Automatica 55 (2015) 217–225

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Constrained predictive control synthesis for quantized systems with Markovian data loss[☆]

Yuanyuan Zou [a,1], James Lam [b], Yugang Niu [a], Dewei Li [c]

[a] Key Lab of Advanced Control and Optimization for Chemical Processed of Ministry of Education, East China University of Science & Technology, Shanghai, China

[b] Department of Mechanical Engineering, The University of Hong Kong, Hong Kong

[c] Department of Automation, Shaohai Jiao Tong University, Shanghai, China

ARTICLE INFO

Article history:
Received 25 July 2014
Received in revised form
5 January 2015
Accepted 6 March 2015

Keywords:
Constrained control
Missing data
Model predictive control
Quantization
Stochastic stability

ABSTRACT

This paper investigates the predictive control synthesis problem for constrained feedback control systems with both missing data and quantization. By introducing a missing data compensation strategy and an augmented Markov jump linear model with polytopic uncertainties, the effects of data loss and quantization on the system performance are considered simultaneously. A robust predictive control synthesis approach involving data missing and recovering probabilities is developed by minimizing an upper bound on the expected value of an infinite horizon quadratic performance objective at each sampling instant. Additional conditions to satisfy the input constraint in the presence of multiple missing data are also incorporated into the model predictive control (MPC) synthesis. Furthermore, both the recursive feasibility of the proposed MPC algorithm and the closed-loop mean-square stability are proved. Simulation results are given to illustrate the effectiveness of the proposed approach.

1. Introduction

As a popular control technique, model predictive control (MPC), has received much attention in recent years for its widespread applications in industrial processes such as chemical processing, robotics, and energy systems. The essence of MPC is to solve a finite horizon optimization control problem based on the current measurements at each sampling time; the first control action of the optimal control sequence is implemented to the plant; the problem is solved again at next sampling time based on the new measurements. In particular, MPC can incorporate the input/output constraints into the on-line optimization and achieve approximately optimal control performance. These features have drawn great research interest in the analysis and synthesis of MPC and motivated the development of MPC towards new directions (Li, Li, & Xi, 2014; Li, Xi, & Gao, 2013; Lješnjanin, Quevedo, & Nešić, 2014; Lu, Li, & Xi, 2013; Mayne, Rawlings, Rao, & Scokaert, 2000; Pin & Parisini, 2011).

With the development of computer and information technologies, communication networks play a more and more important role in large-scale or complex industrial systems, by which a tremendous amount of information is sensed, processed, and transmitted. The insertion of communication networks brings some advantages, such as low cost, reduced system wiring, and simple installation and maintenance (Baillieul & Antsaklis, 2007; Niu & Ho, 2010). However, it also yields some detrimental effects on practical feedback control systems. Owing to the limited capacities of communication network, the input signals are usually quantized before being transmitted. Moreover, in the process of transmission to the actuator, the quantized signals may be lost. Thus, the controlled systems will inevitably be subjected to the effects of quantization error and missing data, so that the conventional control theories may not work effectively in providing the expected closed-loop performance. The literature presented regarding the stability conditions and the performance analysis for networked control systems (NCSs) by taking quantization or/and

☆ This work is partially supported by an RGC grant HKU 7140/11E, National Natural Science Foundation of China (61273073, 61374107, 61374110), the Fundamental Research Funds for the Central Universities (222201314013), and the Cheung Kong Chair Professor Program, Ministry of Education, China. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Tongwen Chen under the direction of Editor Ian R. Petersen.

E-mail addresses: yyzou@ecust.edu.cn (Y. Zou), james.lam@hku.hk (J. Lam), acniuyg@ecust.edu.cn (Y. Niu), dwli@sjtu.edu.cn (D. Li).

