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Brief paper

Predictor based control of linear systems with state, input and output delays ☆

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ABSTRACT

In this paper we develop a predictor-based controller for linear systems with state, input and output delays. First, a state predictor is developed for state feedback control. This predictor is formulated recursively over the prediction time by partitioning the input delay into sections smaller than the state delays. The partitioning of the input delay ensures that the resulting predictor equation only depends on the past values of the state and input. It is shown that the proposed predictor gives an exact prediction of the future states. This recursive predictor is then reformulated into a cascade form in order to reduce the number of redundant calculations and to simplify the predictor equation for practical implementation. We construct a predictor based state feedback control law and show that the spectrum of the closed-loop time-delay system under the constructed control law is the same as that of an equivalent time-delay system without input delay and under the nominal state feedback control. Therefore, the proposed predictor based solution can stabilize the delay system if a stabilizing state feedback control law exists for the input delay free system. These state feedback results are then extended to the case of delayed output feedback. The theoretical derivation is verified through numerical examples.

1. Introduction

The control of dynamic systems with time-delay has been a topic of active research for many years, motivated by the unavoidable presence of time delays in nearly all modern control applications. In some cases the time delay is relatively small and has a negligible effect on the closed-loop dynamics, but in many other cases the time delay may lead to significant degradation of the closed-loop performance and stability. Common sources of time delay in control applications include transportation and transmission lags, communication delays, and chemical and biological processes with large time constants.

Reviews of recent results on the control of time-delay systems can be found in Gu and Niculescu (2003) and Richards (2003). More detailed surveys of the many control methods available for the stabilization of time-delay systems can be found in Dugard and Verriest (1998) and Gu, Kharitonov, and Chen (2003) for linear systems, and Krstic (2009) for nonlinear systems. The stability of linear systems with time delay in the state and saturation in the input was studied in Cao, Lin, and Hu (2002) and Chen, Wang, and Lu (1988). For the stabilization of uncertain systems with state delays, a min–max control law was developed in Cheres, Gutman, and Palmor (1989), a model predictive control algorithm was presented in Jeong and Park (2005), and a model reference adaptive control law was studied in Mirkin and Gutman (2005). In the case where time delay is in both the state and the input, the authors of Du, Lam, and Shu (2010) developed a static output feedback law and an integral output feedback law for an unknown but bounded time delay. Al-Shamali, Crisalle, and Latchman (2003) also considered linear systems with both input and state delays, and presented a sliding mode control scheme for achieving stabilization. When the state and input delays are time varying, a stability condition of the closed-loop system under a static state feedback control law was presented in Zhang, Wu, She, and He (2005).

For systems with delays in the input only, the predictor feedback control is a popular approach that has been studied extensively in the literature since the classical Smith predictor method was introduced for stable linear plants in Smith (1959). The most

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common predictor-type controllers considered in the literature are based on the Artstein model reduction technique (Artstein, 1982) and the finite spectrum assignment technique (Manitius & Olbrot, 1979). In both of these predictor based methods, a finite integral over the input history is utilized in order to transform the delayed plant equation into a delay-free system (Sharma, Bhasin, Wang, & Dixon, 2011). When the delay is also present in the state, the model reduction technique is no longer applicable and the finite spectrum assignment technique is limited to commensurate delays. In particular, when the prediction time is larger than the state delay, a predictor formulated as in Artstein (1982) becomes non-causal, and future state information is required to compute the prediction.

Many variations of the predictor based control (Artstein, 1982; Manitius & Olbrot, 1979) have been reported in the literature for systems with input delay. For Euler–Lagrange systems with parametric uncertainty, a predictor based controller was proposed in Sharma et al. (2011). A predictor for systems with time-varying input delays was presented in Krstic (2010). Similarly, predictor based control laws were proposed in Bekiaris-Liberis and Krstic (2010) and Bresch-Pietri and Krstic (2010) for systems with unknown delays. A finite dimensional feedback control that is truncated from the traditional predictor feedback was developed in Lin and Fang (2007) based on the low gain feedback design (Lin, 1988). This truncated predictor was extended to systems with time-varying input delay in Zhou, Lin, and Duan (2012), and later to exponentially unstable plants in Yoon, Anantachaisilp, and Lin (2013) and Yoon and Lin (2013).

