Automatica 55 (2015) 197–203

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Flocking with connectivity preservation of multiple double integrator systems subject to external disturbances by a distributed control law ☆

Yi Dong, Jie Huang [1]

Department of Mechanical and Automation Engineering, The Chinese University of Hong Kong, Shatin, N.T., Hong Kong

ARTICLE INFO

Article history:
Received 2 December 2013
Received in revised form
13 July 2014
Accepted 25 February 2015

Keywords:
Multi-agent systems
Connectivity maintenance
Flocking
Disturbance rejection

ABSTRACT

This paper studies the flocking with connectivity preservation problem of multiple double integrator systems subject to a class of external disturbances. We solve the problem by a distributed dynamic position feedback control law under the standard assumptions. Our approach is a combination of the potential function technique and observer design. Our result also includes some existing results as special cases.

1. Introduction

Since Reynolds proposed three heuristic rules: separation, alignment, and cohesion for flocking model in Reynolds (1987), the flocking problem has attracted some attention in control community. It was shown that the connectivity of the communication graph is the key for the flocking behavior in Olfati-Saber (2006) and Su, Wang, and Lin (2009). Different assumptions on the communication graph have been proposed to guarantee flocking, for example, Tanner, Jadbabaie, and Pappas (2003) studied flocking problem under the assumption that the graph is connected for all time, while Zhang, Zhai, and Chen (2011) provided a general analysis for flocking problem under a jointly connected assumption on the communication graph.

In many real applications such as rendezvous and flocking, the graph is defined dynamically and state-dependent. It is more practical to enable a control law to preserve the connectivity of the graph instead of assuming the connectivity of the graph. Such a problem is called connectivity preservation problem. There are mainly two techniques to preserve the connectivity of the graph. The first one is to maximize the second smallest eigenvalue of Laplacian matrix associated with the graph using convex optimization and subgradient descent algorithms (Zavlanos, Egerstedt, & Pappas, 2011), while the second one is to employ potential functions to maintain the connectivity of the initially connected graph (Dimarogonas & Johansson, 2010; Ji & Egerstedt, 2007; Su, Wang, & Chen, 2009; Zavlanos, Jadbabaie, & Pappas, 2007; Zavlanos, Tanner, Jadbabaie, & Pappas, 2009). The connectivity preservation is mainly studied for multiple single or double integrator systems where the communication graph is defined by the distance of various agents. The purpose of preserving the connectivity among the agents is to ensure the position consensus among the agents. Such problem is also called rendezvous problem in some literature. If the objective of collision avoidance is also imposed, then the problem can be further called flocking. Depending whether or not a multi-agent system has a leader, the rendezvous/flocking problem can be further classified into the leaderless rendezvous/flocking problem (Ajorlou, Momeni, & Aghdam, 2010; Dimarogonas & Johansson, 2010; Ji & Egerstedt, 2007; Zavlanos et al., 2007, 2009), and leader-following rendezvous/flocking problem (Cao & Ren, 2012; Dong & Huang, 2013, 2014). In Su, Wang, Chen et al. (2009) and Su, Wang, and Chen (2010), both leaderless and leader-following rendezvous/flocking problems were studied.

In this paper, we study the leaderless flocking with connectivity preservation problem for a group of double integrator systems subject to external disturbances. Our problem is different from existing works (Su, Wang, Chen et al., 2009; Zavlanos et al., 2007, 2009)

☆ This work has been supported in part by the Research Grants Council of the Hong Kong Special Administration Region under Grant No. 412612, and in part by the National Natural Science Foundation of China under Grant No. 61004010. The material in this paper was partially presented at the 2014 IEEE Multi-Conference on Systems and Control, October 8–10, 2014, Antibes, France. This paper was recommended for publication in revised form by Associate Editor Wei Ren under the direction of Editor Frank Allgöwer.

E-mail addresses: ydong@mae.cuhk.edu.hk (Y. Dong), jhuang@mae.cuhk.edu.hk (J. Huang).