1 Tel.: +86 21 64253581; fax: +86 21 64253325.

http://dx.doi.org/10.1016/j.automatica.2015.03.016
0005-1098/

data loss into account is rich; e.g., Fu and Xie (2006), Gao and Chen (2008), Niu and Ho (2014), Xiong and Lam (2007), Xia, Fu, and Liu (2011), You and Xie (2011) and You, Sui, and Fu (2014). In particular, a potential model-based NCS architecture was presented in Montestruque and Antsaklis (2003) to deal with data loss. The plant model was used to predict the plant state between transmission times and generate the corresponding control actions. Necessary and sufficient conditions for stability were derived by simple eigenvalue tests of a well-structured test matrix, constructed in terms of the update time and the parameters of the plant. Further, both static quantizer and dynamic quantizer were considered in this model-based architecture (Montestruque & Antsaklis, 2007).

A lot of research has recently been done on control systems with missing data or quantization in a MPC scheme (BenGaid & Çela, 2010; Ding, 2011; Li et al., 2013; Quevedo, Østergaard, & Nešić, 2011; Tang & Ding, 2013; Xue, Li, Li, & Zhu, 2010; Zou & Niu, 2013; Zou, Niu, Chen, & Jia, 2013). In BenGaid and Çela (2010), an MPC approach for the dynamic assignment of the quantization precision and update rate of control inputs was proposed. In Xue et al. (2010) and Zou et al. (2013), robust MPC strategies were presented to stabilize the quantized feedback control systems. For deterministic bounded missing data process which takes values in a finite set arbitrarily, Refs. (Ding, 2011; Tang & Ding, 2013) considered predictive control synthesis approaches, by which the input and state constraints can be handled in a systematic manner. In many situations, however, data loss can be more accurately described as random processes. For example, Quevedo et al. (2011) proposed a packetized predictive control design approach for networks affected by data loss described via a Bernoulli process. An infinite horizon predictive control synthesis method was investigated in Zou and Niu (2013) for systems with measurements possibly missing from sensor to controller according to a Bernoulli process and the constraints under expectation are considered. To further capture the correlation of fading communication channel gains and network congestion levels, more generally, a Markov process was adopted to model the data loss (Huang & Dey, 2007; Xiong & Lam, 2007; You & Xie, 2011). Despite the widespread use of the stochastic features in the network, to the authors' best knowledge, few works on predictive control synthesis approach, in which closed-loop stability can be guaranteed whenever the optimization problem is feasible, have been investigated for constrained control systems with both quantization and Markovian missing data. In particular, when it comes to designing MPC synthesis under explicit physical constraints, how to apply the stochastic characteristics of data loss to achieve the desired performance is still an open problem.

In this paper, we focus on the constrained control systems where controller output data is transmitted over a communication network. Both Markovian data loss and logarithmic quantizers are considered in the network. A novel MPC synthesis problem in a *stochastic* framework is investigated. The main contributions of this paper are as follows: (i) an estimation method to compensate the multiple missing data and a quantization matrix to model the quantized system are presented, then the closed-loop system can be modeled as a Markov jump linear system with polytopic uncertainties by introducing an appropriate augmented state; (ii) a robust probability-based predictive control synthesis approach is proposed, in which the additional conditions to satisfy the system constraints in spite of multiple missing data and quantization errors are incorporated. Meanwhile, the closed-loop systems with the proposed approach is proven to be stochastically stable.

The organization of this paper is as follows. Section 2 introduces the modeling of constrained control systems with quantization and missing data and the MPC problem formulation. In Section 3, the constrained MPC optimization problem is presented for quantized control system involving data loss, and the stability of the resulting closed-loop systems is proved. Section 4 provides two examples to illustrate the effectiveness of the proposed approach. Finally, some conclusions are drawn in Section 5.

