Automatica 53 (2015) 23–29

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Distributed control for uniform circumnavigation of ring-coupled unicycles☆

Ronghao Zheng [a], Zhiyun Lin [b,1], Minyue Fu [c,b,d], Dong Sun [a,2]

[a] Department of Mechanical and Biomedical Engineering, City University of Hong Kong, 83 Tat Chee Avenue, Kowloon, Hong Kong, China

[b] State Key Laboratory of Industrial Control Technology, College of Electrical Engineering, Zhejiang University, 38 Zheda Road, 310027 Hangzhou, PR China

[c] School of Electrical Engineering and Computer Science, University of Newcastle, Callaghan, NSW 2308, Australia

[d] Department of Control Science and Engineering, Zhejiang University, 388 Yuhangtang Road, 310058 Hangzhou, PR China

ARTICLE INFO

Article history:
Received 20 August 2013
Received in revised form
16 May 2014
Accepted 1 November 2014

Keywords:
Multi-agent formation
Distributed control
Nonholonomic dynamics
Uniform circumnavigation

ABSTRACT

The paper studies the general circumnavigation problem for a team of unicycle-type agents, with the goal of achieving specific circular formations and circling on different orbits centered at a target of interest. A novel distributed solution is proposed, in which the control laws are heterogeneous for the agents such that some agents are repellant from the target while attractive to its unique neighbor and some agents are attractive to the target while repellant from its neighbor. A systematic procedure is developed to design the control parameters according to the specific radii of the orbits and the formations that the agents are desired to converge to. Moreover, this control scheme uses a minimum number of information flow links between the agents and local measurements of relative position only. Based on the block diagonalization of circulant matrices by a Fourier transform, asymptotic convergence properties are analyzed rigorously. The validity of the proposed control algorithm is also demonstrated through numerical simulations.

1. Introduction

The circumnavigation problem, in which the task is to circumnavigate a target of interest by one agent or a network of autonomous agents, has many applications in security and surveillance (Shames, Dasgupta, Fidan, & Anderson, 2012), satellite formation flying (Milam, Petit, & Murray, 2001), source seeking (Moore & Canudas-de Wit, 2010), etc. In a space mission, a formation made up of numerous micro satellites is desired to orbit around a large satellite for the tasks of monitoring, maintenance and repairing, in which the fundamental issue is the autonomous formation by the micro satellites and also the relative orbit formation of these micro satellites with respect to the large satellite. In a security and surveillance mission, it is often required to have a single or a group of agents to circle around a target or an area of interest for monitoring or gathering information. Depending on different applications, there may be different control specifications, but in general there is a fundamental requirement of achieving circular formations on specific orbits around a target of interest.

Many circumnavigation algorithms have been studied recently in the literature for a single agent. For holonomic agents, a continuous-time nonlinear periodically time-varying algorithm is developed in Deghat, Shames, Anderson, and Yu (2010) and Shames et al. (2012), which adaptively estimates the position of the target and moves the agent to a trajectory encircling it based on the distance or bearing measurement to the target. The bearing-based circumnavigation algorithm is also extended to the unicycle-like agent in Deghat et al. (2012). Moreover, for the unicycle-like agent, a range-only strategy is presented in Matveev, Teimoori, and Savkin (2011) using a sliding mode approach. The study of circumnavigation by a team of autonomous agents has also attracted a lot of attention recently. Compared with the single-agent case, besides circumnavigating around the target on a specific orbit, the agents should also achieve an optimal configuration surrounding the target through group coordination, e.g., to uniform the spacing next to each other. A big challenge here is how to achieve coordination in a distributed way. Some of the early works in this line include (Yamaguchi, 1999) where a group of holonomic mobile agents are used.

☆ The material in this paper was partially presented at the 9th Asian Control Conference, June 23–26, 2013, Istanbul, Turkey. This paper was recommended for publication in revised form by Associate Editor Hideaki Ishii under the direction of Editor Frank Allgöwer.

