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Brief paper

The pole assignment for the regular triangular decoupling problem ☆

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ABSTRACT

In this paper, by applying the canonical decomposition of the right invertible system $\{C, A, B\}$ obtained in Wei, Cheng and Wang (2010), we derive all explicit solutions of the regular triangular decoupling problem, and then characterize all attainable transfer function matrices for the decoupling and pole assignment problem.

1. Introduction

Consider the system $\{C, A, B\}$ described by

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t), \\ y(t) = Cx(t), \end{cases} \tag{1}$$

where $x(t)$ is an n-dimensional state vector, $u(t)$ is an p-dimensional input vector, $y(t)$ is an m-dimensional output vector, and the related closed-loop transfer matrix function is

$$T(s) \overset{\text{def}}{=} C(sI_n - A)^{-1}B.$$

For the control system (1), we associate with the following state feedback control rule

$$u(t) = Fx(t) + Gv(t), \tag{2}$$

where $v(t)$ is a new m-dimensional input, and the related closed-loop transfer matrix function is

$$T_{F,G}(s) \overset{\text{def}}{=} C(sI_n - A - BF)^{-1}BG. \tag{3}$$

The triangular decoupling problem (TDP) via state variable feedback can be stated as follows: Find matrices F and G, so that the system in (1) together with the state feedback law (2) results in a closed-loop transfer function (3), which has a nonsingular lower triangular form up to row permutation.

Compared with the row-by-row decoupling problem (RRDP), the TDP requires less restrictive conditions. It can be shown that when a system may be row-by-row decoupled, it may also be put in a triangular form (the converse statement is false). In many cases the row-by-row decoupling controller yields more prohibitive efforts on the control variables than those obtained with a triangular one. So if no particular specifications are imposed, we would prefer to choose the latter. On the other hand, even if a system can be row-by-row decoupled, dynamic extension may be required for either realization or stabilization. Since for such systems a stable triangular structure can always be achieved without additional dynamics, it is clear that a triangular decoupling controller is still competitive.

The TDP was first formulated and solved by Morse and Wonham in Morse and Wonham (1970), their approach emphasized particularly geometric characterizations. In their paper, they gave a geometric formulation of the state feedback triangular decoupling problem and presented necessary and sufficient conditions for the existence of decoupling matrices. Under the conditions that the system was completely controllable, they outlined a procedure for simultaneously realizing a triangular structure and assigning the poles of the closed-loop system transfer function matrix. Then in Desusse and Lizarzaburu (1979), Descusse and Lizarzaburu were concerned with the following system:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t), \\ y(t) = Cx(t) + Du(t). \end{cases}$$

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In their paper, (A, B) and (A, B, C, D) were assumed to be controllable and output controllable, respectively. They proposed another approach, based on the structure algorithm of Silverman and Payne (1971), which was a completely general but algebraic one, and was easily tractable on computers. They were concerned with an algebra formulation of the state feedback TDP in the case $D \neq 0$. They presented necessary and sufficient conditions for the existence of decoupling matrices for an (A, B, C, D) quadruple and outlined a procedure for simultaneously realizing a triangular structure and assigning any desired spectrum to the closed loop system. Their main interest was that it was suited for computer implementation, which was not the case with a geometric approach.

The TDP for systems over principal ideal domains or in Hilbert spaces were discussed in Inaba and Otsuka (1989) and Ito and Inaba (1997), respectively, which were a natural generalization of systems over the field of real numbers. In Mutoh and Nikiforuk (1992), Mutoh and Nikiforuk presented a simple method on how to generate the state feedback which made the closed-loop transfer matrix the inverse of the interactor matrix. They also considered an arbitrary pole assignment which preserved the lower left triangular form of the inverse of the interactor matrix under the condition that (A, B, C) was completely controllable and observable.

Recently, numerical reliable methods were proposed for various problems related to the system decoupling problems in Chu and Mehrmann (2001), Chu and Tan (2002a) and Chu and Tan (2002b). Furthermore, in Chu and Tan (2002b), Chu et al. obtained new solvability conditions and parameterized all the solutions for the regular TDP based on a condensed form. Triangular decoupling problem with stability (TDPS) was also discussed in Chu and Tan (2002b).

For the right invertible system (that is, $\text{rank}_g[C(sI - A)^{-1}B] = m$) (1), if $p > m$, (1) is called a non-regular system; if $p = m$, (1) is called a regular system, which was also an invertible system. In Wei, Cheng, and Wang (2010), Wei, Cheng and Wang (2010) proposed a canonical decomposition of the right invertible system $\{C, A, B\}$. By this canonical decomposition, they studied the Smith form of the matrix pencil $P(s) = \begin{pmatrix} A - sI & B \\ C & 0 \end{pmatrix}$, the range of the ranks of $P(s)$ for $s \in \mathbb{C}$ and the invariant quantities of the right invertible system $\{C, A, B\}$. In another article (Wei, Wang & Cheng, 2010), Wei, Wang, Cheng studied the necessary and sufficient conditions of the regular RRDP and the non-regular TDP. It turned out that this canonical decomposition of the right invertible system $\{C, A, B\}$ was suitable for studying problems involving all three matrices A, B, C. Recently in Wei and Shen (2013), we deduced equivalent conditions of controllability, stability and observability of the right invertible system $\{C, A, B\}$. We also derived three equivalent sufficient solvability conditions of the non-regular RRDP, which made the base of our further studying the non-regular RRDP. In our another article (Shen & Wei, 2013), we applied this canonical decomposition to derive a general formula of all solutions to the regular RRDP. Based on this formula we characterized all attainable transfer function matrices for the decoupling and pole assignment problem in general cases.

