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Brief paper

Coordinated output regulation of heterogeneous linear systems under switching topologies[☆]

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ABSTRACT

In this paper, we construct a framework to describe and study the coordinated output regulation problem for multiple heterogeneous linear systems. Each agent is modeled as a general linear multiple-input multiple-output system with an autonomous exosystem which represents the individual offset from the group reference for the agent. The multi-agent system as a whole has a group exogenous state which represents the tracking reference for the whole group. Under the constraints that the group exogenous output is only locally available to each agent and that the agents have only access to their neighbors' information, we propose observer-based feedback controllers to solve the coordinated output regulation problem using output feedback information. A high-gain approach is used and the information interactions are allowed to be switching over a finite set of networks containing both graphs that have a directed spanning tree and graphs that do not. Simulations are shown to validate the theoretical results.

1. Introduction

Coordinated control of multi-agent systems has recently drawn large attention due to its broad applications in physical, biological, social, and mechanical systems (Bai, Arcak, & Wen, 2011; Chopra & Spong, 2009; Cortes, Martinez, & Bullo, 2006; Meng, Dimarogonas, & Johansson, 2014; Meng et al., 2013; Tanner, Jadbabaie, & Pappas, 2007). The key idea of a coordination algorithm is to realize a global emergence using only local information interactions (Jadbabaie, Lin, & Morse, 2003; Olfati-Saber, Fax, & Murray, 2007). The coordination problem of a single-integrator network has been fully studied with an emphasis on the system robustness to the input time delays and switching communication topologies (Blondel, Hendrickx, Olshevsky, & Tsitsiklis, 2005; Jadbabaie et al., 2003; Olfati-Saber et al., 2007; Ren & Beard, 2005), discrete-time dynamical models (Moreau, 2005; You & Xie, 2011), nonlinear couplings (Lin, Francis, & Maggiore, 2007), convergence speed (Cao, Morse, & Anderson, 2008), and leader–follower tracking (Shi, Hong, & Johansson, 2012). The coordination of multiple general linear dynamic systems has recently been studied. For example, the authors of Wieland, Kim, and Allgöwer (2011) generalize the coordination of multiple single-integrator systems to the case of multiple linear time-invariant high-order systems. For a network of neutrally stable systems and polynomially unstable systems, the author of Tuna (2009) proposes a design scheme for achieving synchronization. The case of switching communication topologies is considered in Scardovi and Sepulchre (2009) and a so-called consensus-based observer is proposed to guarantee leaderless synchronization of multiple identical linear dynamic systems under a jointly connected communication topology. Similar problems are also considered in Ni and Cheng (2010) and Wang, Cheng, and Hu (2008), where a frequently connected communication topology is studied in Wang et al. (2008) and an assumption on the neutral stability is imposed in Ni and Cheng (2010). The authors of Li, Duan, Chen, and Huang (2010) propose a neighbor-based observer to solve the synchronization problem for general linear time-invariant systems. In addition, the classical Laplacian matrix is generalized in Yang, Roy, Wan, and Saberi (2011) to a so-called interaction matrix and a D-scaling approach is used to stabilize this interaction

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matrix. Synchronization of multiple heterogeneous linear systems has been investigated under both fixed and switching communication topologies (Alvergue, Pandey, Gu, & Chen, 2013; Grip, Yang, Saberi, & Stoorvogel, 2012; Lunze, 2012; Wieland, Sepulchre, & Allgöwer, 2011). In Grip et al. (2012), a high-gain approach is proposed to dominate the non-identical dynamics of the agents. The cases of frequently connected and jointly connected communication topologies are studied in Kim, Shim, Back, and Seo (2013) and Vengertsev, Kim, Shim, and Seo (2010), respectively, where a slow switching condition and a fast switching condition are presented. Recently, the generalizations of coordination of multiple linear dynamic systems to the cooperative output regulation problem are studied in Ding (2013), Kim, Shim, and Seo (2011), Su and Huang (2012), Wang, Hong, Huang, and Jiang (2012) and Xiang, Wei, and Li (2009). In addition, the study on the synchronization of homogeneous and heterogeneous networks with nonlinear couplings is considered in Cao, Chen, and Li (2008), Cao, Wang, and Sun (2007) and He, Du, Qian, and Cao (2013).