E-mail addresses: rzheng3-c@my.cityu.edu.hk (R. Zheng), linz@zju.edu.cn (Z. Lin), minyue.fu@newcastle.edu.au (M. Fu), medsun@cityu.edu.hk (D. Sun).

1 Tel.: +86 571 8795 1637; fax: +86 571 8795 2152.

2 Tel.: +86 852 3442 8405; fax: +86 852 3442 0172.

http://dx.doi.org/10.1016/j.automatica.2014.11.012
0005-1098/

A cyclic pursuit-based strategy is proposed in Kim and Sugie (2007) which achieves uniform circumnavigation by decoupling the target tracking and inter-agent coordination tasks. Moving-target enclosing strategy is proposed in Guo, Yan, and Lin (2010) by Guo et al. For nonholonomic agents, Ceccarelli et al. present a switching control scheme to drive a group of unicycle-type vehicles to achieve circular motions around a static target in Ceccarelli, Di Marco, Garulli, and Giannitrapani (2008). However, this control law does not result in an even spacing formation, which is achieved in Lan, Yan, and Lin (2010) by using a hybrid control strategy. Moreover, there are also a lot of works on circular formations (Chen & Zhang, 2011; Marshall, Broucke, & Francis, 2004, 2006; Sinha & Ghose, 2007). Although in these works the orbit center depends on the agents' initial states rather than the target, the ideas developed are still very helpful in dealing with the circumnavigation problem if the agents also interact with the target.

However, most aforementioned works assume that the goal is to achieve a circular formation on the same orbit for all the agents. But in many applications such as satellite formation flying, the micro satellites may need to stay on different orbits to form a large aperture; In the target enclosing problem, the agents may need to stay on different orbits to perform different missions, for example, the inner agents surveil the target while the outer agents protect the inner ones against possible attacks. In this paper, we study the general circumnavigation problem with provable stability properties for desired circular formations in which autonomous agents can circle on different orbits around the target. A new distributed control strategy is proposed, which combines attraction/repulsion from its pursuing agent as well as the target of interest. The strategy was originally developed in Zheng, Lin, Fu, and Sun (2013) to achieve uniform distribution on the same orbit when circling about a specific target. But in order to solve the coordinated circumnavigation problem on different orbits, this paper generalizes the idea by considering non-identical control laws for the agents. That is, under the proposed control strategy, some agents may be repellant from the target while attractive to the pursuing agent, and some agents may be attractive to the target while repellant from the pursuing agent. We then show how the control parameters are designed in a systematic way according to the specific radii of the orbits and the formations that the agents are desired to converge to. Moreover, we show that among all equilibrium formations achieving uniform circumnavigation, only two of them are asymptotically stable, corresponding to the clockwise or counterclockwise motions around the target. The stability analysis technique is based on the block diagonalization of circulant matrices by a Fourier transform. Simulations are also given to demonstrate the effectiveness of the proposed distributed control strategy to achieve coordinated circumnavigation formation.

The work is a generalization of cyclic pursuit strategies studied in Marshall et al. (2004), Zheng, Lin, and Yan (2009) and cyclic repelling strategies studied in Zheng et al. (2013) so that the combination of attraction and repulsion can be used to realize general uniform circumnavigation around a target of interest on different orbits. Moreover, different from the distributed circumnavigation control laws for multiple unicycles developed in Ceccarelli et al. (2008) and Lan et al. (2010) that are switching over time, the control law in this paper is time-invariant and continuous. In addition, it uses only local information of relative positions and is simple and easily implementable, which is important from the engineering standpoint.

2. Problem formulation

Consider a team of N autonomous unicycle-like agents moving in the plane $\mathbb{R}^2$ and suppose in the plane there is a point-like stationary target or beacon $\mathcal{T}$. Our objective is to make all agents asymptotically converge to a formation while navigating on concentric orbits centered at the target. The task should be carried out based on locally available information which can be obtained by individual agents through onboard sensors, e.g., an omnidirectional camera.

