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Brief paper

On robust synchronization of heterogeneous linear multi-agent systems with static couplings☆

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ABSTRACT

This paper addresses cooperative control problems in heterogeneous groups of linear dynamical agents that are coupled by diffusive links. We study networks with parameter uncertainties, resulting in heterogeneous agent dynamics, and we analyze the robustness of their output synchronization. The networks under consideration consist of non-identical double-integrators and harmonic oscillators. The geometric approach to linear control theory reveals structural requirements for non-trivial output synchronization in such networks. Furthermore, a clock synchronization problem and a circular motion coordination problem are discussed as applications corresponding to these two network types. The results are illustrated by numerical simulations.

1. Introduction

Consensus and synchronization problems in networks of dynamical agents are typically solved with diffusive couplings, i.e., distributed control laws based on the output differences of neighboring agents. Well-known examples are the classical consensus protocol (Olfati-Saber & Murray, 2004; Ren & Beard, 2005) and its extensions to double-integrators (Ren & Atkins, 2007), harmonic oscillators (Ren, 2008), and general linear agents (Scardovi & Sepulchre, 2009; Wieland, Kim, & Allgöwer, 2011). In this context, a major challenge is robust synchronization in heterogeneous linear networks, i.e., multi-agent systems consisting of non-identical linear agents (Grip, Yang, Saberi, & Stoorvogel, 2012; Lunze, 2012; Wieland & Allgöwer, 2009; Wieland, Sepulchre, & Allgöwer, 2011; Wu & Allgöwer, 2012). In Wieland and Allgöwer (2009) and Wieland, Sepulchre et al. (2011), a necessary condition for synchronization in heterogeneous linear networks is presented. The result is formulated as an internal model principle for synchronization and states that the agents have to embed a common internal model in order to be able to synchronize.

In this paper, we study cooperative control problems in heterogeneous linear networks, i.e., in diffusively coupled multi-agent systems with general high-order linear dynamics subject to parameter perturbations, which cause non-identical agent dynamics. In particular, we focus on output synchronization problems. The main goal is to develop a deeper understanding of the effects of heterogeneity in the agent dynamics on the dynamic behavior of the diffusively coupled multi-agent system and its implications for distributed control design. The contributions are the following.

We analyze the dynamic behavior of selected heterogeneous linear multi-agent systems. For each network, we discuss the implications of the internal model principle for synchronization, highlight the importance of the network topology, and assess the robustness of synchronization with respect to parameter uncertainties in the agent dynamics. Firstly, we consider a network of non-identical double-integrators, which achieves output synchronization if the output is position only, in Section 4. Afterwards, in Section 5, we study state synchronization in the same network. The structural requirements for synchronization are not met in this case, but it turns out that the synchronization error remains small,

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depending on the graph topology and the heterogeneity in the network. Secondly, in Section 6, we consider a network of harmonic oscillators with perturbed frequencies. We show that the internal model condition is not satisfied and that static diffusive couplings have a stabilizing effect in such networks. In particular, the network is rendered asymptotically stable if and only if there are oscillators with different frequencies in a certain region of the network. A preliminary version of these results has been presented in Seyboth, Dimarogonas, Johansson, and Allgöwer (2012). Moreover, we present two application examples: a clock synchronization problem and a motion coordination problem for mobile robots. The latter shows that heterogeneity may significantly impair the performance of cooperative control strategies designed for identical agents.