In this paper we are concerned with the regular TDP. Based on the canonical decompositions of the right invertible system, we not only derive all explicit expressions of regular TDP up to row permutation but also completely describe the closed-loop structure of a decoupling system. The set of assignable poles as well as the set of fixed decoupling poles are determined in a more general case without any restricted condition. So the regular TDPS in Chu and Tan (2002b) can be solved directly.

The paper is organized as follows. In Section 2, we provide some preliminary results; in Section 3, we derive all solutions of the regular TDP; in Section 4, we study the decoupling and pole assignment for the regular TDP; in Section 5, we provide some numerical examples; and finally in Section 6, we conclude this paper with some remarks.

2. Preliminaries

Throughout this paper, we use the following notation. $\mathbb{R}^{m\times n}$ ($\mathbb{C}^{m\times n}$) is the set of $m \times n$ matrices with real (complex) entries, $\mathbb{R}^m$ ($\mathbb{C}^m$) is the set of m-dimensional real (complex) vectors. $\mathbb{R}_r^{m\times n}$ ($\mathbb{C}_r^{m\times n}$) is the subset of $\mathbb{R}^{m\times n}$ ($\mathbb{C}^{m\times n}$), in which every matrix has rank r. I_k denotes the identity matrix of order k, $0_{l\times m}$ the l by m matrix of all zero entries. For a matrix A, A^T, A^H, $\text{rank}(A)$, $\lambda(A)$ and $\mathcal{R}(A)$ are the transpose, conjugate transpose, rank, spectrum and range of A, respectively. $Re(\lambda(A))$ denotes the real part of the eigenvalues of the matrix A. $\text{rank}_g[A(s)]$ denotes the generic rank of a matrix function $A(s)$, i.e. $\text{rank}_g[A(s)] = \max_{s\in\mathbb{C}}\{\text{rank}[A(s)]\}$.

To begin with, we state a result of the canonical decomposition of the right invertible system of Theorem 1.1 in Wei, Cheng et al. (2010).

Theorem 2.1 (Wei, Cheng et al., 2010). *Suppose that $\{C, A, B\}$ is a right invertible system with $A \in \mathbb{C}^{n\times n}$, $B \in \mathbb{C}_p^{n\times p}$, $C \in \mathbb{C}_m^{m\times n}$. Then there exists a parameter $\widehat{s} \in \mathbb{C}$, such that $\text{rank}(A_0) = n$ with $A_0 = A - \widehat{s}I_n$. Furthermore, there exist two invertible matrices $Q \in \mathbb{C}^{n\times n}$, $T \in \mathbb{C}^{p\times p}$, an $m \times m$ permutation matrix P, and a nonnegative integer k called the index of the system, such that when $k = 0$,*

$$C_m \equiv PCQ = (I_m, 0), \qquad B_m \equiv Q^{-1}BT = \begin{pmatrix} I_p \\ 0 \end{pmatrix}$$
$$A_m \equiv Q^{-1}A_0Q = \left(\begin{array}{cc|c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline D_0^{(1)} & D_1^{(1)} & D_2^{(1)} \end{array}\right)$$

in which A_{00}, A_{11}, $D_2^{(1)}$ are square matrices of order m, $p-m$, $n-p$, respectively; and when $k \geq 1$,

$$C_m \equiv PCQ = \begin{array}{c} \left(\begin{array}{ccc} I_{r_0} & 0 & 0 \\ D_0^{(0)} & 0 & I_{t_0} \end{array}\right)\begin{array}{c} r_0 \\ t_0 \end{array} \\ \begin{array}{ccc} r_0 & n-m & t_0 \end{array} \end{array},$$
$$B_m \equiv Q^{-1}BT = \begin{pmatrix} I_p \\ 0 \end{pmatrix},$$
$$A_m \equiv Q^{-1}A_0Q = \begin{array}{c} \left(\begin{array}{c|c} X_{11} & X_{12} \\ \hline X_{21} & X_{22} \end{array}\right)\begin{array}{c} p \\ n-p \end{array} \\ \begin{array}{cc} p & n-p \end{array} \end{array},$$

in which X_{11}, X_{12}, X_{21} and X_{22} are given in Box I, where

$$r_0 = \text{rank}(CB), \qquad t_0 = m - r_0,$$
$$t_j = t_{j+1} + r_{j+1} > 0 \quad \text{for } j = 0 : k-1, \ t_k = 0,$$
$$m = \sum_{i=0}^{k} r_i, \qquad l = n - p - \sum_{j=0}^{k-1} t_j.$$

When C, A, B are real matrices, we can take $\widehat{s} \notin \lambda(A)$ a real number, and the matrices Q, T, C_m, A_m, B_m are real.

By Theorem 2.1, we know that $D_{k+2}^{(k+1)}$ is an $l \times l$ square invertible matrix. Suppose that $\lambda_1, \ldots, \lambda_q$ are different nonzero eigenvalues of $D_{k+2}^{(k+1)}$, and $D_{k+2}^{(k+1)}$ has a Jordan canonical form

$$D_{k+2}^{(k+1)} \sim J = \text{diag}(J_1(\lambda_1), \ldots, J_q(\lambda_q))$$
$$J_\delta(\lambda_\delta) = \text{diag}(J_{\delta,1}(\lambda_\delta), \ldots, J_{\delta,n_\delta}(\lambda_\delta)) \quad \text{for } \delta = 1 : q, \tag{4}$$

in which $J_{\delta,j}(\lambda_\delta)$ are Jordan blocks,

$$J_{\delta,j}(\lambda_\delta) = (\lambda_\delta) \quad \text{for } \delta_j = 1;$$