, when the maximum sampling partition diameter tends to zero. A particular emphasis is put on the fact that the observation error converges exponentially to the origin in the absence of uncertainties and noise measurements. Of particular interest, the expression of the underlying decay rate is given.

The paper is organized as follows. In the next section, the class of systems under consideration and the notations employed throughout this paper are introduced together with concise convergence results on the continuous time high gain observer on which the proposed continuous–discrete time observer is based upon. The output noise measurements are taken into account in the provided convergence analysis. A technical lemma with its proof are also given in this section and this lemma is used to establish the main result of the paper. Section 3 is devoted to the main contribution of the paper, namely the design of the continuous–discrete time observer. The fundamental result is given with a comprehensive proof thanks to the technical lemma derived in Section 2. In Section 4, the effectiveness of the proposed observer is highlighted via simulation results involving the estimation of the reactions rates in a bioreactor involving an ethanolic fermentation. Finally, some concluding remarks are given in Section 5.

Throughout the paper, I_p and 0_p will denote the p-dimensional identity and zero matrices respectively and $\|\cdot\|$ denotes the euclidean norm; λ_M (resp. λ_m) is the maximum (resp. minimum) eigenvalue of a certain Symmetric Positive Definite (SPD) matrix P and $\sigma = \sqrt{\lambda_M/\lambda_m}$ is its conditioning number.

2. Problem statement and preliminaries

Consider the class of multivariable nonlinear systems that are diffeomorphic to the following bloc triangular form:

$$\begin{cases} \dot{x}(t) = Ax(t) + \varphi(u(t), x(t)) + B\varepsilon(t) \\ y(t_k) = Cx(t_k) + w(t_k) = x^1(t_k) + w(t_k) \end{cases} \tag{1}$$

with

$$x = \begin{pmatrix} x^1 \\ \vdots \\ x^{q-1} \\ x^q \end{pmatrix} \in \mathbb{R}^n, \quad \varphi(u,x) = \begin{pmatrix} \varphi^1(u,x^1) \\ \varphi^2(u,x^1,x^2) \\ \vdots \\ \varphi^{q-1}(u,x^1,\dots,x^{q-1}) \\ \varphi^q(u,x) \end{pmatrix}$$

$$A = \begin{pmatrix} 0_{(q-1)p,p} & I_{(q-1)p} \\ 0_{p,p} & 0_{p,(q-1)p} \end{pmatrix}, \qquad B = \begin{pmatrix} I_p & 0_p & \cdots & 0_p \end{pmatrix}^T,$$

$$C = \begin{pmatrix} I_p & 0_p & \cdots & 0_p \end{pmatrix} \tag{2}$$

where $x^i \in \mathbb{R}^p$ for $i \in [1,\ q]$ are the state variables blocks, $u(t) \in U$ a compact subset of $\mathbb{R}^m$ denotes the system input and $y \in \mathbb{R}^p$ denotes the system output which is available only at the sampling instants that satisfy $0 \le t_0 < \cdots < t_k < t_{k+1} < \cdots$ with time-varying sampling intervals $\tau_k = t_{k+1} - t_k$ and $\lim_{k\to\infty} t_k = +\infty$; $w(t)$ is the output noise and $\varepsilon : \mathbb{R}^+ \mapsto \mathbb{R}^p$ is an unknown function describing the system uncertainties and may depend on the state, the input and uncertain parameters.

As it is mentioned in the introduction, our main objective is to design a continuous–discrete time observer providing a continuous time estimation of the full state of system (1) by using the output measurements that are available only at the sampling instants. Such a design will be carried out under the following assumptions:

A1. The state $x(t)$ is bounded, i.e. there exists a compact set $\Omega \in \mathbb{R}^n$ such that $\forall t \ge 0,\ x(t) \in \Omega$.

A2. The functions φ^i for $i \in [1,\ q]$ are Lipschitz with respect to x uniformly in u, i.e.

$$\forall \rho > 0;\ \exists L > 0;\ \forall u \text{ s.t. } \|u\| \le \rho;$$
$$\forall (x,\bar{x}) \in \Omega \times \Omega : \|\varphi^i(u,x) - \varphi^i(u,\bar{x})\| \le L\|x - \bar{x}\|. \tag{3}$$

A3. The unknown function ε is essentially bounded, i.e.

$$\exists \delta_\varepsilon > 0; \quad \sup_{t\ge 0} \text{Ess}\|\varepsilon(t)\| \le \delta_\varepsilon.$$

A4. The noise signal w is essentially bounded, i.e.

$$\exists \delta_w > 0; \quad \sup_{t\ge 0} \text{Ess}\|w(t)\| \le \delta_w.$$

Furthermore, one naturally assumes that the time intervals τ_k's are bounded away from zero by τ_m and are upperly bounded by the upper bound of the sampling partition diameter τ_M, i.e.

$$0 < \tau_m \le \tau_k = t_{k+1} - t_k \le \tau_M, \quad \forall k \ge 0. \tag{4}$$

There are two remarks that are worth to be pointed out. Firstly, the class of systems described by (1) may seem very restrictive since it assumes a non prime dimension ($n = pq$) and the state blocks x^k have the same dimension p. This is not the case since it is shown in Hammouri and Farza (2003) that in the uncertainties-free case, system (1) is a normal form which characterizes a class of uniformly observable nonlinear systems that can be put under this form via an injective map (see e.g. Farza, M'Saad, Maatoug, & Kamoun, 2008 and Hammouri & Farza, 2003 for more details). Secondly, since the system state trajectory lies in a bounded set Ω, one can extend the nonlinearities $\varphi(u,x)$ in such a way that this extension becomes globally Lipschitz on the entire state space