Predictor based control for systems with state, input and output delays has rarely been explored in the literature, mainly due to the high complexity of the predictor formulation for systems with multiple delays. The finite spectrum assignment technique has been studied for systems with commensurate delays (Manitius & Olbrot, 1979; Watanabe, 1986). For linear systems with input and state delays, a predictor was constructed in Kharitonov (2014) by using the properties of transition matrices. A predictor was constructed in Jankovic (2010) for linear, block-feedforward systems with input, output and state delays. The author employed the cascading structure of the state equation to relate, through spectral equivalence, the stability of the time-delay closed-loop system under the predictor feedback control to the stability of an equivalent delay-free system. The predictor proposed in Jankovic (2010) was formulated recursively over the states, using a backstepping-like approach to take advantage of the triangular structure of the system state space equation. In each step of the recursive procedure, the state delay is treated as an input delay. The idea of Jankovic (2010) was extended for nonlinear systems with time varying delays in Bekiaris-Liberis and Krstic (2011).

In this paper, we propose a predictor based control law for linear systems with state, input and output delays. First, we consider the state feedback case, and propose a predictor that is formulated recursively over the prediction time. The recursive formulation partitions the input delay into segments smaller than the state delay, which results in a causal predictor equation. One advantage of our predictor formulation over the solution proposed in Jankovic (2010) is that our predictor does not require any assumption on the structure of the plant state space equation. On the other hand, when compared to the work in Kharitonov (2014), in this paper we provide an explicit expression for our recursive predictor, which is potentially useful in the study of robustness characteristics of the closed-loop system. The explicit predictor also allows us to exploit the well-known observer–controller duality property to construct predictors for output feedback control.

In order to simplify the predictor equation for practical implementation, and to reduce the number of calculations, the recursive predictor proposed in this paper is reformulated in a cascade form. It is shown that this new formulation of the recursive predictor becomes exact, *i.e.*, the prediction error is zero in a finite time. A state feedback control law is then built based on the proposed predictor for systems with state and input delays. We demonstrate that the spectrum of the closed-loop system under the proposed predictor feedback control is the same as that of an equivalent system without input delay and under a nominal state feedback law. Therefore, the proposed predictor based controller can stabilize the time-delay system if there exist a state feedback law stabilizing the equivalent input-delay free system. Finally, a state predictor is built for output feedback control through the dual observer–controller formulation and the recursive predictor for state feedback control. It is shown that the separation principle holds, and the spectrum of the closed-loop system equals the combined spectra of the state feedback closed-loop system and the dynamics of the state observation error.

The remainder of this paper is structured as follows. Problem definitions and objectives are stated in Section 2, where we also present a brief discussion on the limitations of the traditional predictor formulation in systems with both input and state delays. In Section 3, a state predictor is developed to overcome the previously discussed limitations by formulating the predictor equation recursively over the prediction time. The proposed recursive predictor is then redefined in Section 4, where the predictor equation is formulated in a cascade form. Based on the recursive predictor, a predictor based state feedback control law is introduced in Section 5, and a state predictor is constructed for output feedback control in Section 6. The effectiveness of the proposed control laws is demonstrated through numerical examples in Section 7, and we draw our conclusions in Section 8.

2. Problem definition

We first consider a linear time-invariant system with state and input time delays,

$$\dot{x}(t) = \sum_{i=0}^{N} A_i x(t-\tau_i) + Bu(t-\tau_u), \tag{1}$$

for an integer $N \geq 0$. The time-delay system (1) has the state vector $x \in \mathbb{R}^n$ and the input vector $u \in \mathbb{R}^m$. The time delays in the state and input are represented by the real scalars $\tau_i \geq 0$ and $\tau_u > 0$, respectively. Without loss of generality, we will assume that $\tau_0 = 0$, and the state delays are ordered such that $\tau_i < \tau_j$ for $i < j$.

The predictor feedback approach has been studied extensively for the control of input delayed systems,

$$\dot{x}(t) = A_0 x(t) + Bu(t-\tau_u), \tag{2}$$

which corresponds to system (1) with $N = 0$. In the predictor feedback, the future state $x(t+\tau_u)$ is computed and used in the feedback control calculation in order to neutralize the effect of the delay in the input signal. In the absence of state delays, the explicit expression of such a predictor is causal, *i.e.*, the predictor equation can be solved from the current state measurement and the input history $u(\theta), \theta \in [t-\tau_u, t]$.

When a state delay is present, as in (1), a prediction of the future state can be obtained from the solution of the delayed state space equation as

$$x(t+\tau_u) = \mathrm{e}^{A_0\tau_u}x(t) + \int_t^{t+\tau_u} \mathrm{e}^{A_0(t+\tau_u-\sigma)}Bu(\sigma-\tau_u)\mathrm{d}\sigma + \sum_{i=1}^{N}\int_t^{t+\tau_u} \mathrm{e}^{A_0(t+\tau_u-\sigma)}A_i x(\sigma-\tau_i)\mathrm{d}\sigma. \tag{3}$$

We observe that the causality of the above predictor equation may be lost for some combinations of state and input delays, resulting in a control law that requires future state information. To illustrate