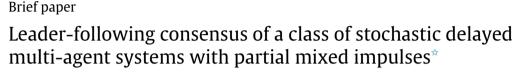
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## Yang Tang<sup>a,b,c,1</sup>, Huijun Gao<sup>d,e</sup>, Wenbing Zhang<sup>f</sup>, Jürgen Kurths<sup>b,c,g</sup>

<sup>a</sup> The Key Laboratory of Advanced Control and Optimization for Chemical Processes, Ministry of Education,

East China University of Science and Technology, Shanghai 200237, China

<sup>b</sup> Institute of Physics, Humboldt University of Berlin, Berlin 12489, Germany

<sup>c</sup> Potsdam Institute for Climate Impact Research, Potsdam 14473, Germany

<sup>d</sup> Research Institute of Intelligent Control and Systems, Harbin Institute of Technology, Harbin 150080, China

<sup>e</sup> King Abdulaziz University, Jeddah 21589, Saudi Arabia

<sup>f</sup> Department of Mathematics, Yangzhou University, Jiangsu 225002, China

g Department of Control Theory, Nizhny Novgorod State University, Gagarin Avenue 23, Nizhny Novgorod 606950, Russia

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#### ABSTRACT

In this paper, the exponential leader-following consensus problem is investigated for a class of nonlinear stochastic networked multi-agent systems with partial mixed impulses and unknown time-varying but bounded delays. The main feature of partial mixed impulses is that time-varying impulses are not only composed of synchronizing and desynchronizing impulses simultaneously but they are also injected into a fraction of nodes in multi-agent systems. Three kinds of partial mixed impulses are proposed: fixed partial mixed impulses, periodic partial mixed impulses, and try-once-discard-like partial mixed impulses. By means of the Lyapunov function theory and the comparison principle, conditions are derived for ensuring global exponential leader-following consensus under the presented three kinds of partial mixed impulses. Simulations of leader-following consensus of robotic systems are provided to validate the effectiveness of the proposed results and to show the advantages of the proposed partial mixed impulses.

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### 1. Introduction

Coordination, as a very important topic in collective cooperative motion, which is one of the most common and spectacular manifestation of coordinated behavior in nature and plays an important role in various contexts, such as biological networks, power networks, transportation networks, climate networks, social networks and technical networks (Jadbabaie, Lin, & Morse,

<sup>1</sup> Tel.: +49 33128820768; fax: +49 30209399188.

http://dx.doi.org/10.1016/j.automatica.2015.01.008 0005-1098/© 2015 Elsevier Ltd. All rights reserved. 2003; Ren & Cao, 2011; Tang, Qian, Gao, & Kurths, 2014; Wielanda, Sepulchre, & Allgöwer, 2011). In modeling complex networks and multi-agent systems with self-dynamics, inherent time-delays (Gao, Chen, & Lam, 2008; Hespanha, Naghshtabrizi, & Xu, 2007; Saber, Fax, & Murray, 2007; Tang, Gao, & Kurths, 2014) and stochastic disturbances (Lu, Kurths, Cao, Mahdavi, & Huang, 2012; Tang, Qian et al., 2014) are widely observed in implementations of electronic networks, and genetic regulatory networks and injections of control inputs in large-scale networked systems. On the other hand, the states of various dynamical networks and/or multi-agent systems such as communication networks, large-scale chemical process networks and biological networks often suffer from instantaneous disturbances and undergo abrupt changes at certain instants, which may arise from switching phenomena, control requirements or frequency change, i.e., systems exhibit impulsive effects including stabilizing and destabilizing effects (Hespanha, Liberzon, & Teel, 2008; Liu & Hill, 2011; Teel, Subbaramana, & Sferlazza, 2014; Yang, 2001; Yang & Xu, 2007; Zhang, Tang, Miao, & Du, 2013).

In recent years, pinning control, like leader-following consensus or controllability (Tang, Qian et al., 2014), has sparked interests



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*E-mail addresses*: tangtany@gmail.com, yangtangde@126.com (Y. Tang), hjgao@hit.edu.cn (H. Gao), zwb850506@126.com (W. Zhang), Juergen.Kurths@pik-potsdam.de (J. Kurths).

of many researchers, since there exists a common requirement to regulate the behavior of large ensembles of interacting units from engineering, social and biological systems by a rather small effort (Chen, Liu, & Lu, 2007; Hong, Hu, & Gao, 2006; Liu, Lu, & Chen, 2013; Lu et al., 2012; Ren & Cao, 2011). Pinning control has been investigated by utilizing various control techniques, such as state feedback control (Chen et al., 2007), adaptive control (Chen et al., 2007; Tang, Gao et al., 2014; Tang, Qian et al., 2014) and impulsive control (Liu et al., 2013; Lu et al., 2012). Unfortunately, up to now, coordination results for networked multi-agent systems or dynamical networks with both synchronizing and desynchronizing impulses, in which only a part of the nodes experience impulsive effects, have been widely overlooked in the literature primarily due to the difficulty in a mathematical derivation. This remains an important challenge in modeling time-varying properties of networked multi-agent systems and its generality of including the impulsive pinning strategy as a special case.

