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Formation control using binary information ☆

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ABSTRACT

In this paper, we study the problem of formation keeping of a network of strictly passive systems when very coarse information is exchanged. We assume that neighboring agents only know whether their relative position is larger or smaller than the prescribed one. This assumption results in very simple control laws that direct the agents closer or away from each other and take values in finite sets. We show that the task of formation keeping while tracking a desired velocity and rejecting matched disturbances is still achievable under the very coarse information scenario. In contrast with other results of practical convergence with coarse or quantized information, here the control task is achieved exactly.

1. Introduction

Distributed motion coordination of mobile agents has attracted increasing attention in recent years owing to its wide range of applications from biology and social networks to sensor/robotic networks. Distributed formation keeping control is a specific case of motion coordination which aims at reaching a desired geometrical shape for the positions of the agents while tracking a desired velocity. This problem has been addressed with different approaches e.g. Arcak (2007), Bai, Arcak, and Wen (2011), Bullo, Cortés, and Martınez (2009), Mesbahi and Egerstedt (2010), Ren and Beard (2007) and Ren and Cao (2011). In problems of formation control, an important component, besides the dynamics of the agents and the graph topology, is the flow of information among the agents. In fact, the usual assumption in the literature on cooperative control is that a continuous flow of perfect information is exchanged among the agents. However, due to the coarseness of sensors and/or to communication constraints, the latter might be a restrictive requirement. To cope with the problem, the use of distributed quantized feedback control has been proposed in the literature both for the discrete-time (Kashyap, Başar, & Srikant, 2007) and the continuous-time dynamic agents (Ceragioli, De Persis, & Frasca, 2011; Chen, Lewis, & Xie, 2011; Cortés, 2006; Dimarogonas & Johansson, 2010; Xargay, Choe, Hovakimyan, & Kaminer, 2012). In fact, in formation control by quantized feedback control, information is transmitted among the agents whenever measurements cross the thresholds of the quantizer. At these times, the corresponding quantized value taken from a discrete set is transmitted. This allows to deal naturally both with the continuous-time nature of the agents' dynamics and the discrete nature of the information transmission process without the need to rely on sampled-data models (Ceragioli et al., 2011; De Persis & Jayawardhana, 2012).

Main contribution. In this paper, we study the problem of distributed position-based formation keeping of a group of agents with strictly passive dynamics which exchange binary information. The binary information models a sensing scenario in which each agent detects whether or not the components of its current distance vector from a neighbor are above or below the prescribed distance and apply a force (in which each component takes a binary value) to reduce or respectively increase the actual distance. A similar coarse sensing scenario was considered in Yu, LaValle, and Liberzon (2012) in the context of the so-called "minimalist" robotics.

Remarkably, despite such a coarse information and control action, we show that the control law guarantees exact achievement of the desired formation. This is an interesting result, since statically quantized control inputs typically generate practical convergence, namely the achievement of an approximate formation in which the distance from the actual desired formation depends on

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the quantizer resolution (De Persis & Jayawardhana, 2012). Here the use of binary information allows us to conclude asymptotic convergence without the need to dynamically update the quantizer resolution.

Motivation. There are two main aspects that triggered our interest for the problem above.

(i) The use of binary information in coordination problems (Chen et al., 2011; Cortés, 2006) has been proven useful to the design and real-time implementation of distributed controls for systems of first- or second-order agents in a cyber–physical environment (see e.g. De Persis & Frasca, 2013, De Persis, Frasca, & Hendrickx, 2013 and Nowzari & Cortés, 2012). We envision that a similar role will be played by the results in this paper for a larger class of coordination problems (see De Persis et al., 2013 for an early result in this respect).

(ii) The resulting control laws are implemented by very simple directional commands (such as "move north", "move north-east", "stay still", etc). We also show that the presence of coarse information does not affect the ability of the proposed controllers to achieve the formation in a leader–follower setting in which the desired reference velocity is only known to the leader.

This paper adopts a similar setting as in De Persis and Jayawardhana (2012) but controllers and analyses are different. Moreover, the paper investigates the formation control problem with unknown reference velocity tracking and matched disturbance rejection that was not considered in De Persis and Jayawardhana (2012). Compared with Yu et al. (2012), where also coarse information was used for rendezvous, the results in our contribution apply to a different class of systems and to a different cooperative control problem. Early results with the same sensing scenario but for a formation of double integrators have been presented in Jafarian and De Persis (2013).

The paper is organized as follows: Section 2 introduces the problem statement along with some motivations and notations. Analysis of the formation keeping problem with coarse data in the case of known/unknown reference velocity is studied in Section 3. Section 4 investigates the problem of formation keeping with coarse data in the presence of matched input disturbances. Related simulations are presented in Section 6. The paper is summarized in Section 7.

Notation. Given two sets S_1, S_2, the symbol $S_1 \times S_2$ denotes the Cartesian product of two sets. This can be iterated. The symbol $\times_{k=1}^{m} S_k$ denotes $S_1 \times S_2 \times \cdots \times S_m$. For a set S, $\mathrm{card}(S)$ denotes the cardinality of the set S. Given a matrix M of real numbers, we denote by $\mathcal{R}(M)$ and $\mathcal{N}(M)$ the range and the null space, respectively. The symbols $\mathbf{1}, \mathbf{0}$ denote vectors or matrices of all 1 and 0 respectively. Sometimes the size of the matrix is explicitly given. Thus, $\mathbf{1}_N$ is the N-dimensional vector of all 1. I_p is the $p \times p$ identity matrix. Given two matrices A, B, the symbol $A \otimes B$ denotes the Kronecker product. $A = \mathrm{block.diag}\{A_1, \dots, A_N\}$ denotes a diagonal matrix $A_{N\times N}$ such that A_i is its ith diagonal element.

