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Multiple window moving horizon estimation ☆

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ABSTRACT

Long horizon lengths in Moving Horizon Estimation are desirable to reach the performance limits of the full information estimator. However, the conventional MHE technique suffers from a number of deficiencies in this respect. First, the problem complexity scales at least linearly with the horizon length selected, which restrains from selecting long horizons if computational limitations are present. Second, there is no monitoring of constraint activity/inactivity which results in conducting redundant constrained minimizations even when no constraints are active. In this study we develop a Multiple-Window Moving Horizon Estimation strategy (MW-MHE) that exploits constraint inactivity to reduce the problem size in long horizon estimation problems. The arrival cost is approximated using the unconstrained full information estimator arrival cost to guarantee stability of the technique. A new horizon length selection criterion is developed based on the sensitivity between remote states in time. The development will be in terms of general causal descriptor systems, which includes the standard state space representation as a special case. The potential of the new estimation algorithm will be demonstrated with an example showing a significant reduction in both computation time and numerical errors compared to conventional MHE.

1. Introduction

Inequality constraints in estimation problems can arise from physical insight into known boundaries on the state trajectories and can be viewed as additional a-priori information. The added value of inequality constraints in state estimation is well known and demonstrated in many fields, see for example the studies given in Haseltine and Rawlings (2005) and Robertson and Lee (2002). Inequality constraints may also arise in convex filtering problems as in ℓ_1 trend filtering and total variation de-noising (Chu, Keshavarz, Gorinevsky, & Boyd, 2012) or when other densities with finite support describe the system and/or measurement noise (Aravkin, Burke, & Pillonetto, 2013; Goodwin, Seron, & Doná, 2005; Robertson & Lee, 2002).

Unfortunately inequality constraints in the estimation problem generally prevent the use of recursive solutions for finding the estimates (Rao, 2000; Rawlings & Mayne, 2009). The moving horizon estimate (MHE), on the other hand, is found by limiting the estimation problem to a window of measurements and system dynamic updates that slides with time while partially accounting for past measurements through an extra penalty cost term, referred to as an arrival cost (Rao, 2000). The horizon length is selected based on many factors, including computational limitations, system observability and model accuracy. Higher estimation accuracy, in terms of mean square error, may be achieved by using long horizon lengths or alternatively, finding more accurate arrival cost approximations, provided that the model uncertainties are well accounted for Rawlings and Mayne (2009). Efforts to improve arrival cost approximations can be found in Chu et al. (2012) and Rao (2000) for linear state space systems and in Lopez-Negrete, Patwardhan, and Biegler (2011), Qu and Hahn (2009), Rao (2000), Ungarala (2009) and Zavala (2010) for non-linear state space systems.

The conventional sliding window technique in Moving Horizon Estimation, however, can become computationally inefficient. For example, at certain times the solution of the inequality constrained state estimation problem may be identical to the solution of the unconstrained problem; i.e. dropping the inequality constraints from the sliding window minimization problem for these states has no effect on the value of the estimates. For linear systems, these unconstrained state estimates can be determined using simple recursive solutions and hence there is no added value in including these states in the minimization problem as they create an

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unnecessary computational burden. Moreover, numerical errors associated with window minimizations increase with the size of the sliding window used.

Recently, in Chu et al. (2012), an approximation hypothesis was used to derive a simple arrival cost update for general staged QP problems with sufficiently large horizon lengths by assuming that the active and inactive state constraints of the last state in the moving horizon window remain respectively active or inactive indefinitely after exiting the window. Consequently, equality constraints corresponding to the active inequality constraints were included in the arrival cost update. However, no stability analysis was provided using this method, nor means for selecting the sufficiently large horizons.

This technique seems very attractive but can cause problems when the horizon is not chosen large enough to satisfy the active constraint hypothesis. If short horizons are used, for example, then estimator divergence may result if the presumably indefinite active constraint is not really active after smoothing the state (i.e. after more measurements are collected). This overweighting of past data may result in neglecting new data and potentially can cause estimator divergence if the coupling between the states in time is strong (Rawlings & Mayne, 2009). On the other hand, dropping inequality constraints from the minimization problem once they pass outside the sliding window has no destabilizing effect, as the estimator will possess the guaranteed convergence and stability properties of the unconstrained estimator.

In view of the above, a numerical algorithm that accounts for active constraints over large horizons without compromising stability or efficiency is developed. This is achieved by using an arrival cost approximation that guarantees stability while exploiting regions of constraint inactivity to automatically reformulate the objective function into a reduced form in every iteration. In other words, the 'sparsity' of active constraints is exploited to enable efficient long horizon estimation. The new algorithm is called Multiple Window Moving Horizon Estimation (MW-MHE) and has been analyzed in the context of descriptor systems. A complete convergence and stability analysis for our selection of the arrival cost for descriptor MHE is given in the Appendix using analogies with the presentation given in Rao (2000) and Rawlings and Mayne (2009) for state space systems.