2. Preliminaries: notation and graph theory

For a vector $v \in \mathbb{R}^n$, $\operatorname{diag}(v)$ and $\operatorname{diag}(v_1, \ldots, v_n)$ both denote the diagonal matrix with the entries v_i, $i = 1, \ldots, n$, of v on the diagonal. The all ones and all zeros vectors are denoted by $\mathbf{1}$ and $\mathbf{0}$, respectively, and $I = \operatorname{diag}(\mathbf{1})$ is the identity matrix. The null space and image of a linear map defined by a matrix M are denoted by $\ker(M)$ and $\operatorname{im}(M)$, respectively. The norm $\|\cdot\|$ is understood as 2-norm for vectors and induced 2-norm for matrices. The spectrum of a square matrix M is denoted by $\sigma(M)$, which is to be understood as the set of roots of the characteristic polynomial of M, i.e., it respects the multiplicity of the eigenvalues. For symmetric matrices $M = M^\mathsf{T}$, $M > 0$ ($M \geq 0$) stands for positive (semi-)definiteness, while $M < 0$ ($M \leq 0$) stands for negative (semi-)definiteness. For a complex number $z \in \mathbb{C}$, $\mathbf{Re}(z)$ is the real part and $\mathbf{Im}(z)$ the imaginary part of z. The closed right-half complex plane is denoted by $\bar{\mathbb{C}}^+$. Let $\dot{x} = Ax, x \in \mathbb{R}^n$, be a linear dynamical system. A subspace $\mathcal{U} \subseteq \mathbb{R}^n$ is called invariant with respect to $\dot{x} = Ax$, or shortly A-invariant, if $x(0) \in \mathcal{U}$ implies $x(t) \in \mathcal{U}$ for all t. For convergence to a subspace $\mathcal{U}$, we write $x(t) \to \mathcal{U}$ as $t \to \infty$ as shorthand notation for $\forall \epsilon > 0 \exists \tau > 0 \forall t \geq \tau$: $\operatorname{dist}(x(t), \mathcal{U}) < \epsilon$, where $\operatorname{dist}(x(t), \mathcal{U}) = \inf_{\zeta \in \mathcal{U}} \|x(t) - \zeta\|$.

The network topology is modeled by a time-invariant directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A_\mathcal{G})$. Each vertex v_k in the set $\mathcal{V} = \{v_1, \ldots, v_N\}$ corresponds to a dynamical subsystem (agent) k in the network. There is a directed edge from vertex v_j to v_k, i.e., $(v_j, v_k) \in \mathcal{E}$, if and only if v_k is influenced by (receives information from) v_j. A consecutive sequence of directed edges is called a directed path. The adjacency matrix $A_\mathcal{G} \in \mathbb{R}^{N\times N}$ describes the graph structure and edge weights, i.e., $a_{kj} > 0 \Leftrightarrow (v_j, v_k) \in \mathcal{E}$ and $a_{kj} = 0$ otherwise. A graph $\mathcal{G}$ is called undirected if $(v_j, v_k) \in \mathcal{E} \Leftrightarrow (v_k, v_j) \in \mathcal{E}$ and $a_{kj} = a_{jk}$. The Laplacian matrix $L \in \mathbb{R}^{N\times N}$ is defined as $L = \operatorname{diag}(A_\mathcal{G}\mathbf{1}) - A_\mathcal{G}$. By construction, L is a Metzler matrix and has zero row sums, i.e., $L\mathbf{1} = \mathbf{0}$. The vector of ones $\mathbf{1}$ is the eigenvector corresponding to the zero eigenvalue $\lambda_1(L) = 0$. All eigenvalues of L are contained in the closed right-half plane. The zero eigenvalue $\lambda_1(L) = 0$ is simple and all other eigenvalues have positive real parts $\mathbf{Re}(\lambda_k(L)) > 0$, $k \in \{2, \ldots, N\}$, if and only if $\mathcal{G}$ is connected (Ren & Beard, 2005). An induced subgraph of $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is a graph $\tilde{\mathcal{G}} = (\tilde{\mathcal{V}}, \tilde{\mathcal{E}})$ with $\tilde{\mathcal{V}} \subseteq \mathcal{V}$ and $\tilde{\mathcal{E}} = \{(v, w) \in \mathcal{E} : v, w \in \tilde{\mathcal{V}}\}$.

Definition 2.1 (*(Strongly) Connected Graph*)**.** A graph $\mathcal{G}$ is called connected if it contains a directed spanning tree, i.e., if there exists a vertex v_k such that there is a path from v_k to every other vertex $v_j \in \mathcal{V}$. A graph $\mathcal{G}$ is called strongly connected if there exists a directed path from any vertex to any other vertex in $\mathcal{V}$.

Definition 2.2 (*iSCC, Wieland, 2010*)**.** An independent strongly connected component (iSCC) of a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is an induced subgraph $\tilde{\mathcal{G}} = (\tilde{\mathcal{V}}, \tilde{\mathcal{E}})$ which is maximal, subject to being strongly connected, and satisfies $(v, \tilde{v}) \notin \mathcal{E}$ for any $v \in \mathcal{V} \setminus \tilde{\mathcal{V}}$ and $\tilde{v} \in \tilde{\mathcal{V}}$.

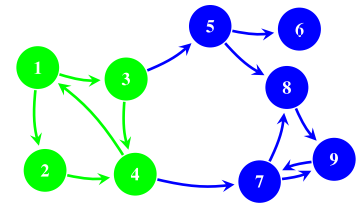

Fig. 1. A connected directed graph $\mathcal{G}$.