In this paper, we generalize the classical output regulation problem of a single linear system to the coordinated output regulation problem of multiple heterogeneous linear systems. We consider the case where each agent has an individual offset and simultaneously there is a group tracking reference. The individual offset and the group reference are generated by autonomous systems (*i.e.,* systems without inputs). Each individual offset is available to its corresponding agent while the group reference can be obtained only through constrained communication among the agents, *i.e.,* the group reference trajectory is available to only a subset of the agents. Our goal is to find an observer-based feedback controller for each agent such that the output of each agent converges to a given trajectory determined by the combination of the individual offset and the group reference. Motivated by the approach in Grip et al. (2012), we propose a unified observer to solve the coordinated output regulation problem of multiple heterogeneous general linear systems, where the open-loop poles of the agents can be exponentially unstable and the dynamics are allowed to be different both with respect to dimensions and parameters. This relaxes the common assumption of identical dynamics (Li et al., 2010; Ni & Cheng, 2010; Scardovi & Sepulchre, 2009; Su & Huang, 2012; Tuna, 2009; Vengertsev et al., 2010; Xiang et al., 2009), or open-loop poles at most polynomially unstable (Ni & Cheng, 2010; Scardovi & Sepulchre, 2009; Su & Huang, 2012; Wieland, Sepulchre et al., 2011), or relative degree and minimum phase requirement (Kim et al., 2011). In addition, in this work, the information interaction is allowed to be switching from a graph set containing both a directed spanning tree set and a disconnected graph set. This extends the existing works considering fixed communication topologies (Grip et al., 2012; Kim et al., 2011; Li et al., 2010; Tuna, 2009; Wang et al., 2012).

The remainder of the paper is organized as follows. In Section 2, we give some basic definitions on the network model. In Section 3, we formulate the problem of coordinated output regulation of multiple heterogeneous linear systems. We then propose the state feedback control law with a unified observer design in Section 4. Numerical studies are carried out in Section 5 to validate our design and a brief concluding remark is drawn in Section 6.

2. Network model

We use graph theory to model the communication topology among agents. A directed graph G consists of a pair $(\mathbf{V}, \mathbf{E})$, where $\mathbf{V} = \{\nu_1, \nu_2, \ldots, \nu_n\}$ is a finite, nonempty set of nodes and $\mathbf{E} \subseteq \mathbf{V} \times \mathbf{V}$ is a set of ordered pairs of nodes. An edge (ν_i, ν_j) denotes that node ν_j can obtain information from node ν_i. All neighbors of node ν_i are denoted as $N_i := \{\nu_j | (\nu_j, \nu_i) \in \mathbf{E}\}$. For an edge (ν_i, ν_j) in a directed graph, ν_i is the parent node and ν_j is the child node. A directed path in a directed graph is a sequence of edges of the form $(\nu_i, \nu_j), (\nu_j, \nu_k), \ldots$. A directed tree is a directed graph, where every node has exactly one parent except for one node, called the root, which has no parent, and the root has a directed path to every other node. A directed graph has a directed spanning tree if there exists at least one node having a directed path to all other nodes.

For a leader–follower graph $\bar{G} := (\bar{\mathbf{V}}, \bar{\mathbf{E}})$, we have $\bar{\mathbf{V}} = \{\nu_0, \nu_1, \ldots, \nu_n\}$, $\bar{\mathbf{E}} \subseteq \bar{\mathbf{V}} \times \bar{\mathbf{V}}$, where ν_0 is the leader and $\nu_1, \nu_2, \ldots, \nu_n$ denote the followers. The leader–follower adjacency matrix $\bar{A} = [a_{ij}] \in \mathbb{R}^{(n+1)\times(n+1)}$ is defined such that a_{ij} is positive if $(\nu_j, \nu_i) \in \bar{\mathbf{E}}$ while $a_{ij} = 0$ otherwise. Here we assume that $a_{ii} = 0$, $i = 0, 1, \ldots, n$, and the leader has no parent, *i.e.*, $a_{0j} = 0$, $j = 0, 1, \ldots, n$. The leader–follower "grounded" Laplacian matrix $L = [l_{ij}] \in \mathbb{R}^{n\times n}$ associated with $\bar{A}$ is defined as $l_{ii} = \sum_{j=0}^{n} a_{ij}$ and $l_{ij} = -a_{ij}$, where $i \neq j$.

