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Brief paper

Restricted dynamic observer error linearizability ☆

Hong-Gi Lee [1], Kyung-Duk Kim, Hong-Tae Jeon

School of Electrical and Electronic Engineering, Chung-Ang University, Dongjak-Ku, Seoul, 156-756, Republic of Korea

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ABSTRACT

Following the general dynamic observer error linearization problem, state equivalence to a triangular nonlinear observer canonical form with index d and the restricted dynamic observer error linearization problem are defined. In this paper, the necessary and sufficient conditions for the above two problems are given. Since our proofs are constructive, a desired state transformation can also be found in the theorem.

1. Introduction

One of the nonlinear observer design methods is to find a new state coordinate (and an output transformation) that transforms the given system into a nonlinear observer canonical form (NOCF), as first suggested in Bestle and Zeitz (1983) and Krener and Isidori (1983). Then, a simple Luenberger-like observer design is feasible for a NOCF, as in Remark 1. We call this problem *observer error linearization* or the state equivalence to a NOCF which has been studied by many researchers in Besancon (1999), Califano, Monaco, and Normand-Cyrot (2003, 2009), Chung and Grizzle (1990), Hou and Pugh (1999), Huijberts (1999), Keller (1987), Krener and Respondek (1985), Lee, Arapostathis, and Marcus (2008), Lee and Hong (2011), Lee and Nam (1991), Lin and Byrnes (1995), Xia and Gao (1988) and Xia and Gao (1989). The observer error linearization problem is a sort of dual concept of the feedback linearization problem. Thus, following the dual concept of the dynamic feedback linearization problem, the dynamic observer error linearization problem can also be defined to enlarge the class of the nonlinear systems for Luenberger-like observer designs. Now we define the dynamic observer error linearization problem and then we give the recent developments in the research of this area.

Consider an autonomous system of the form

$$\dot{x}(t) = F\big(x(t)\big); \qquad y(t) = h(x(t)) \tag{1}$$

with $f(0) = 0$, $h(0) = 0$, state $x \in \mathbb{R}^n$, and output $y \in \mathbb{R}$. Throughout the paper, we assume the observability rank condition on the neighborhood of the origin:

$$\dim \operatorname{span}\{dh(x), dL_F h(x), \dots, dL_F^{n-1}h(x)\} = n. \tag{2}$$

Thus, we can assume, without loss of generality, that

$$F(x) = [x_2 \quad \cdots \quad x_n \quad \alpha(x)]^{\mathsf{T}}; \qquad h(x) = x_1. \tag{3}$$

Definition 1. System (1) is said to be state equivalent to a triangular nonlinear observer canonical form (TNOCF) with index d, if there exists a smooth diffeomorphism $S : V_0 \to \mathbb{R}^n$, defined on some neighborhood of the origin $V_0 \subset \mathbb{R}^n$, which transforms (1), in the variable $z = S(x)$, to

$$\dot{z} = A_0 z + \gamma(z_1, \dots, z_{d+1}); \qquad y = C_0 z \tag{4}$$

where $\gamma : \mathbb{R}^{d+1} \to \mathbb{R}^n$ is a smooth function with $\frac{\partial \gamma_i}{\partial z_j} = 0$ for $j > i+1$, $C_0 = \begin{bmatrix} 1 & O_{1\times(n-1)} \end{bmatrix}$, and $A_0 = \begin{bmatrix} O_{(n-1)\times 1} & I_{(n-1)\times(n-1)} \\ 0 & O_{1\times(n-1)} \end{bmatrix}$.

TNOCF (4) is useful for a Luenberger-like observer design, when $\{\dot{y}, \ddot{y}, \dots, y^{(d)}\}$ are available in addition to y. Following Back, Yu, and Seo (2006), we define the restricted dynamic observer problem as follows.

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E-mail addresses: hglee@cau.ac.kr (H.-G. Lee), perfumere2@gmail.com (K.-D. Kim), htjeon@cau.ac.kr (H.-T. Jeon).

[1] Tel.: +82 2 820 5317; fax: +82 2 817 0292.

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Definition 2. System (1) is said to be restricted dynamic observer error linearizable (RDOEL) with index d, if there exists the restricted dynamic system with index d (called auxiliary dynamics)

$$\dot{w}_i = \begin{cases} w_{i+1}, & 1 \le i \le d-1 \\ y, & i = d \end{cases} \triangleq p(w, y) \tag{5}$$

such that the extended system

$$\dot{\xi} \triangleq \begin{bmatrix} \dot{w} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} p\big(w, h(x)\big) \\ F(x) \end{bmatrix} \triangleq f(\xi) \tag{6}$$
$$y_a = w_1$$

is state equivalent to a TNOCF with index d.