Denote $\mathbb{N} = \{1, 2, \dots, N\}$. The kinematic model of the unicycle-like agent is described as follows subject to nonholonomic constraints. For each agent $i \in \mathbb{N}$,

$$\dot{x}_i = v_i \cos\theta_i, \qquad \dot{y}_i = v_i \sin\theta_i, \qquad \dot{\theta}_i = \omega_i, \tag{1}$$

where (x_i, y_i) is the Cartesian coordinate of the ith agent in the world frame and θ_i gives the orientation of agent i, also in the world frame. The longitudinal velocity v_i and angular velocity ω_i are the control inputs which we need to design.

To accomplish the mission, we make the following assumptions:

(1) The agents interact each other forming a directed ring with minimum information exchange. That is, agent i detects agent $(i \mod N) + 1$ in its local frame. Throughout the paper, modulo N operation is used to identify agents, i.e., agent $N + i$ is the same as agent i.
(2) The global posture information (x_i, y_i, θ_i) is unavailable. The difference of orientations $\theta_j - \theta_i$ is also unavailable. Each agent i can only measure the relative position $\mu_b^{(i)}$ of the target and the relative position $\mu_+^{(i)}$ of agent $i + 1$ in its own local frame.
(3) Each agent i knows the predefined radius r_i of its circular orbit and the one for its neighbor r_{i+1}, obtained for example, through local communication.

Remark 1. For simplicity, in the following analysis, it is assumed that the target can be detected by all the agents all the time. This assumption, however, can be relaxed in practice. For example, when an agent cannot detect the target, it can just pursue its neighboring agent. If some of the agents detect the target, then eventually all the agents can be steered to the vicinity of the target. In that case, it is reasonable to assume that all the agents can detect the target simultaneously.

Denote by (x_b, y_b) the coordinate of the target in the world frame. The relationship between the relative measurements in local frames and the global coordinate in the world frame is given as

$$\mu_b^{(i)} = R(\theta_i)\begin{bmatrix} x_b - x_i \\ y_b - y_i \end{bmatrix}, \qquad \mu_+^{(i)} = R(\theta_i)\begin{bmatrix} x_{i+1} - x_i \\ y_{i+1} - y_i \end{bmatrix},$$

where $R(\theta_i) = \begin{bmatrix} \cos\theta_i & \sin\theta_i \\ -\sin\theta_i & \cos\theta_i \end{bmatrix}$ is the rotation matrix. We define $\psi_i := \text{atan2}(y_{i+1} - y_b, x_{i+1} - x_b) - \text{atan2}(y_i - y_b, x_i - x_b)$ and also let $\psi_i \in [0, 2\pi)$.[3]

Now we are ready to give the formal problem statement for the uniform circumnavigation problem studied in the paper.

The uniform circumnavigation problem *For given radii $r_1, r_2, \dots, r_N$, design a local control law of v_i and ω_i for each agent i with model (1) such that*

(1) $\lim_{t\to\infty} \|(x_i, y_i) - (x_b, y_b)\| = r_i,\ i = 1, \dots, N,$
(2) $\lim_{t\to\infty} \omega_1 = \cdots = \lim_{t\to\infty} \omega_N = \bar{\omega},$
(3) $\lim_{t\to\infty} v_1/r_1 = \cdots = \lim_{t\to\infty} v_N/r_N = \bar{v},$
(4) $\lim_{t\to\infty} \psi_1 = \cdots = \lim_{t\to\infty} \psi_N = \bar{\psi},$

where $\bar{v}$ and $\bar{\omega}$ are some constants and $\bar{\psi} = \frac{2d\pi}{N}$ for some $d \in \{1, 2, \dots, N - 1\}$.

Remark 2. In the uniform circumnavigation problem, formations are not unique to satisfy the specifications. In general, there are $N - 1$ possible formations because $\bar{\psi}$ can be any value $\frac{2d\pi}{N}$ for $d \in \{1, \dots, N - 1\}$. See Fig. 1 for an example of all possible four formations of five agents.

[3] The function $\text{atan2}(y, x)$ represents a two-argument arctangent function returning the angle of point (x, y) as a numeric value in $[0, 2\pi)$.