[1] Tel.: +852 39438473; fax: +852 26036002.

http://dx.doi.org/10.1016/j.automatica.2015.03.006
0005-1098/

in at least two ways. First, our model as given by (1) in the next section is subject to external disturbances generated by a class of autonomous systems called exosystem. Second, we introduce a simplified bounded potential function that not only guarantees the connectivity of the graph but also avoids the collision. It is noted that the leader-following rendezvous problem for multiple double integrator systems subject to disturbances has also been studied by ourselves in Dong and Huang (2013, 2014). However, since our system in this paper does not have a leader, we do not have to employ a distributed observer. Thus, as will be elaborated in Remark 3.3, our analysis is quite different from that in Dong and Huang (2013, 2014), and our control law is much simpler. Moreover, our control law here not only achieves consensus in velocity but also avoids collisions.

The rest of this paper is organized as follows. In Section 2, we will formulate our problem. In Section 3, we will present our main result, which will be illustrated by an example in Section 4. Finally, we will close this paper in Section 5 with some concluding remarks.

The following notation will be used throughout this paper: given the column vectors a_i, $i = 1, \ldots, s$, we define $\mathrm{col}(a_1, \ldots, a_s) = [a_1^T, \ldots, a_s^T]^T$.

2. Problem formulation

Consider a collection of double integrator systems of the following form

$$\begin{aligned} \dot{q}_i &= p_i \\ \dot{p}_i &= u_i + d_i, \quad i = 1, \ldots, N \end{aligned} \tag{1}$$

where $q_i \in \mathbb{R}^n$ denotes the position, $p_i \in \mathbb{R}^n$ denotes the velocity, $u_i \in \mathbb{R}^n$ is the input and $d_i \in \mathbb{R}^n$ is the external disturbance generated by

$$\dot{w}_i = S_i w_i, \qquad d_i = D_i w_i \tag{2}$$

where $w_i \in \mathbb{R}^{s_i}$, $S_i \in \mathbb{R}^{s_i \times s_i}$ and $D_i \in \mathbb{R}^{n \times s_i}$ are known constant matrices. Without loss of generality, we assume the pair (D_i, S_i) is detectable.

Put our system (1) in the state space form:

$$\begin{aligned} \dot{x}_i &= Ax_i + Bu_i + E_i w_i \\ y_i &= Cx_i \end{aligned} \tag{3}$$

where, for $i = 1, \ldots, N$, $x_i = \begin{bmatrix} q_i \\ p_i \end{bmatrix} \in \mathbb{R}^{2n}$, $y_i \in \mathbb{R}^n$ are the state and measurement output, respectively. $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \otimes I_n$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \otimes I_n$, $E_i = \begin{bmatrix} 0_{n \times s_i} \\ D_i \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \end{bmatrix} \otimes I_n$.

We view the system (1) as a multi-agent system of N agents with the N subsystems of (1) as N followers. With respect to the system (1), we can define a digraph[2] $\mathcal{G}(t) = (\mathcal{V}, \mathcal{E}(t))$ where $\mathcal{V} = \{1, \ldots, N\}$ with $i = 1, \ldots, N$, associated with the ith subsystem of (1), and $\mathcal{E}(t) \subseteq \mathcal{V} \times \mathcal{V}$. The set $\mathcal{V}$ is called the node set of $\mathcal{G}(t)$ and the set $\mathcal{E}(t)$ is called the edge set of $\mathcal{G}(t)$. We use $\mathcal{N}_i(t)$ to denote the neighbor set of the node i for $i = 1, \ldots, N$.

Let us now characterize the edge set $\mathcal{E}(t)$, which is in spirit similar to that in Dong and Huang (2013, 2014).

Given any $r > 0$, $\epsilon_2 \in (0, r)$, and $\epsilon_1 \in (0, r - \epsilon_2)$, for any $t \geq 0$, $\mathcal{E}(t) = \{(i, j) \mid i, j \in \mathcal{V}\}$ is defined such that

(1) $\mathcal{E}(0) = \{(i, j) \mid \epsilon_1 < \|q_i(0) - q_j(0)\| < (r - \epsilon_2), i, j = 1, \ldots, N\}$;
(2) if $\|q_i(t) - q_j(t)\| \geq r$, then $(i, j) \notin \mathcal{E}(t)$;
(3) for $i = 1, \ldots, N, j = 1, \ldots, N$, if $(i, j) \notin \mathcal{E}(t^-)$ and $\|q_i(t) - q_j(t)\| < (r - \epsilon_2)$, then $(i, j) \in \mathcal{E}(t)$.
(4) for $i = 1, \ldots, N, j = 1, \ldots, N$, if $(i, j) \in \mathcal{E}(t^-)$ and $\|q_i(t) - q_j(t)\| < r$, then $(i, j) \in \mathcal{E}(t)$.