Actually, for a class of nonlinear stochastic networked multiagent systems with partial time-varying impulses and unknown time-varying delays, it is theoretically challenging and practically difficult to establish easy-to-verify criteria for ensuring consensus. The inherent features of the leader-following consensus (tracking control) problem for stochastic delayed networked multi-agent systems with partial mixed impulses pose some fundamental difficulties: (1) How can we properly define partial timevarying/mixed impulses, in which the time-varying features reside in two aspects: only a fraction of nodes are subjected to impulses and the set of nodes injected with impulses is also time-varying? (2) Is it possible for us to establish a connection of partial mixed impulses with networked-induced constraints in networked control systems such as time-varying sampling intervals and competition of multiple nodes (Goebel, Sanfelice, & Teel, 2009; Nešić & Teel, 2004; Zhang, Gao, & Kaynak, 2013)? (3) How can we develop an effective technique to obtain mathematically verifiable leader-following consensus criteria and quantify consensus regions against such kind of partial mixed impulses? The answers to these questions may well explain why the tracking control problem for networked multi-agent systems with or without partial mixed impulses is still open.

Motivated by the above discussion, three kinds of partial mixed impulses are proposed and studied here, i.e., fixed partial mixed impulses, periodic partial mixed impulses, and try-once-discardlike partial mixed impulses are presented and discussed in detail. Based on the proposed three types of partial mixed impulses, the global mean square leader-following consensus problem is investigated for a class of stochastic networked multi-agent systems with unknown time-varying delays. Compared with the works of impulsive effects in uncoupled dynamical systems (Liu, Shen, & Zhang, 2011; Yang & Xu, 2007), complex networks (Guan, Liu, Feng, & Wang, 2010; Liu & Hill, 2011; Liu et al., 2013; Lu et al., 2012; Zhang, Tang et al., 2013) and networked control systems (Goebel et al., 2009; Nešić & Teel, 2004; Zhang, Gao et al., 2013), the main contributions of this paper are mainly threefold: (1) a novel concept of partial mixed impulses is proposed for the first time, which can encompass several well-known impulses (Guan et al., 2010; Liu & Hill, 2011; Liu et al., 2011; Lu et al., 2012; Yang & Xu, 2007); (2) we establish a connection between the proposed impulses and networked-induced constraints like time-varying sampling intervals and competition of multiple nodes in networked control systems (Goebel et al., 2009; Nešić & Teel, 2004; Zhang, Gao et al., 2013); and (3) three different kinds of partial mixed impulses are investigated and compared; meanwhile, effects of systems' parameters on the size of consensus regions are characterized. This paper is organized as follows. In Section 2, some preliminaries regarding the model, partial mixed impulses and assumptions are briefly outlined. In Section 3, leader-following consensus conditions are presented by means of the comparison principle. In Section 4, simulations are exploited to show the effectiveness of the obtained results. The conclusions are given in Section 5.

**Notations.** Let  $\mathbb{N}_+ = \{1, 2, 3, \ldots\}$ .  $\|\cdot\|$  is the Euclidean vector norm in  $\mathbb{R}^n$ .  $\lambda_{max}(\cdot)$  is the maximum eigenvalue of a matrix.  $\#\mathcal{D}$ denotes the element number of the finite set  $\mathcal{D}$  composed of the vertices to be controlled. PC(m) denotes the class of piecewise right continuous function  $\varphi : [t_0 - \tau, +\infty) \rightarrow \mathbb{R}^m$  with the norm defined by  $\|\varphi(t)\|_{\tau} = \sup_{-\tau \leq s \leq 0} \|\varphi(t+s)\|$ . For  $x : \mathbb{R}^n \rightarrow \mathbb{R}^n$ , denote  $x(t^+) = \lim_{s \to 0^+} x(t+s)$  and  $x(t^-) = \lim_{s \to 0^-} x(t+s)$ . Let  $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathcal{P})$  be a complete probability space with filtration  $\{\mathcal{F}_t\}_{t\geq 0}$  satisfying the usual conditions (i.e., the filtration contains all  $\mathcal{P}$ -null sets and is right continuous). Denote by  $L^2_{\mathcal{F}_0}([-\tau, 0]; \mathbb{R}^n)$  the family of all  $\mathcal{F}_0$ -measurable  $PC([-\tau, 0]; \mathbb{R}^n)$ valued random variables  $\xi = \{\xi(s) : -\tau \leq s \leq 0\}$  such that  $\sup_{-\tau \leq s \leq 0} \mathbb{E} \|\xi(s)\|^2 < \infty$ , where  $\mathbb{E}\{\cdot\}$  stands for the mathematical expectation operator with respect to a given probability measure  $\mathcal{P}$ .  $A \setminus B$  represents the set difference from set A to set B.

