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Brief paper

On optimal input design for networked systems☆

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A R T I C L E I N F O

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A B S T R A C T

The topic of this paper is optimal input signal design for identification of interconnected/networked dynamic systems. We consider the case when it is only possible to design some of the input signals, while the rest of the inputs are only measurable. This is most common in industrial applications, where external excitation can only be applied to some subsystems. One example is feed-forward control from measurable disturbances. The optimal input signal will be correlated with the measured signals. The main purpose of this paper is to reveal how to re-formulate the input design problem for networked systems as an input design problem for feedback control systems. We can then use the powerful partial correlation approach for optimal closed loop input design. This means that the corresponding networked optimal input design problem can be formulated as a semi-definite program, for which there are efficient numerical methods. We evaluate this approach using two numerical examples with important applications. The result reveals some non-trivial interesting properties of the optimal input signals.

1. Introduction

System identification is concerned with the estimation and validation of mathematical models of dynamical systems from experimental data. Identification has mainly been studied in a classical open-loop or closed-loop setting with inputs, outputs and unmeasurable disturbances. However, networks of interconnected dynamical systems are becoming more and more important in many fields of engineering. A networked system consists of interconnected subsystems; see Fig. 1 for an example. A common objective of the system identification experiment is to model one subsystem or a part of the networked system. The structure of this problem raises many interesting system identification questions.

The first question is how to determine the structure of the underlying network that generated the measurements. Early work addressing this problem can be found in Caines (1976) and Granger (1969). Some recent work on topology identification of networked systems can be found in Materassi and Innocenti (2010), Sanandaji, Vincent, and Wakin (2011) and Yuan, Stan, Warnick, and Goncalves (2011). If the network is sparsely interconnected, regularization ideas could be applied; see Chiuso and Pillonetto (2012) and Seneviratne and Solo (2012).

In many applications the structure of the network is known but the dynamics of the subsystem building up the network are not. The classical Prediction Error Method (PEM), Ljung (1999), is often applicable in this case. However, identifiability of the subsystems could be an issue. For example, in a cascade of two systems where only the output of the second system is measured, without prior knowledge it is impossible to say which dynamic belongs to which system. In Van den Hof, Dankers, Heuberger, and Bombois (2013) the conditions for consistent identification of closed-loop systems with PEM are extended to the network settings and conditions on the interconnection structure, the presence of noise sources and excitation signals are derived. The follow-up paper (Dankers, Van den Hof, Bombois, & Heuberger, submitted for publication) asks which signals must be included in the predictor model to guarantee consistency. Even if the estimate of a model is consistent, the quality of the identified model could be inadequate. Hence, the next question asks what affects the quality of the identified model. Some initial work in this is Hägg, Wahlberg, and Sandberg (2011) and Wahlberg, Hjalmarsson, and Mårtensson (2009) where the basic building blocks of a networked system, the cascade and the parallel are analyzed. The effects of sensor placement, input signals and common dynamics of the subsystems on the asymptotic properties of the identified models are discussed.

One vital question is how should one design the excitation signal used during the experiment to obtain as much information

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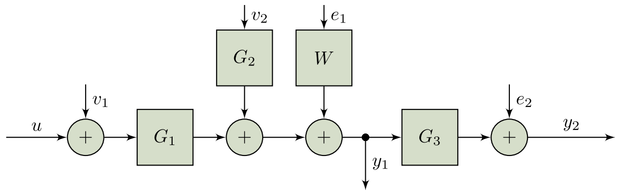

Fig. 1. Example of a network of interconnected subsystems.

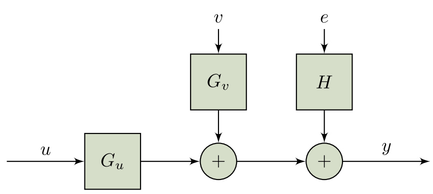

Fig. 2. Considered networked system.

as possible about the subsystems in the network? Here, we will consider the excitation design problem using the Application Oriented Input Design framework, Hjalmarsson (2009), where the system identification experiment is designed such that the identified system satisfies some application requirements.

However, some special attention is required to apply this framework to networked systems. For example, we need to take into consideration that although some of the signals are measurable we cannot excite them, something that is inherent in many industrial applications. In this case it is possible to correlate the input with the measured disturbances or, equivalently, feed-forward the disturbance to the input. In this paper we will show how to formulate a large class of networked input design problems in this framework and how to formulate the problem as a semi-definite program (SDP) that can be solved efficiently using numerical methods. This paper is a generalization of the preliminary results in Hägg and Wahlberg (2013) where optimal input design for feed-forward control was studied.

The main contributions of this paper include:

- Develop a framework for optimal input design for networked systems, particularly for systems where some of the input signals are measurable but cannot be designed. Furthermore we show how to formulate the optimization problem as a semi-definite program.
- Show how to use the framework in two simulation examples with practical applicability which reveals some interesting properties of the optimal input signals.