As in Ji and Egerstedt (2007), Su, Wang, Chen et al. (2009) and Su et al. (2010), ϵ_2 is used to introduce the hysteresis effect for adding the new links, and the role of $\epsilon_1 \in (0, r - \epsilon_2)$ is the same as proposed in Wen, Duan, Su, Chen, and Yu (2012).

We will consider the dynamic control law in the following form:

$$\begin{aligned} u_i &= h_i(q_i - q_j, \zeta_i, \zeta_j), \quad i = 1, \ldots, N \\ \dot{\zeta}_i &= g_i(q_i, q_j, \zeta_i, \zeta_j), \quad j \in \mathcal{N}_i(t) \end{aligned} \tag{4}$$

where h_i, g_i are sufficiently smooth functions to be specified later, and $\zeta_i \in R^{(2n+s_i)}$ is used to estimate $\mathrm{col}(q_i, p_i, w_i)$.

Remark 2.1. If we allow the dimension of ζ_i, $i = 1, \ldots, N$, to be zero and assume the velocity information is available, then (4) reduces to the static state feedback control law. A typical static feedback control law is as follows:

$$u_i = -\sum_{j \in \mathcal{N}_i(t)} \nabla_{q_i} \psi(\|q_i - q_j\|) - \sum_{j \in \mathcal{N}_i(t)} (p_i - p_j) \tag{5}$$

where $\psi(\cdot)$ is the so-called potential function. By designing different potential functions, a control law of the form (5) can be used to solve rendezvous with connectivity preservation problem (Su et al., 2010), flocking with connectivity preservation problem (Zavlanos et al., 2007, 2011, 2009) for systems of the form (1) with the external disturbance $d_i = 0$. However, this static control law cannot handle the external disturbance d_i. Therefore, we need to introduce the dynamic control law of the form (4). Since the control law (4) depends neither on the velocity nor on the external disturbance, we call it the dynamic distributed position feedback control law.

Our problem is called flocking with connectivity preservation and is described as follows:

Definition 2.1. Given the multi-agent system composed of (1) and (2), $r > 0$, $r_0 \in (0, r)$, $\epsilon_1 \in (0, r_0)$, $\epsilon_2 \in (0, r - r_0)$, and arbitrary positive real numbers ρ_i, κ_i, $i = 1, \ldots, N$, find a distributed control law of the form (4) such that, for all initial conditions $w_i(0), q_i(0), p_i(0), \zeta_i(0)$, $i = 1, \ldots, N$, that make $\mathcal{G}(0)$ connected, and satisfy $\|q_i(0) - q_j(0)\| > \epsilon_1$, $\|p_i(0) - p_j(0)\| \leq \rho_i$, and $\|\zeta_i(0) - \mathrm{col}(q_i(0), p_i(0), w_i(0))\| \leq \kappa_i$, the closed-loop system has the following properties:

(1) $\mathcal{G}(t)$ is connected for all $t \geq 0$;
(2) $\lim_{t \to \infty}(p_i - p_j) = 0, i, j = 1, \ldots, N$;
(3) Collision can be avoided among agents, that is, $\|q_i(t) - q_j(t)\| > 0$, $i, j = 1, \ldots, N$, for all $t \geq 0$.

As in Dong and Huang (2013, 2014), the numbers ρ_i and κ_i are to define some closed balls in which the initial states of the system are allowed to stay. They are introduced so that the parameters of our control law will be independent of the initial conditions of the system.

3. Main result

The flocking problem has been considered by various potential function techniques. A bounded potential function was proposed in Wen et al. (2012) as follows.

$$\psi(s) = \frac{(s - r_0)^2 (r - s)}{s + \frac{r_0^2 (r - s)}{\kappa_1 + Q \max}} + \frac{s(s - r_0)^2}{r - s + \frac{s(r - r_0)^2}{\kappa_2 + Q \max}}, \quad 0 \leq s \leq r \tag{6}$$

where $\kappa_1 \geq 0$, $\kappa_2 \geq 0$ and $r_0 \in (\epsilon_1, r - \epsilon_2)$.

[2] See Appendix in Dong and Huang (2014) for a summary of digraph.