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Brief paper

Minimum variance unbiased FIR filter for discrete time-variant systems☆

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A B S T R A C T

This paper is concerned with the minimum variance unbiased (MVU) finite impulse response (FIR) filtering problem for linear system described by discrete time-variant state-space models. An MVU FIR filter is derived by minimizing the variance from the unbiased FIR (UFIR) filter. The relationship between the filter gains of MVU FIR, UFIR and optimal FIR (OFIR) filters is derived analytically, and the mean square errors (MSEs) of different FIR filters are compared to provide an insight into the estimation performance. Simulations provided verify that errors in the MVU FIR filter are in between the UFIR and OFIR filters. It is also shown that the MVU FIR filter can offer optimal estimates without a prior knowledge of the initial state, and exhibits better robustness against temporary modeling uncertainties than the Kalman filter.

1. Introduction

State estimation is one of the key problems of interest in control and signal processing. In general, the existing estimation methods can be divided into two classes: estimators with infinite impulse response (IIR) structure implying unlimited memory and the finite impulse response (FIR) ones having limited memory. The best known IIR algorithm is inarguably the Kalman filter (KF), which provides linear least-mean-squares estimates for linear state-space model (Gelb, 1963; Shaked & de Souza, 1995). As it is simple, accurate and fast, KF has played a significant role in various fields, and considerable efforts have been devoted to extending KF to suit specific practical situations. A central premise in the KF theory is that the noise sources are white noise processes having known statistics, and the underlying state-space model is known. It has also been shown that the performance of KF can be deteriorated or even unstable in the presence of poor models. Contrary to the IIR structures, FIR estimators utilize finite measurements over the most recent time interval and have some inherent good engineering features such as bounded input/bounded output (BIBO) stability, robustness against temporary model uncertainties, and round-off errors (Shmaliy, 2010), making it competitive in applications. It was concluded in Jazwinski (1970) that the limited memory filter appears to be the only device to prevent divergence in the system with unbounded perturbation.

In the last three decades, many successful developments of FIR estimators have been achieved under various conditions. In Jazwinski (1968), the maximum likelihood criterion was used to derive a linear optimal FIR (OFIR) filter, while the FIR filters in one and two dimensions were developed by the weighted least square technique in Algazi, Suk, and Rim (1986). Later, the OFIR filter was proposed for discrete time-invariant system in Kwon, Kwon, and Lee (1989), and for continues time-varying systems in Kwon, Lee, and Kwon (1994). Other related results can be found in the works of Ling and Lim (1999), Liu and Liu (1994), Pei and Shyu (1996), Wang (1991), Yuan and Stuller (1994) and Zhu, Ahmad, and Swamy (1994). However, until 1999, when the problem was solved by combining the receding horizon strategy with the KF Kwon, Kim, and Park (1999), the literatures seemed to lack a systematic way of designing the OFIR filter. To this end, the unbiased FIR (UFIR) filter

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was proposed for discrete-time system model in Kwon, Kim, and Han (2002), and a fixed-lag FIR smoother was developed in Kwon, Han, Kwon, and Kwon (2007), where the variance of estimation error was minimized with the unbiased constraint. Quite recently, a real-time UFIR filter ignoring noise statistics was derived and realized iteratively in Shmaliy (2009) and Shmaliy (2011), and then further extended to nonlinear systems using the linearization technique in Shmaliy (2012). In Shmaliy (2008), the p-shift OFIR estimator (smoother, filter and predictor) was obtained for time-invariant state space model. For more details, one can refer to Choi, Han, and Cioffi (2008), Shmaliy (2010), Shmaliy and Manzano (2012) and Simon and Shmaliy (2013).

Although the methods aforementioned provide different FIR-type estimators, some well-recognized solutions such as the minimum variance unbiased (MVU) FIR filter still remain unconsidered. In this paper, we minimize the variance of the UFIR filter provided in Shmaliy (2011), and propose an MVU FIR filter. The remaining part of the paper is organized as follows. In Section 2, we describe the systems and formulate the problem. The MVU FIR filter is derived in Section 3. In Section 4, we compare the MVU FIR filter with the OFIR and UFIR filters analytically. The mean square errors (MSEs) of the different FIR-type approaches including the proposed method are also compared analytically. Simulations are given in Section 5, and concluding remarks are drawn in Section 6.