System (1) is said to be RDOEL, if system (1) is RDOEL with some index d. If we use a general nonlinear dynamic system $\dot{w} = \bar{p}(w, y)$ in Definition 2, system (1) is said to be DOEL with index d.

Remark 1. If system (1) is RDOEL with index d, then there exists a smooth diffeomorphism $S : V_0 \to \mathbb{R}^n \times \mathbb{R}^d$, defined on some neighborhood of the origin $V_0 \subset \mathbb{R}^n \times \mathbb{R}^d$, which transforms (6), in the variable $z = S(x, w) = S(\xi)$, to a TNOCF with index d given by

$$\dot{z} = Az + \gamma(z_1, \ldots, z_{d+1}); \qquad y_a = Cz \tag{7}$$

where $\gamma : \mathbb{R}^{d+1} \to \mathbb{R}^{n+d}$ is a smooth vector function, $C = \begin{bmatrix} 1 & O_{1\times(n+d-1)} \end{bmatrix}$, and $A = \begin{bmatrix} O_{(n+d-1)\times 1} & I_{(n+d-1)} \\ 0 & O_{1\times(n+d-1)} \end{bmatrix}$. Thus, choosing $L \in \mathbb{R}^{(n+d)\times 1}$ such that $(A - LC)$ is Hurwitz, we can design a state estimator

$$\begin{aligned} &\dot{w} = p(w, y), \quad w \in \mathbb{R}^d \\ &\dot{\hat{z}} = (A - LC)\hat{z} + \gamma(w, y) + Lw_1, \quad \hat{z} \in \mathbb{R}^{n+d} \\ &\hat{\xi} \triangleq \begin{bmatrix} \hat{w}^{\mathsf{T}} & \hat{x}^{\mathsf{T}} \end{bmatrix}^{\mathsf{T}} = S^{-1}(\hat{z}) \end{aligned} \tag{8}$$

that yields an asymptotically vanishing error, i.e., $\lim_{t\to\infty} \|\xi(t) - \hat{\xi}(t)\| = 0$ or $\lim_{t\to\infty} \|x(t) - \hat{x}(t)\| = 0$.

An observer for system (1) using dynamic observer error linearization is system (5) and system (8). Thus, the dimension of the observer state is $n + 2d$. Another interesting observer design problem with a dynamic compensator, using immersion instead of the dual concept of dynamic feedback linearization compensator, can also found in Back and Seo (2006), Jouan (2003) and Levine and Marino (1986). The relation between these two problems can be found in Section 3 of Back et al. (2006).

The RDOEL problem has been studied in Lee and Hong (2012) and Noh, Jo, and Seo (2004) with using w_1 only in the observer design. But, the observer design in Definition 2, which is first suggested in Back et al. (2006), is more powerful, since w and y are used. The sufficient problems of RDOEL problem have been investigated by Back et al. (2006) and Boutat and Busawon (2011). Very recently, Califano and Moog (2014) have found the necessary and sufficient conditions for system (1) to be DOEL by using the multi-output scheme $y_a = \begin{bmatrix} w_1 & \cdots & w_d & y \end{bmatrix}^{\mathsf{T}}$ in Definition 2. The purpose of this paper is to obtain the necessary and sufficient conditions for system (1) to be RDOEL with the standard single output scheme in Definition 2. As we see in Example 4, our conditions cannot be applied for (general) DOEL problem, while the conditions of Califano and Moog (2014) can be. If system is RDOEL, then our solution is different from the one of Califano and Moog (2014), as mentioned in Remark 4. Our conditions for RDOEL are, as in the Appendix, simply implemented by a MATLAB programming, because the conditions of Theorem 1 contain only the Lie brackets of the vector fields. Finally, we obtain the necessary and sufficient conditions for the state equivalence to a TNOCF with index d.