#### 2. Preliminaries

In this section, some preliminaries about the model and necessary assumptions are given. The problem formulation is briefly outlined. Three kinds of partial mixed impulses are proposed.

Consider the following reference state or the leader state:

$$ds(t) = [As(t) + B\tilde{f}_1(s(t), t) + C\tilde{f}_2(s(t - \tau(t)))]dt + \tilde{f}_3(s(t), s(t - \tau(t)), t)dw(t),$$

where A, B and C are constant matrices which are defined on  $\mathbb{R}^{n \times n}$ ;  $\tilde{f}_1(s(t), t) = [\tilde{f}_{11}(s(t), t), \dots, \tilde{f}_{1n}(s(t), t)]^T$  and  $\tilde{f}_2(s(t - \tau(t))) = [\tilde{f}_{21}(s(t - \tau(t))), \dots, \tilde{f}_{2n}(s(t - \tau(t)))]^T$  are continuous nonlinear vector functions;  $\tilde{f}_3(.,.,.)$  is the noise intensity function; w(t) is a scalar Brownian motion defined on  $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathcal{P})$  satisfying  $\mathbb{E}\{dw(t)\} = 0$  and  $\mathbb{E}\{[dw(t)]^2\} = dt$ ;  $\tau(t)$  is an unknown but bounded time-varying delay satisfying  $0 < \tau(t) \leq \tau$ , which is named by internal time-delay.

We consider the following nonlinear networked multi-agent system with stochastic disturbances and unknown time-varying delays, which can be forced to the leader state s(t):

$$dx_{i}(t) = [Ax_{i}(t) + B\tilde{f}_{1}(x_{i}(t), t) + C\tilde{f}_{2}(x_{i}(t - \tau(t)))]dt$$
  
-  $dk_{i}(x_{i}(t) - s(t))dt + d\sum_{j=1}^{N} g_{ij}x_{j}(t)dt$   
+  $\tilde{f}_{3}(x_{i}(t), x_{i}(t - \tau(t)), t)dw(t), \quad i = 1, 2, ..., N, \quad (1)$ 

where  $x_i(t) \in \mathbb{R}^n$  is the state vector of the *i*th node; *d* stands for the control gain;  $L = (g_{ij})_{N \times N}$  is the undirected coupling matrix representing the coupling topology, which is defined as follows: if there is a connection between nodes *i* and *j*  $(i \neq j)$ ,  $g_{ij} = g_{ji} =$ 1 > 0; otherwise  $g_{ij} = g_{ji} = 0$   $(i \neq j)$ . For diagonal elements of *L*,  $g_{ii} = -\sum_{j=1, j\neq i}^{N} g_{ij}$ , i = 1, 2, ..., N. Assume that only one node *j* in the network has the information from the reference state s(t), i.e.,  $k_j = 1, j \in \{1, 2, ..., N\}$  and  $k_i = 0$  for  $i \in$  $\{1, 2, ..., N\} \setminus \{j\}$ . Here, *L* is assumed to be connected. According to (Chen et al., 2007; Hong et al., 2006), all the eigenvalues of  $M = L - K = (m_{ij})_{N \times N}$  are negative, where *K* is a  $N \times N$  diagonal matrix whose *j*th diagonal element is  $k_j$  and the others are zero. For multiple nodes having the information from the reference state s(t), the results in the following still hold. The initial conditions of system (1) are assumed to be  $x_i(t) = \vartheta_i(t), -\tau \leq t \leq$ 0, i = 1, 2, ..., N, where  $\vartheta_i(t) \in L^2_{\mathcal{F}_0}([-\tau, 0], \mathbb{R}^n)$ . In our model, only the internal delay is considered, which is widely observed Download English Version:

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