If $\mathcal{G}$ is connected, then $\mathcal{G}$ has exactly one iSCC (Wieland, 2010). Furthermore, in this case, $\operatorname{rank}(L) = N-1$ and the null space of L^T is spanned by a non-negative vector $p \in \mathbb{R}^N$, i.e., $p \geq \mathbf{0}$ element-wise. The k-th element p_k is positive, if and only if $v_k \in \mathcal{V}_{\text{iSCC}}$ (Wieland, 2010). The vector p is the left eigenvector of L corresponding to eigenvalue zero, i.e., $p^\mathsf{T}L = \mathbf{0}^\mathsf{T}$. We normalize p such that $p^\mathsf{T}\mathbf{1} = 1$. If $\mathcal{G}$ is strongly connected, then $\mathcal{V}_{\text{iSCC}} = \mathcal{V}$ and $p > \mathbf{0}$ element-wise. Fig. 1 shows an example of a directed graph which is connected but not strongly connected. Its iSCC consists of $\mathcal{V}_{\text{iSCC}} = \{v_1, v_2, v_3, v_4\}$, and any vertex in $\mathcal{V}_{\text{iSCC}}$ is the root of a spanning tree. For further details, see Godsil and Royle (2001), Wieland (2010) and Wieland, Kim et al. (2011).

3. Synchronization in heterogeneous linear networks

It has been shown in Wieland and Allgöwer (2009) and Wieland, Sepulchre et al. (2011) that the geometric approach to linear systems theory (Basile & Marro, 1992; Wonham, 1985) is useful for the analysis of synchronization problems in networks of linear systems. In this section, we review the main result of Wieland and Allgöwer (2009), i.e., the internal model principle for synchronization. We consider a heterogeneous group of N linear agents,

$$\begin{aligned} \dot{x}_k &= A_k x_k + B_k u_k \\ y_k &= C_k x_k, \end{aligned} \tag{1}$$

with state $x_k \in \mathbb{R}^{n_k}$, input $u_k \in \mathbb{R}^{q_k}$, and output $y_k \in \mathbb{R}^p$, for $k \in \mathcal{N}$, where $\mathcal{N}$ is the index set $\mathcal{N} = \{1, \ldots, N\}$. The agents are interconnected by static diffusive couplings

$$u_k = K_k \sum_{j=1}^{N} a_{kj}(y_j - y_k), \tag{2}$$

where $K_k \in \mathbb{R}^{q_k\times p}$ is a coupling gain matrix and a_{kj} are the elements of the adjacency matrix $A_\mathcal{G}$ of the underlying communication graph $\mathcal{G}$. The network of N agents (1) with couplings (2) is said to reach output synchronization, if

$$y_j(t) - y_k(t) \to 0 \quad \text{as } t \to \infty$$

for all pairs $j, k \in \mathcal{N}$. Furthermore, non-trivial output synchronization is reached if, additionally, the closed-loop system has no asymptotically stable equilibrium set on which $y_k(t) = 0$ for all $k \in \mathcal{N}$. We impose the following standing assumption.

Assumption 3.1. (A_k, C_k) is detectable for all $k \in \mathcal{N}$.

The closed-loop system (1), (2) can be compactly written as $\dot{x} = (\hat{A} - \hat{B}\hat{K}(L \otimes I_p)\hat{C})x$, where $x = [x_1^\mathsf{T} \cdots x_N^\mathsf{T}]^\mathsf{T} \in \mathbb{R}^{\hat{n}}$ and $\hat{n} = \sum_{k=1}^{N} n_k$ is the state dimension of the overall network, and with the block diagonal matrices $\hat{A} = \operatorname{diag}(A_1, \ldots, A_N)$, $\hat{B} = \operatorname{diag}(B_1, \ldots, B_N)$, $\hat{C} = \operatorname{diag}(C_1, \ldots, C_N)$, and $\hat{K} = \operatorname{diag}(K_1, \ldots, K_N)$. Output synchronization is reached if all solutions $x(t)$ converge to the synchronous subspace $\mathcal{S} \subseteq \mathbb{R}^{\hat{n}}$, which is defined as the subspace on which the outputs $y_k = C_k x_k$ of all agents are identical, i.e., $\mathcal{S} = \{x \in \mathbb{R}^{\hat{n}} : C_1x_1 = \cdots = C_nx_n\}$. The internal model principle for synchronization is a necessary condition for non-trivial output