Automatica 53 (2015) 320–328

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Bounded droop controller for parallel operation of inverters[☆]

George C. Konstantopoulos[a,1], Qing-Chang Zhong[a,b], Beibei Ren[c], Miroslav Krstic[d]

[a] Department of Automatic Control and Systems Engineering, The University of Sheffield, Sheffield, S1 3JD, UK
[b] Department of Electrical and Computer Engineering, Illinois Institute of Technology, Chicago, IL 60616, USA
[c] Department of Mechanical Engineering, Texas Tech University, Lubbock, TX 79409, USA
[d] Department of Mechanical and Aerospace Engineering, University of California, San Diego, CA 92093, USA

ARTICLE INFO

Article history:
Received 6 November 2013
Received in revised form
19 November 2014
Accepted 21 December 2014

Keywords:
Droop control
Non-linear systems
Stability
Parallel operation of inverters
Proportional load sharing

ABSTRACT

In this paper, the stability of parallel-operated inverters in the sense of boundedness is investigated. At first, the non-linear model of parallelled inverters with a generic linear or non-linear load is obtained by using the generalised dissipative Hamiltonian structure and then the robust droop controller, recently proposed in the literature for parallel operation of inverters, is implemented in a way to produce a bounded control output. The proposed controller is called the bounded droop controller (BDC). It introduces a zero-gain property and can guarantee the boundedness of the closed-loop system solution. Therefore, for the first time, the closed-loop stability in the sense of boundedness is guaranteed for parallelled inverters feeding generic non-linear/linear loads. The controller structure is further improved to increase its robustness with respect to initial conditions, numerical errors or external disturbances while maintaining the stability property. Moreover, the controller is tuned to avoid any possible limit cycles in the voltage dynamics. Real-time simulation results for two single-phase inverters operated in parallel loaded with a non-linear load are presented to verify the effectiveness of the proposed BDC.

1. Introduction

The penetration of renewable energy sources into electrical networks has increased in the last decades due to environmental, technical and economical reasons. Their integration is accomplished by using suitable power electronic devices (inverters), thus forming distributed generation units. The integration of renewable sources along with energy storage devices and local loads form a microgrid, which has been extensively studied in the literature (Guerrero, Chandorkar, Lee, & Loh, 2013; Guerrero, Loh, Lee, & Chandorkar, 2013; Guerrero, Vasquez, Matas, Castilla, & de Vicuna, 2009; Iyer, Belur, & Chandorkar, 2010; Weiss, Zhong, Green, & Liang, 2004; Zhong & Hornik, 2013). In microgrids, due to the limited availability of high current power electronic devices, inverters are often operated in parallel. In order to avoid circulating currents among the inverters, the droop control method (Barklund, Pogaku, Prodanovic, Hernandez-Aramburo, & Green, 2008; Guerrero, Chandorkar et al., 2013; Guerrero, Loh et al., 2013; Guerrero, Vasquez, Matas, Garcia de Vicuna, & Castilla, 2011), which does not require any external communication mechanism among the inverters (Chandorkar, Divan, & Adapa, 1993; Tuladhar, Jin, Unger, & Mauch, 1997), is often adopted. However, secondary control is often used to restore the microgrid voltage and frequency to the desired level (Guerrero, Chandorkar et al., 2013; Guerrero et al., 2009, 2011).

One of the main issues in microgrid operation is the accurate power sharing among the parallelled inverters in accordance to their power ratings, which should be maintained in both grid-connected and stand-alone operation. Especially in the stand-alone mode, load sharing according to each inverter capacity under different operating conditions is a challenging task (Sao & Lehn, 2005), which is usually achieved using droop control techniques. However, conventional droop controllers introduce inherent limitations in accurate real and reactive power sharing as noted in Zhong (2013) and Zhong and Hornik (2013). Additionally, the inverter output impedance plays a key role in accurate load sharing, since inverters equipped with the conventional droop control are

☆ The financial support from the EPSRC, UK, under Grant No. EP/J01558X/1 is greatly appreciated. The material in this paper was partially presented at the 2014 American Control Conference 2014, June 4–6, 2014, Portland, OR, USA. This paper was recommended for publication in revised form by Associate Editor Huaguang Zhang under the direction of Editor Toshiharu Sugie.