2. Preliminaries

2.1. The multi-agent system

In this subsection, we review the passivity-based approach to coordination control (Arcak, 2007, see also Bai et al., 2011, Bürger, Zelazo, & Allgöwer, 2011, De Persis & Jayawardhana, 2012 and Munz, Papachristodoulou, & Allgöwer, 2011). A network of N agents in $\mathbb{R}^p$ are considered. For each agent i, $x_i \in \mathbb{R}^p$ represents its position. The communication topology of the network is assumed

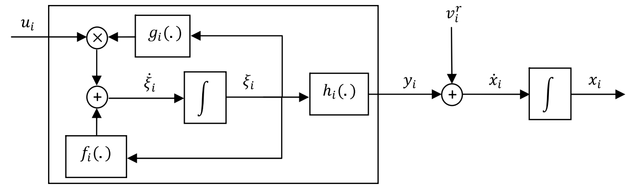

Fig. 1. The dynamics of each agent is assumed to be composed of a strictly passive system and an integrator. The system $\mathcal{H}_i$ is strictly passive from an external input u_i to the velocity error y_i.

to be modeled by a connected and undirected graph $G = (V, E)$, where V is a set of N nodes and $E \subseteq V \times V$ is a set of M edges connecting the nodes. Label one end of each edge in E with a positive sign and the other end with a negative sign. We define the relative position z_k as follows

$$z_k = \begin{cases} x_i - x_j & \text{if node } i \text{ is the positive end of the edge k} \\ x_j - x_i & \text{if node } j \text{ is the positive end of the edge k} \end{cases}$$

where $x_i \in \mathbb{R}^p$ is the position of agent i expressed in an inertial frame. We define the $N \times M$ incidence matrix B associated with the graph G as follows

$$b_{ik} = \begin{cases} +1 & \text{if node } i \text{ is the positive end of the edge k} \\ -1 & \text{if node } j \text{ is the positive end of the edge k} \\ 0 & \text{otherwise.} \end{cases}$$

By definition of B, we can represent the relative position variable z, with $z \triangleq [z_1^T \dots z_M^T]^T$, $z \in \mathbb{R}^{pM}$, as a function of the position variable x, namely

$$z = (B^T \otimes I_p)x, \tag{1}$$

which implies that z belongs to the range space $\mathcal{R}(B^T \otimes I_p)$.

We denote the network desired reference velocity by $v^*(t)$. This is the reference velocity to which the velocities of all the agents should converge. Moreover, each agent is expected to track its own desired (time-varying) reference velocity denoted by $v_i^r(t)$. If the desired network velocity is known to all of the agents, we have $v_i^r(t) = v^*(t)$. Otherwise, each agent will be equipped with an appropriate controller (as in Section 3.2) to recover $v^*(t)$, that is to guarantee that $v_i^r(t)$ converges to $v^*(t)$. This implies that by tracking $v_i^r(t)$, each agent eventually tracks $v^*(t)$.

We assume that the velocity of each agent is given by

$$\dot{x}_i = \mathcal{H}_i\{u_i\} + v_i^r, \tag{2}$$

where $\mathcal{H}_i\{u_i\}$ represents a system with dynamics (as in Fig. 1)

$$\mathcal{H}_i : \begin{cases} \dot{\xi}_i = f_i(\xi_i) + g_i(\xi_i)u_i \\ y_i = h_i(\xi_i) \end{cases} \tag{3}$$

where $\xi_i \in \mathbb{R}^{n_i}$ is the state variable, $u_i \in \mathbb{R}^p$ is the control input, $y_i \in \mathbb{R}^p$ is the velocity error, and the exogenous signal $v_i^r \in \mathbb{R}^p$ is the reference velocity for agent i. The maps f_i, g_i and h_i are assumed to be locally Lipschitz such that $f_i(\mathbf{0}) = \mathbf{0}$, $h_i(\mathbf{0}) = \mathbf{0}$, and $g_i(\mathbf{0})$ is full column-rank.

Notice that the output y_i of (3) is the velocity tracking error $\dot{x}_i - v_i^r$, namely

$$y_i = \dot{x}_i - v_i^r. \tag{4}$$

The system $\mathcal{H}_i$ is assumed to be strictly passive from the input u_i to the velocity error y_i. Since the system $\mathcal{H}_i$ is strictly passive, there is a continuously differentiable storage function $S_i : \mathbb{R}^{n_i} \to \mathbb{R}_+$ which is positive definite and radially unbounded and satisfies

$$\frac{\partial S_i}{\partial \xi_i}[f_i(\xi_i) + g_i(\xi_i)u_i] \le -W_i(\xi_i) + {y_i}^T u_i \tag{5}$$

where W_i is a continuous positive function and $W_i(\mathbf{0}) = \mathbf{0}$.