Notation. Throughout this paper, I is the identity matrix with appropriate dimensions, and the notation $P > 0(\geq 0)$ means that P is real symmetric and positive definite (semi-definite). In block symmetric matrices, we use an asterisk $(*)$ to represent a term that is induced by symmetry. diag$\{\cdots\}$ stands for a block-diagonal matrix and Co$\{\cdots\}$ denotes the convex hull, that is, if $\Omega = \mathrm{Co}\{A_1, A_2, \ldots, A_L\} = \left\{\sum_{i=1}^{L} a_i A_i \mid \sum_{i=1}^{L} a_i = 1,\ a_i \geq 0\right\}$. $\mathcal{E}$ is the expectation operator and $\mathcal{E}_x$ denotes conditional expectation with respect to x. The notation $\psi(k+j \mid k)$ (where ψ may represent u, x or z) denotes the prediction of ψ at future time $k+j$ based on the current state $x(k)$ and the control input $u_d(k-1)$.

2. Problem formulation

Consider the following system:

$$x(k+1) = Ax(k) + Bu(k), \tag{1}$$

where $u(k) \in \mathbf{R}^m$ is the control input, and $x(k) \in \mathbf{R}^n$ is the state vector; A and B are constant matrices of appropriate dimensions. In industrial processes, the physical limitations inherited in control equipment invariably impose hard constraints on the manipulated variables, which are described as

$$|u_j(k)| \leq \bar{u}_j, \quad j = 1, \ldots, m. \tag{2}$$

In this work, we are interested in communication networks situated between the controller output and the plant input. Thus, all the data to be transmitted are to be quantized and may be missing. The quantizer at time instant k is modeled by

$$u(k) = f(v(k)), \tag{3}$$

where $f(\cdot)$ is a logarithmic quantizer. For a scalar input $v(\cdot) \in \mathbf{R}$, $f(v)$ is given by

$$f(v) = \begin{cases} \nu_i, & \text{if } \dfrac{1}{1+\tau}\nu_i < v \leq \dfrac{1}{1-\tau}\nu_i,\ v > 0, \\ 0, & \text{if } v = 0, \\ -f(-v), & \text{if } v < 0, \end{cases} \tag{4}$$

where $\tau = \frac{1-\rho}{1+\rho}$. The associated set of quantized levels has the following form: $\mathcal{V} = \{\pm\nu_i, \nu_i = \rho^i \nu_0,\ i = \pm 1, \pm 2, \ldots\} \cup \{\pm\nu_0\} \cup \{0\}, 0 < \rho < 1, \nu_0 > 0$. The quantizer $f(\cdot)$ maps the whole segment of v to the quantization level $\mathcal{V}$ and each quantization level ν_i corresponds to a segment of v. From Fu and Xie (2006), a sector-bound expression on any $f(\cdot)$ can be given as $f(v) = (1+\xi)v$, where $\xi \in [-\tau, \tau]$. For the discrete-time system in (1) with $u(k) \in \mathbf{R}^m$, the quantized control signals with individual quantizer f_j for channel j $(j = 1, \ldots, m)$ become

$$\begin{aligned} u(k) &= f(v(k)) \\ &= \begin{bmatrix} f_1(v_1(k)) & f_2(v_2(k)) & \cdots & f_m(v_m(k)) \end{bmatrix}^T \\ &= \Lambda(k)\, v(k), \end{aligned} \tag{5}$$

where $\Lambda(k) = \mathrm{diag}\{1+\xi_1(k), 1+\xi_2(k), \ldots, 1+\xi_m(k)\} \in \Pi = \mathrm{Co}\{\Lambda^{(1)}, \Lambda^{(2)}, \ldots, \Lambda^{(2^m)}\}$, $\xi_j(k) \in \left[-\tau_j, \tau_j\right]$, $j = 1, \ldots, m$, $\Lambda^{(l)}$ is a diagonal matrix with entries being $1-\tau_j$ or $1+\tau_j$, and the 2^m combinations of $1-\tau_j$ and $1+\tau_j$ generate all $\Lambda^{(l)}$.

When data are transmitted from the controller to the actuator, they may be lost. Here a stochastic variable $\gamma(k) \in \mathbf{R}$ is introduced to denote the data status for time instant k (1 for transmitted data, 0 for missing data). It is assumed that the data loss process is a