E-mail addresses: g.konstantopoulos@sheffield.ac.uk (G.C. Konstantopoulos), zhongqc@ieee.org (Q.-C. Zhong), beibei.ren@ttu.edu (B. Ren), krstic@ucsd.edu (M. Krstic).

[1] Tel.: +44 114 222 6080; fax: +44 114 22 25683.

http://dx.doi.org/10.1016/j.automatica.2015.01.012

required to have the same per-unit output impedance (Guerrero, de Vicuna, Matas, Castilla, & Miret, 2005). Therefore, recently, several control designs have been proposed in order to achieve accurate power sharing among the inverters (Guerrero et al., 2006; Guerrero, Matas, de Vicuna, Castilla, & Miret, 2007; Li & Kao, 2009; Majumder, Chaudhuri, Ghosh, Ledwich, & Zare, 2010; Mohamed & El-Saadany, 2008; Zhong, 2013). Among these techniques, the robust droop controller (RDC) proposed in Zhong (2013) has been proven to achieve accurate load sharing even if numerical computational errors, disturbances, noises, parameter drifts and component mismatches occur.

Although a lot of research has been done in the field of load sharing, the stability properties of the proposed techniques have not been adequately exploited. Most of the stability analysis has been focused on small-signal modelling and linearisation methods (Coelho, Cortizo, & Garcia, 2002; Majumder et al., 2010; Marwali, Jung, & Keyhani, 2007; Mohamed & El-Saadany, 2008), which are only valid around a specific equilibrium point (local stability). Several conditions of the local exponential stability for frequency droop control are exploited in Simpson-Porco, Dörfler, and Bullo (2013), where however fixed or bounded voltage magnitudes and a purely inductive network are considered. Due to the non-linear structure of the droop controller, it becomes obvious that the non-linear stability analysis is essential for investigating the behaviour of parallel inverters. Recently the $\mathcal{L}_\infty$ stability of the conventional droop control has been proven in Schiffer, Ortega, Astolfi, Raisch, and Sezi (2014) where asymptotic stability of lossless microgrids is also achieved. In this work, the Kron-reduced network is considered and instantaneous frequency regulation is assumed for the analysis. It should be noted that the Kron-reduced network approach considers all loads in the linear form. Non-linear load dynamics can be suitably investigated only using the inverter currents model.

To the best of the authors' knowledge, the non-linear stability of a robust droop control technique, which achieves accurate load sharing, independently from the type of the load (linear or non-linear) has not been solved yet. In this paper, two parallel single-phase inverters feeding a local load are considered. The load is given in the generalised dissipative Hamiltonian form, which represents the general case of a power electronic-driven dynamic system (Konstantopoulos & Alexandridis, 2013; Ortega, Loria, Nicklasson, & Sira-Ramirez, 1998). For this system, the robust droop controller proposed in Zhong (2013) can be considered, since it is proven to achieve the most robust performance and introduces a dynamic voltage droop opposed to the conventional droop controllers. Particularly, in the present work, in order to analyse the stability of inverters operated in parallel, the RDC is implemented in a way to ensure that the control input stays within a predefined range, without changing the main concept of the initial control design. The controller performance is extensively analysed using non-linear Lyapunov methods and is proven to achieve a bounded performance. Using $\mathcal{L}_\infty$ stability analysis and the small-gain theorem (Khalil, 2001), the first proof of stability in the sense of boundedness is presented for the non-linear closed-loop system using the important zero-gain property of the proposed controller. Further investigation of the controller structure leads to the final form of the proposed bounded droop controller (BDC) which is robust to external disturbances and guarantees the desired performance. This represents a significant superiority with respect to the existing techniques since robust accurate load sharing is achieved with a guaranteed stability for a general load case using a dynamic droop controller. Extensive real-time simulation results for two inverters in parallel operation with a non-linear load are illustrated to verify the effectiveness of the proposed BDC compared to the RDC using an OPAL-RT real-time digital simulator.