2. Networked systems

Consider a networked system in Fig. 2 on the form

$$y(t) = G_u(q)u(t) + G_v(q)v(t) + H_0(q)e(t), \qquad (1)$$

where $u(t) = [u_1(t) \ \cdots \ u_p(t)]^T$ is the input vector, $v(t) = [v_1(t) \ \cdots \ v_r(t)]$is the vector of *measurable or known* disturbances, $e(t) = [e_1(t) \ \cdots \ e_m(t)]^T$ the *unmeasurable* disturbances and $y(t) = [y_1(t) \ \cdots \ y_m(t)]^T$ are the measured outputs. The measurement noise is assumed to be zero mean white noise with covariance $\operatorname{diag}(\lambda_1, \ldots, \lambda_m)$ while the measurable disturbances are modeled as stationary stochastic processes with known spectral properties, *i.e.,* $v(t)$ can be written as $v(t) = M(q)s(t)$, where $s(t)$ is a zero mean Gaussian process with covariance Σ_s. The spectrum of $v(t)$ can hence be written as $\Phi_v(\omega) = M(e^{j\omega})\Sigma_s M(e^{-j\omega})^T$. Furthermore we assume that the disturbances are independent of the measurement noise $e(t)$.

For ease of notation, we will sometimes omit the time or frequency argument when there is no risk of confusion.

We will now give two examples to show that systems of the form (1) can represent a quite broad class of networked systems.

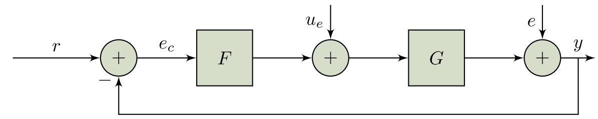

Fig. 3. Feedback with excitation.

2.1. Networked system

The networked system in Fig. 1 can be written on the form (1) as

$$y = \begin{bmatrix} G_1 \\ G_1G_3 \end{bmatrix} u + \begin{bmatrix} G_1 & G_2 \\ G_1G_3 & G_2G_3 \end{bmatrix} v + \begin{bmatrix} W & 0 \\ G_3W & 1 \end{bmatrix} e.$$

Here we can only design one of the inputs, namely u, while v_1 and v_2 are given from the application. This is a common setup in many industrial problems. The goal could be to identify the subsystems $G_1(q)$ and $G_3(q)$ for a subsequent control design. The question is then how should we excite the system so that this identification is as good as possible, taking into account the known properties and the measurements of the disturbances v?

2.2. Reference feed-forward

Consider the feedback system in Fig. 3.

The system operates in a closed loop with a given controller $F(q)$ and a given reference signal $r(t)$. For example, we want to estimate the system $G(q)$ to re-tune the controller $F(q)$. During the system identification experiment we can excite the system by u_e while still running the system in a closed loop with a given reference, so that a minimum of production is lost during the experiment. The objective could then be to minimize the output variance during the experiment, *i.e.,* to disturb the process as little as possible during the experiment while the identified model satisfies some quality constraints. Since the reference $r(t)$ is known we can correlate the input u_e with r using, for example, a feed-forward controller. The closed-loop system can be written on the form (1) as

$$y(t) = \frac{G}{1+GF}u_e(t) + \frac{GF}{1+GF}r(t) + \frac{1}{1+GF}e(t). \qquad (2)$$

3. System identification

The goal is to identify the dynamics of the subsystems in the network using a Prediction Error Method (Ljung, 1999).

The model set is given by $G_u(q,\theta)$, $G_v(q,\theta)$, $H(q,\theta)$, where $\theta \in R^n$ is the model parameter vector that we want to estimate and we assume that the true system can be described by the model with a parameter vector denoted by θ_0. Furthermore, we will assume that the identifiability conditions in Van den Hof et al. (2013) are satisfied so that the system is identifiable and can be consistently identified.

The optimal one step ahead predictor of the model is given by

$$\hat{y}(t,\theta) = H^{-1}(q,\theta)\begin{bmatrix} G_u(q,\theta) & G_v(q,\theta) \end{bmatrix} \begin{bmatrix} u(t) \\ v(t) \end{bmatrix} + \left[I - H^{-1}(q,\theta)\right] y(t);$$

see Ljung (1999) for details. The idea of the prediction error method is to find an estimate of θ such that the error between the prediction and the measured data is as small as possible.

The model parameter vector estimated with a Prediction Error Method from N data points of the inputs, measurable disturbances and outputs is given by

$$\hat{\theta}_N = \arg\min_{\theta} \det \left[\frac{1}{N} \sum_{t=1}^{N} \left(y(t) - \hat{y}(t)\right)^T \left(y(t) - \hat{y}(t)\right) \right].$$