The following notations are used in this paper: $\mathbb{R}^K$ denotes the K dimensional Euclidean space, $E\{\cdot\}$ represents the statistical expectation, $\mathbf{I}$ refers to an identity matrix of proper dimensions, $\mathrm{tr}(\cdot)$ is the trace operation, and $\mathrm{diag}(a_1 \cdots a_m)$ denotes a diagonal matrix with diagonal elements $a_1, \ldots, a_m$.

2. Problem formulation and preliminaries

Consider a linear discrete time-variant system represented by a state space model as follows:

$$\mathbf{x}_n = \mathbf{A}_n \mathbf{x}_{n-1} + \mathbf{B}_n \mathbf{w}_n, \tag{1}$$

$$\mathbf{y}_n = \mathbf{C}_n \mathbf{x}_n + \mathbf{D}_n \mathbf{v}_n, \tag{2}$$

where n is the discrete time index, $\mathbf{x}_n \in \mathbb{R}^K$ is the state vector, $\mathbf{y}_n \in \mathbb{R}^M$ is the measurement vector, $\mathbf{A}_n \in \mathbb{R}^{K\times K}$, $\mathbf{B}_n \in \mathbb{R}^{K\times P}$, $\mathbf{C}_n \in \mathbb{R}^{M\times K}$ and $\mathbf{D}_n \in \mathbb{R}^{M\times M}$ are known system matrices. The process noise $\mathbf{w}_n \in \mathbb{R}^P$ and measurement noise $\mathbf{v}_n \in \mathbb{R}^M$ are mutually uncorrelated with zero mean, i.e., $E\{\mathbf{w}_n\} = \mathbf{0}$ and $E\{\mathbf{v}_n\} = \mathbf{0}$, and have arbitrary distributions and known covariances.

To derive an FIR filter with the measurements collected from $m = n - N + 1$ to n, where N is the estimation horizon length, we reorganize the state and measurement equations as

$$\mathbf{X}_{n,m} = \mathbf{A}_{n,m}\mathbf{x}_m + \mathbf{B}_{n,m}\mathbf{W}_{n,m}, \tag{3}$$

$$\mathbf{Y}_{n,m} = \mathbf{C}_{n,m}\mathbf{x}_m + \mathbf{H}_{n,m}\mathbf{W}_{n,m} + \mathbf{D}_{n,m}\mathbf{V}_{n,m}. \tag{4}$$

Here, $\mathbf{X}_{n,m} = \left[\mathbf{x}_n^T \cdots \mathbf{x}_m^T\right]^T \in \mathbb{R}^{NK\times 1}$, $\mathbf{Y}_{n,m} = \left[\mathbf{y}_n^T \cdots \mathbf{y}_m^T\right]^T \in \mathbb{R}^{NM\times 1}$, $\mathbf{W}_{n,m} = \left[\mathbf{w}_n^T \cdots \mathbf{w}_m^T\right]^T \in \mathbb{R}^{NP\times 1}$, and $\mathbf{V}_{n,m} = \left[\mathbf{v}_n^T \cdots \mathbf{v}_m^T\right]^T \in \mathbb{R}^{NM\times 1}$. The extended model matrix $\mathbf{A}_{n,m} \in \mathbb{R}^{NK\times K}$, process noise matrix $\mathbf{B}_{n,m} \in \mathbb{R}^{NK\times NP}$, observation matrix $\mathbf{C}_{n,m} \in \mathbb{R}^{NM\times K}$, auxiliary process noise matrix $\mathbf{H}_{n,m} \in \mathbb{R}^{NM\times NP}$ and measurement noise matrix $\mathbf{D}_{n,m} \in \mathbb{R}^{NM\times NM}$ are time-variant, and can be specified by, respectively,

$$\begin{aligned}
\mathbf{A}_{n,m} &= \left[\mathcal{A}_{n,m+1}^T, \mathcal{A}_{n-1,m+1}^T, \ldots, \mathcal{A}_{m+1,m+1}^T, \mathbf{I}\right]^T, \\
\mathbf{B}_{n,m} &= \begin{bmatrix} \mathbf{B}_n & \mathcal{A}_{n,n}\mathbf{B}_{n-1} & \cdots & \mathcal{A}_{n,m+1}\mathbf{B}_m \\ \mathbf{0} & \mathbf{B}_{n-1} & \cdots & \mathcal{A}_{n-1,m+1}\mathbf{B}_m \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{B}_m \end{bmatrix}, \\
\mathbf{C}_{n,m} &= \bar{\mathbf{C}}_{n,m}\mathbf{A}_{n,m}, \\
\mathbf{H}_{n,m} &= \bar{\mathbf{C}}_{n,m}\mathbf{B}_{n,m}, \\
\mathbf{D}_{n,m} &= \mathrm{diag}\left(\mathbf{D}_n \mathbf{D}_{n-1} \cdots \mathbf{D}_m\right), \\
\bar{\mathbf{C}}_{n,m} &= \mathrm{diag}\left(\mathbf{C}_n \mathbf{C}_{n-1} \cdots \mathbf{C}_m\right),
\end{aligned} \tag{5}$$