The paper is organised as follows. In Section 2, the dynamic model of the system consisting of two single-phase inverters and a load is obtained along with its properties and an overview of the robust droop controller is presented. In Section 3, the bounded droop controller is proposed and its performance is investigated. Furthermore, the boundedness of the closed-loop system is proven using non-linear analysis. The controller structure is further improved to increase its robustness with respect to computational errors or disturbances and guarantee a desired operation. In Section 4, extensive real-time simulation results are provided to certify the effectiveness of the proposed bounded control scheme and, finally, in Section 5, some conclusions are drawn.

2. Parallel operation of inverters

2.1. System modelling

Fig. 1 represents the schematic diagram of two single-phase inverters connected in parallel feeding a common load. An *LC* filter is assumed at the output of each inverter where L_1, L_2 and C_1, C_2 are the filter inductances and capacitances respectively for each inverter. In practice, each inductor and capacitor introduces parasitic resistances represented as R_1 and R_2 in series with the inductors (typically very small) and r_{C1} and r_{C2} in parallel with the capacitors (typically very large). Variables v_{r1}, v_{r2} and i_1, i_2 are the inverter output voltages and currents, respectively, while v_o and i_L are the load voltage and current, respectively. The filter capacitors along with the parasitic resistances can be regarded as a part of the load and therefore, C_1, C_2, r_{C1} and r_{C2} can represent some of the load characteristics as well (Zhong, 2013; Zhong & Hornik, 2013). The dynamic equations of the system are

$$
\begin{aligned}
L_1\frac{di_1}{dt} &= -R_1 i_1 - v_o + v_{r1},\\
L_2\frac{di_2}{dt} &= -R_2 i_2 - v_o + v_{r2}, \qquad (1)\\
(C_1+C_2)\frac{dv_o}{dt} &= i_1 + i_2 - \frac{r_{C1}+r_{C2}}{r_{C1}r_{C2}}v_o - i_L.
\end{aligned}
$$

Although a lot of research has been done for linear loads (resistive, resistive–inductive), in a typical application the load is usually non-linear. This increases the difficulty of the control design and the analysis. However, since most of the loads are fed by power electronic devices (power converters), by using average analysis (Ortega et al., 1998), the load can be represented by the generalised dissipative Hamiltonian form Konstantopoulos and Alexandridis (2013) and Ortega et al. (1998)

$$M\dot{w} = (J(w,\mu) - R)\,w + Gv_o, \qquad (2)$$

where $w = \begin{bmatrix} i_L & w_1 & w_2 \ldots & w_{m-1}\end{bmatrix}^T \in \mathcal{R}^m$ represents the states of the load and μ is a bounded vector in a closed set which describes the duty-ratio signals of the converters. Matrix M is constant and positive definite, J is skew-symmetric, R is constant and positive definite and $G = \begin{bmatrix} 1 & 0_{1\times(m-1)}\end{bmatrix}^T$. For the load equation (2), the load voltage can be considered as an input to the load system (in fact this is usually the case when for example a voltage source device is connected at the inverter's output). It should also be noted that all non-linearities of the load and the bounded duty-ratio signals μ are restricted into the skew-symmetric matrix J. This is a common issue in power systems, especially for power converter-fed loads (Karagiannis, Mendes, Astolfi, & Ortega, 2003; Konstantopoulos & Alexandridis, 2013; Ortega et al., 1998). As a result, the complete plant system can be written into the generalised dissipative Hamiltonian form

$$\tilde{M}\dot{\tilde{x}} = \left(\tilde{J}(\tilde{x},\mu) - \tilde{R}\right)\tilde{x} + \tilde{G}u, \qquad (3)$$

where the state vector is $\tilde{x} = \begin{bmatrix} i_1 & i_2 & v_o & w^T\end{bmatrix}^T$, the input vector is $u = \begin{bmatrix} v_{r1} & v_{r2}\end{bmatrix}^T$ and matrices $\tilde{M}$, $\tilde{J}$ and $\tilde{R}$ as defined below retain