Fig. 1 shows an explanation of our new proposed strategy. A short sliding window objective function is used to scan for states with active constraints. The inequalities associated with states that never became constrained inside the sliding window are dropped from future minimizations assuming they will remain unconstrained. On the other hand, inequalities associated with states that were constrained inside the sliding window objective remain in subsequent minimizations and form fixed windows that are augmented to the sliding window. These fixed windows, (within the intervals $[a_1, b_1]$ and $[a_2, b_2]$ as shown in Fig. 1) remain in the estimation problem until their influence on the current state is negligible. This allows reformulating the objective into a significantly smaller minimization problem, especially when periods of constraint inactivity dominate. On the other hand, stability is maintained by using the arrival cost for unconstrained minimization. The algorithm was implemented using the semi-definite programming solver (Toh, Todd, & Tütüncü, 1999) with the CVX parser in Matlab (Grant & Boyd, 2008) and is available online in Al-Matouq (2014).

In conventional MHE, the horizon length is often selected based on computational limitations because of the linear growth of problem complexity with horizon length. If the system to be estimated, however, operates mostly inside the region defined by the inequality constraints, and intermittently operates near the constraints, then the new MW-MHE can be used to exploit regions of constraint inactivity to perform long horizon estimation efficiently. This also promotes selecting horizon lengths based on the sensitivity between remote states in time rather than based on implementation restrictions. Hence, a new tuning method for selecting the horizon length based on a user specified minimum magnitude of acceptable coupling between distant states in time is also developed in this study.

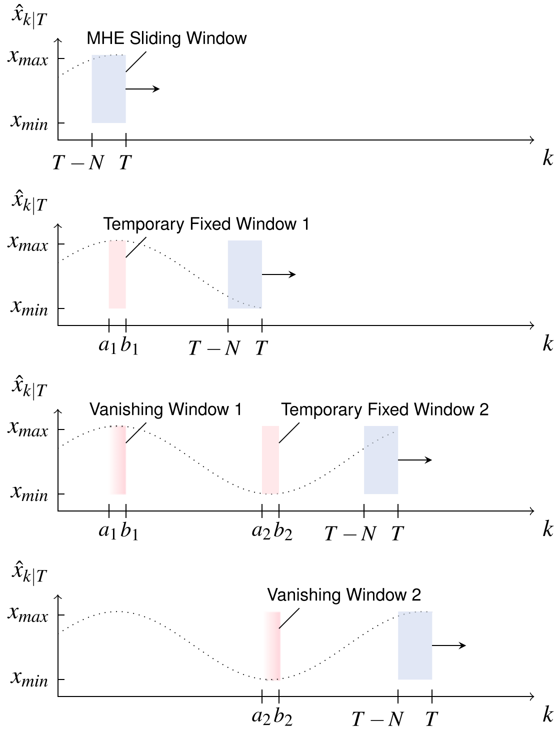

Fig. 1. Multiple window formulation.

The development will be in terms of general causal descriptor systems, which includes the standard state space representation as a special case. Our motivation for descriptor systems is estimation problems that involve differential algebraic models ubiquitous in simulation environments (Biegler, Campbell, & Mehrmann, 2012), and problems that involve singularly perturbed systems (Kumar & Daoutidis, 1999). Descriptor systems have been also used in unknown input estimation in Al-Matouq and Vincent (2014) and Darouach, Zasadzinski, Onana, and Nowakwski (1995) which avoids improvising a random walk model on the input signal. Moreover, other staged QP filtering and estimation problems can benefit from the descriptor system framework, like ℓ_1 trend filtering and total variation de-noising (Chu et al., 2012). Descriptor moving horizon estimation was first considered in Boulkroune, Darouach, and Zasadzinski (2010).

The paper is organized as follows. Section 2 presents the constrained full information estimation problem for descriptor systems followed by the required assumptions. The Moving Horizon approximation is then presented in Section 3 following the theme given in Rao, Rawlings, and Lee (2001), where the relationship between full information and moving horizon estimation was analyzed using dynamic programming. Section 4, which is the main contribution of this study, will present the multiple window moving horizon estimation algorithm and a new tuning parameter based on the coupling between remote states in time. Finally, Section 5 will demonstrate the potential of the new MW-MHE algorithm with an example. The Appendix sections detail the proofs of theorems used in this study.

The following notation is used in this study: $\mathbb{R}$ represents the set of real numbers; $A \in \mathbb{R}^{n\times m}$ is an $n \times m$ matrix with real values; $\|z\|_A := z^T A^{-1} z$; $\|A\|_{i2}$ is the matrix induced two norm for matrix