with

$$\mathcal{A}_{j,i} = \prod_{r=0}^{j-i} \mathbf{A}_{j-r}. \tag{6}$$

It shows that the equations at the starting point m on the horizon are uniquely found with $\mathbf{w}_m$ zero-valued. That is, the initial state $\mathbf{x}_m$ should be known a priori or estimated optimally. At this point, the FIR estimate can be computed with the discrete convolution as

$$\hat{\mathbf{x}}_n = \mathbf{K}_n \mathbf{Y}_{n,m}, \tag{7}$$

where $\hat{\mathbf{x}}_n \triangleq \hat{\mathbf{x}}_{n|n}$ denotes the estimate at n utilizing the measurement vector $\mathbf{Y}_{n,m}$, and $\mathbf{K}_n$ is the filter gain determined by a given cost criterion. Fig. 1 demonstrates the operation principles of the IIR and FIR structures. It shows that only one most recent measurement explicitly appears in IIR (Kalman) filtering, while FIR estimators explicitly employ N most recent measurements. In this way, some nice properties such as BIBO stability and better robustness are achieved.

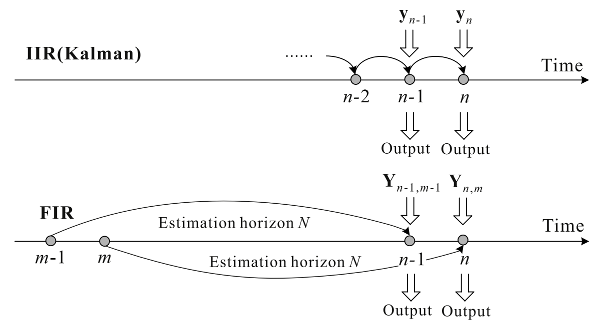

Fig. 1. Operation time diagrams of the IIR and FIR structures.

Provided estimate $\hat{\mathbf{x}}_n$, we define the estimation error at n by $\mathbf{e}_n \triangleq \mathbf{x}_n - \hat{\mathbf{x}}_n$. The problem considered is now formulated as follows: Given the model, (1) and (2), derive a new FIR filter minimizing the variance in the UFIR filter (Shmaliy, 2011) by

$$\tilde{\mathbf{K}}_n = \arg\min_{\tilde{\mathbf{K}}_n} E\left\{\mathbf{e}_n\mathbf{e}_n^T\right\}, \tag{8}$$

with the unbiasedness condition $E\{\mathbf{x}_n\} = E\left\{\hat{\mathbf{x}}_n\right\}$. We also wish to provide a comparison of the UFIR, MVU FIR, and OFIR filters (Shmaliy & Manzano, 2012).

3. Filter design

Before we give the main derivation of the MVU FIR filter, the UFIR and OFIR filters proposed in Shmaliy (2011) and Shmaliy and Manzano (2012) respectively are reviewed. The OFIR filter gain $\hat{\mathbf{K}}_n$ is derived by minimizing the estimation error variance without the unbiasedness condition, which is specified by

$$\hat{\mathbf{K}}_n = \left(\mathcal{A}_{n,m+1}\boldsymbol{\Theta}_{x,m}\mathbf{C}_{n,m}^T + \bar{\mathbf{B}}_{n,m}\boldsymbol{\Theta}_{w,m}\mathbf{H}_{n,m}^T\right)\mathbf{Z}_{x+w+v,m}^{-1}, \tag{9}$$

where $\bar{\mathbf{B}}_{n,m}$ is the first row vector of $\mathbf{B}_{n,m}$,

$$\boldsymbol{\Theta}_{x,m} = E\left\{\mathbf{x}_m\mathbf{x}_m^T\right\},$$

$$\boldsymbol{\Theta}_{w,m} = E\left\{\mathbf{W}_{n,m}\mathbf{W}_{n,m}^T\right\},$$

$$\mathbf{Z}_{x+w+v,m} = \mathbf{Z}_{x,m} + \mathbf{Z}_{w,m} + \mathbf{Z}_{v,m},$$