2. Preliminaries

In this section, we derive the necessary conditions for the dynamic observer error linearizability with index d, which are used in the proof of the main theorem in the next section. We refer the reader to Isidori (1995), Marino and Tomei (1995) and Nijmeijer and van der Schaft (1990) for the basic definitions and results in nonlinear systems and differential geometry. Suppose that system (1) is RDOEL with index d. Then the extended system

$$\dot{\xi} = \begin{bmatrix} \xi_2 & \cdots & \xi_{n+d} & \bar{\alpha}(\xi) \end{bmatrix}^{\mathsf{T}} = f(\xi); \qquad y_a = \xi_1 \tag{9}$$

is state equivalent, via state transformation $z = S(\xi)$, to

$$\begin{aligned} &\dot{z} = \bar{f}(z); \qquad y_a = z_1 \\ &\bar{f}_i = z_{i+1} + \gamma_i(z_1, \ldots, z_{i+1}), \quad 1 \le i \le d \\ &\bar{f}_i = z_{i+1} + \gamma_i(z_1, \ldots, z_{d+1}), \quad d+1 \le i \le n+d-1 \\ &\bar{f}_{n+d} = \gamma_{n+d}(z_1, \ldots, z_{d+1}) \end{aligned} \tag{10}$$

where $[\xi_1 \ \xi_2 \ \xi_3 \ \cdots \ \xi_{n+d}] = [w_1 \ \cdots \ w_d \ x_1 \ \cdots \ x_n]$ and $\bar{\alpha}(\xi) = \alpha(\xi_{d+1}, \ldots, \xi_{n+d})$.

Lemma 1. *If system (9) is state equivalent to TNOCF (10), then system (9) is state equivalent to TNOCF (10) with*

$$\gamma_i(\xi_1, \ldots, \xi_{i+1}) = 0, \quad 1 \le i \le d-1. \tag{11}$$

Proof. Suppose that system (9) is state equivalent to TNOCF (10) via $z = S(\xi)$. Let $\bar{z} = \bar{T}(z)$, where $\bar{z}_1 = z_1, \bar{z}_i = z_i + \gamma_{i-1}$, $2 \le i \le d$, and $\bar{z}_i = z_i$, $d+1 \le i \le n+d$. Then we have

$$\begin{aligned} &\dot{\bar{z}}_i = \bar{z}_{i+1}, \quad 1 \le i \le d-1 \\ &\dot{\bar{z}}_i = \bar{z}_{i+1} + \tilde{\gamma}_{i+1}(\bar{z}_1, \ldots, \bar{z}_{d+1}), \quad 1 \le i \le d-1 \\ &\dot{\bar{z}}_{n+d} = \tilde{\gamma}_{n+d}(\bar{z}_1, \ldots, \bar{z}_{d+1}) \\ &y_a = \bar{z}_1 \end{aligned} \tag{12}$$

where, with a slight abuse of notation, for $d \le i \le n+d$, $\tilde{\gamma}_i \triangleq \gamma_i(\bar{z}_1, z_2(\bar{z}_1, \bar{z}_2), \ldots, z_d(\bar{z}_1, \ldots, \bar{z}_d), \bar{z}_{d+1})$. Therefore, system (9) is state equivalent to TNOCF (12) via $\bar{z} = \bar{T} \circ S(\xi)$.

Lemma 2. *System (9) is state equivalent to TNOCF (10), if and only if*

$$\bar{\alpha}(\xi) = \sum_{i=d}^{n+d} L_f^{n+d-i} \bar{\gamma}_i(\xi_1, \ldots, \xi_{d+1}) \tag{13}$$

where, with a slight abuse of notation, for $d \le i \le n+d$,

$$\bar{\gamma}_i(\xi_1, \ldots, \xi_{d+1}) \triangleq \gamma_i(\xi_1, \ldots, \xi_d, z_{d+1}(\xi_1, \ldots, \xi_{d+1})). \tag{14}$$

Furthermore, the state transformation $z = S(\xi)$ satisfies

$$\begin{aligned} &z_i = \xi_i, \quad 1 \le i \le d \\ &z_{d+i} = \xi_{d+i} - \sum_{k=0}^{i-1} L_f^{i-1-k} \bar{\gamma}_{d+k}, \quad 1 \le i \le n \end{aligned} \tag{15}$$

or

$$\begin{aligned} &\xi_i = z_i, \quad 1 \le i \le d \\ &\xi_{d+i} = z_{d+i} + \sum_{k=0}^{i-1} L_{\bar{f}}^{i-1-k} \gamma_{d+k}, \quad 1 \le i \le n. \end{aligned} \tag{16}$$

Proof. Obvious.

For a vector field τ_1, define $\tau_2 = ad_f \tau_1 = \frac{\partial \tau_1}{\partial \xi} f - \frac{\partial f}{\partial \xi} \tau_1$, and $\tau_i = ad_f^{i-1} \tau_1 = ad_f(ad_f^{i-2} \tau_1)$, $i \ge 3$.