We assume that the leader–follower communication topology $\bar{G}_{\sigma(t)}$ is time-varying and switched from a finite set $\{\bar{G}_k\}_{k\in\Gamma}$, where $\Gamma = \{1, 2, \ldots, \delta\}$ is an index set and $\delta \in \mathbb{N}$ indicates its cardinality. We impose the technical condition that $\bar{G}_{\sigma(t)}$ is right continuous, where $\sigma : [t_0, \infty) \to \Gamma$ is a piecewise constant function of time, i.e., $\bar{G}_{\sigma(t)}$ remains constant for $t \in [t_\ell, t_{\ell+1})$, $\ell = 0, 1, \ldots$ and switches at $t = t_\ell$, $\ell = 1, 2, \ldots$. In addition, we assume that $\inf_\ell (t_{\ell+1} - t_\ell) \geq \tau_d > 0$, $\ell = 0, 1, \ldots$, with $\lim_{\ell\to\infty} t_\ell = \infty$, where τ_d is a constant known as the dwell time (Liberzon & Morse, 1999).

Let the sets $\{\bar{A}_k\}_{k\in\Gamma}$ and $\{L_k\}_{k\in\Gamma}$ be the leader–follower adjacency matrices and leader–follower grounded Laplacian matrices associated with $\{\bar{G}_k\}_{k\in\Gamma}$, respectively. Consequently, the time-varying leader–follower adjacency matrix and time-varying leader–follower grounded Laplacian matrix are defined as $\bar{A}_{\sigma(t)} = [a_{ij}(t)]$ and $L_{\sigma(t)} = [l_{ij}(t)]$.

Other notations in this paper: $\lambda_{\min}(P)$ and $\lambda_{\max}(P)$ denote, respectively, the minimum and maximum eigenvalues of a real symmetric matrix P, P^{T} denotes the transpose of P, I_n denotes the $n \times n$ identity matrix, and $\mathrm{diag}(A_1, A_2, \ldots, A_n)$ denotes a block diagonal matrix with the main diagonal blocks matrices. A square matrix A is called a Hurwitz matrix if every eigenvalue of A has strictly negative real part.

3. Problem formulation

3.1. Agent dynamics

Suppose that we have n agents modeled by the linear multiple-input multiple-output (MIMO) systems for each $\nu_i \in \mathbf{V}$:

$$\dot{x}_i = A_i x_i + B_i u_i, \tag{1}$$

where $x_i \in \mathbb{R}^{n_i}$ is the agent state, $u_i \in \mathbb{R}^{m_i}$ is the control input, $A_i \in \mathbb{R}^{n_i\times n_i}$, $B_i \in \mathbb{R}^{n_i\times m_i}$, and n_i and m_i are positive integers, for all $\nu_i \in \mathbf{V}$.

Also suppose that there is an individual autonomous exosystem for each $\nu_i \in \mathbf{V}$:

$$\dot{\omega}_i = S_i \omega_i, \tag{2}$$

where $\omega_i \in \mathbb{R}^{q_i}$, $S_i \in \mathbb{R}^{q_i\times q_i}$, and q_i is a positive integer, for all $\nu_i \in \mathbf{V}$.

In addition, there is a group autonomous exosystem for the multi-agent system as a whole:

$$\dot{x}_0 = A_0 x_0, \tag{3}$$

where $x_0 \in \mathbb{R}^{n_0}$, $A_0 \in \mathbb{R}^{n_0\times n_0}$, and n_0 is a positive integer.

3.2. Available information for agents

For the individual autonomous exosystem tracking, available output information for each agent $\nu_i \in \mathbf{V}$ is $y_{si} = C_{si} x_i + C_{wi} \omega_i$, where $y_{si} \in \mathbb{R}^{p_1}$, $C_{si} \in \mathbb{R}^{p_1\times n_i}$, $C_{wi} \in \mathbb{R}^{p_1\times q_i}$, and p_1 is a positive integer.