Automatica 53 (2015) 149–154

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

On existence, optimality and asymptotic stability of the Kalman filter with partially observed inputs[☆]

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ARTICLE INFO

Article history:
Received 7 May 2014
Received in revised form
17 September 2014
Accepted 10 December 2014

Keywords:
Asymptotic stability
Existence
Kalman filter
Optimality
Unknown inputs

ABSTRACT

For linear stochastic time-varying systems, we investigate the properties of the Kalman filter with partially observed inputs. We first establish the existence condition of a general linear filter when the unknown inputs are partially observed. Then we examine the optimality of the Kalman filter with partially observed inputs. Finally, on the basis of the established existence condition and optimality result, we investigate asymptotic stability of the filter for the corresponding time-invariant systems. It is shown that the results on existence and asymptotic stability obtained in this paper provide a unified approach to accommodating a variety of filtering scenarios as its special cases, including the classical Kalman filter and state estimation with unknown inputs.

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1. Introduction

State estimation plays an important role in state space modelling and control. It has been applied to a wide range of areas; see Li (2009) and Liang, Chen, and Pan (2010) for some recent applications in network control systems, transportation management, etc.

In the recent decades, state estimation for discrete-time linear stochastic systems with unknown inputs (also termed as unknown input filtering (UIF) problem) has received considerable attention since the original work of Kitanidis (1987) first appeared. Various filters were developed under different assumptions for the systems with unknown inputs; see, e.g., Cheng, Ye, Wang, and Zhou (2009), Darouach and Zasadzinski (1997), Darouach, Zasadzinski, and Xu (1994), Fang and Callafon (2012), Gillijns and De Moor (2007), Hsieh (2000, 2010) and Kitanidis (1987), among many others. Most of these researches used the technique of minimum variance unbiased estimation, hence leading to an unbiased minimum-variance filter (UMVF). Another important research line is state estimation for descriptor systems (Hsieh, 2011, 2013). It has been recently shown in Hsieh (2013) that any linear descriptor systems can be transformed into a linear stochastic system with unknown inputs. This shows a close link between these two kinds of problem. In addition, various properties for these developed filters have been investigated, including the existence condition (Darouach & Zasadzinski, 1997), asymptotic stability (Fang & Callafon, 2012), and global optimality of the UMVF (Cheng et al., 2009 and Hsieh, 2010).

Recently, Li (2013) has developed a Kalman filter for linear systems with partially observed inputs, where the inputs are observed not at the level of interest but rather the input information is available at an aggregate level. It has been shown that the developed filter provides a unified approach to state estimation for linear systems with Gaussian noise. In particular, it includes two important extreme scenarios as its special cases: (a) the filter where all the inputs are completely available (i.e. the classical Kalman filter; see, e.g., Simon, 2006); and (b) the filter where all inputs are unknown (i.e. the filter investigated in Kitanidis, 1987 and many others for the UIF problem). Potentially the proposed filter can be applied to a variety of practical problems in many different areas such as population estimation and traffic control.

So far there is not any study discussing the existence and asymptotic stability issues of this newly proposed unified filter. In this paper we investigate the properties of the Kalman filter with partially observed inputs developed in Li (2013). For linear stochastic time-varying systems with partially observed inputs, we

☆ This work was jointly funded by UK Engineering and Physical Sciences Research Council (EPSRC) and BAE Systems (EP/H501401/1). The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Tongwen Chen under the direction of Editor Ian R. Petersen.

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http://dx.doi.org/10.1016/j.automatica.2014.12.044
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establish the existence condition for a general linear filter. Then we show that the developed filter is optimal in the sense of minimum error covariance matrix. Finally, we consider asymptotic stability of the filter for the corresponding time-invariant systems based on the established existence condition and optimality result.

This paper has provided a unified approach to accommodating existence and asymptotic stability conditions in a variety of filtering scenarios: it includes the results on existence and asymptotic stability for some important filters as its special cases, e.g., the filters developed for the problems where the inputs are completely available and where all the inputs are unknown. Note that the former is the classical Kalman filtering problem and the corresponding existence and asymptotic stability conditions are well established in the literature. For the latter case with unknown inputs, there has been a continuing research interest in existence and asymptotic stability conditions for various discrete-time systems (e.g., Cheng et al., 2009, Darouach & Zasadzinski, 1997, Fang & Callafon, 2012, Kitanidis, 1987) and continuous-time systems (e.g., Bejarano, Floquet, Perruquetti, & Zheng, 2013, Corless & Tu, 1998, Hou & Muller, 1992).

This paper is structured as follows. First, Section 2 is devoted to problem statement. Then we establish the existence condition in Section 3. We focus on the properties of the filter proposed in Li (2013) in Section 4. In Section 5, we investigate asymptotic stability. Finally, this paper concludes in Section 6.

2. Problem statement

Consider a linear stochastic time-varying system:

$$\begin{aligned} x_{k+1} &= A_k x_k + G_k d_k + \omega_k \\ y_k &= C_k x_k + \upsilon_k, \end{aligned} \tag{1}$$

where $x_k \in R^n$ is the state vector, $d_k \in R^m$ is the input vector, and $y_k \in R^p$ is the measurement vector at each time step k with $p \geq m$ and $n \geq m$. The process noise $\omega_k \in R^n$ and the measurement noise $\upsilon_k \in R^p$ are assumed to be mutually uncorrelated with zero-mean and a known covariance matrix, $Q_k = E[\omega_k \omega_k^T] \geq 0$ and $R_k = E[\upsilon_k \upsilon_k^T] > 0$, respectively. A_k, G_k and C_k are known matrices. Without loss of generality, we follow Gillijns and De Moor (2007) and Kitanidis (1987), and assume that G_k has a full column-rank. The initial state x_0 is independent of ω_k and υ_k with a known mean $\hat{x}_0$ and covariance matrix $P_0 > 0$.

We consider the scenario where the input vector d_k is not fully observed at the level of interest but rather it is available only at an aggregate level. Specifically, let D_k be a $q_k \times m$ known matrix with $0 \leq q_k \leq m$ and F_{0k} an orthogonal complement of D_k^T such that $D_k F_{0k} = O_{q_k \times (m-q_k)}$ and $F_{0k}^T F_{0k} = I_{m-q_k}$, where O and I represent the zero matrix and identity matrix of appropriate dimensions. We suppose that the input data is available only on some linear combinations:

$$r_k = D_k d_k, \tag{2}$$

where r_k is available at each time step k. D_k is assumed to have a full row-rank; otherwise the redundant rows can be removed.

As pointed out in Li (2013), the matrix D_k characterizes the availability of input information at each time step k. It includes two extreme scenarios that are usually considered: (a) $q_k = m$ and D_k is an identity matrix, i.e. the complete input information is available; this is case that the classical Kalman filter can be applied; (b) $q_k = 0$, i.e. no information on the input variables is available; this is the problem investigated in Darouach and Zasadzinski (1997), Gillijns and De Moor (2007), Hsieh (2000) and Kitanidis (1987).

Throughout this paper, we use $\lambda(B)$ to denote any eigenvalue of a square matrix B. For any two symmetric matrices A and B with suitable dimensions, the notation $A \geq B$ is used if and only if $A - B$ is non-negative definite. In addition, we use $G_k^{\perp}$ to denote an orthogonal complement of G_k and $\Omega_k = [G_k, G_k^{\perp}]$. Define

$$\Pi_k = \begin{pmatrix} D_{k-1} \\ C_k G_{k-1} \end{pmatrix}. \tag{3}$$

3. Existence condition

To establish the existence condition of a general linear filter for system (1) and (2), we first consider an invertible linear transformation.

3.1. Transformation

Consider the following invertible matrix:

$$M_k = \begin{bmatrix} D_k & O_{q_k \times (n-m)} \\ O_{(n-m)\times m} & I_{n-m} \\ F_{0k}^T & O_{(m-q_k)\times(n-m)} \end{bmatrix} \Omega_k^{-1}.$$

It is straightforward to verify that $M_k G_k d_k$ can be expressed as:

$$\begin{aligned} M_k G_k d_k &= [D_k^T, O_{m\times(n-m)}, F_{0k}]^T d_k \\ &= [(D_k d_k)^T, (O_{(n-m)\times m} d_k)^T, (F_{0k}^T d_k)^T]^T \\ &= [r_k^T, O_{1\times(n-m)}, (F_{0k}^T d_k)^T]^T \\ &= \tilde{r}_k + \tilde{G}_k \delta_k, \end{aligned} \tag{4}$$

where $\tilde{r}_k = [r_k^T, O_{1\times(n-m)}, O_{1\times(m-q_k)}]^T$, $\delta_k = F_{0k}^T d_k$ and $\tilde{G}_k = [O_{(m-q_k)\times q_k}, O_{(m-q_k)\times(n-m)}, I_{m-q_k}]^T$. We note that $\tilde{r}_k$ is completely available due to Eq. (2).

Left-multiplying both sides of Eq. (4) by M_k^{-1}, $G_k d_k$ can be decoupled into two parts:

$$G_k d_k = M_k^{-1} \tilde{r}_k + M_k^{-1} \tilde{G}_k \delta_k. \tag{5}$$

From Eq. (5), the dynamics of x_{k+1} can be rewritten as:

$$\begin{aligned} x_{k+1} &= A_k x_k + M_k^{-1} \tilde{r}_k + M_k^{-1} \tilde{G}_k \delta_k + \omega_k \\ &= A_k x_k + u_k + F_k \delta_k + \omega_k, \end{aligned}$$

where $u_k = M_k^{-1} \tilde{r}_k$ is a known term, and F_k is given by

$$F_k = M_k^{-1} \tilde{G}_k = [G_k, G_k^{\perp}] \begin{bmatrix} F_{0k} \\ O \end{bmatrix} = G_k F_{0k}. \tag{6}$$

Consequently, linear system (1) with the partially observed inputs $r_k = D_k d_k$ can be equivalently represented by the following system:

$$\begin{aligned} x_{k+1} &= A_k x_k + u_k + F_k \delta_k + \omega_k \\ y_k &= C_k x_k + \upsilon_k. \end{aligned} \tag{7}$$

The above manipulation shows that a linear stochastic system with partially observed inputs (2) is equivalent to a linear system with unknown inputs; similar property is also found for linear descriptor systems (Hsieh, 2013).

3.2. Existence condition

In this subsection, we will establish the existence condition of a general, asymptotically stable and unbiased linear filter for system (7) and hence for its equivalent system, Eqs. (1) and (2).

Motivated by the linear filter structure in the literature (e.g. Darouach et al., 1994), we consider a general linear filter for discrete-time linear system (7) of the form

$$\hat{x}_{k+1} = E_k \hat{x}_k + J_k u_k + K_{k+1} y_{k+1}, \tag{8}$$

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