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Technical communique

Distributed command filtered backstepping consensus tracking control of nonlinear multiple-agent systems in strict-feedback form[☆]

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ABSTRACT

This paper investigates the distributed consensus tracking problems of multi-agent systems on undirected graph with a fixed topology. Each follower is assumed to be in strict-feedback form with unknown state-dependent controlling effects. A distributed robust adaptive neural networks-based control scheme is designed to guarantee the consensus output tracking errors between the followers and the leader are cooperatively semi-globally uniformly ultimately bounded. Command filtered backstepping technique is extended to the consensus tracking control problems, which avoids the classical "explosion of complexity" problem in standard backstepping design and removes the assumption that the first n derivatives of the leader's output should be known. The function approximation technique using neural networks is employed to compensate for unknown functions induced from the controller design procedure. Stability analysis and parameter convergence of the proposed algorithm are conducted based on algebraic graph theory and Lyapunov theory.

1. Introduction

In the past few years, the consensus problems of multi-agent systems (MASs) have recently extended to nonlinear MASs (see Du, Li, & Shi, 2012; Nguyen & Tran, 2013; Sedziwy, 2014; Shen, Jiang, & Shi, 2014; Zhang & Lewis, 2012 and their references for details). Because many practical systems have more complicated dynamics with unmatched nonlinearities, the consensus problems of nonlinear MASs have attracted extensive attention. In the latest development on this topic, the consensus problems of MASs in strict-feedback form were studied in Yoo (2013,?), where the virtual and actual controlling effects were assumed to not only be known but also be equal to one. This assumption may lead to conservativeness of the results and limitation in practice because the controlling effects may be dependent on its state (Haimo, 1986). The MASs with state-dependent controlling effects were also considered in Egersted and Hu (2001) and Qu (2009). Hence, it is challenging to handle the consensus problem of the high-order uncertain nonlinear MASs in strict-feedback form with unknown state-dependent controlling effects, which is a motivation of this work. On the other hand, the "explosion of complexity" problem always exists in standard backstepping design procedure. Hence, a lot of efforts have been devoted to avoid the problem, and many methods are proposed, such as dynamic surface control, command filter (Farrell, Polycarpou, Sharma, & Dong, 2009; Zuo, 2012) and so on. To our best knowledge, the problems of command filtered backstepping consensus tracking control for MASs have not been reported, which is important and challenging in both theory and real world applications. In this paper, we consider the leader-following consensus tracking control problem of nonlinear MASs in strict-feedback form, and propose a distributed adaptive neural networks (NNs)-based control scheme to guarantee the consensus tracking errors are cooperatively semi-globally uniformly ultimately bounded (CSUUB). Compared with the existing works, the main contributions and innovation from our paper are as follows: (1) The virtual and actual controlling effects of each follower are assumed to be unknown functions of its own state; (2) Command filtered backstepping control technique is first extended to

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the consensus problem of MASs to avoid the classical "explosion of complexity" problem.

2. Problem formulation and preliminaries

Consider $N(N \geq 2)$ followers. Follower k is described as:

$$\begin{cases} \dot{x}_{k,i} = f_{k,i}(\bar{x}_{k,i}) + g_{k,i}(\bar{x}_{k,i})x_{k,i+1} + \omega_{k,i}(\bar{x}_{k,i}, t) \\ \quad i = 1, \dots, n-1 \\ \dot{x}_{k,n} = f_{k,n}(\bar{x}_{k,n}) + g_{k,n}(\bar{x}_{k,n})u_k + \omega_{k,n}(\bar{x}_{k,n}, t) \\ y_k = x_{k,1} \end{cases} \tag{1}$$

where $k = 1, \dots, N$; $\bar{x}_{k,i} = [x_{k,1}, \dots, x_{k,i}]^T \in R^i$, $i = 1, \dots, n-1$; $\bar{x}_{k,n} = [x_{k,1}, \dots, x_{k,n}]^T \in R^n$, $u_k \in R$ and $y_k \in R$ denote the state, control input and the output, respectively; $f_{k,l}(\cdot) \in R$, $g_{k,l}(\cdot) \in R$ and $\omega_{k,l}(\cdot) \in R$, $l = 1, \dots, n$ denote the uncertainties, unknown state-dependent controlling effects and external bounded disturbances, respectively. Dynamics of the leader node, labeled 0, is described as follows:

$$\begin{cases} \dot{x}_{0,i} = x_{0,i+1}, \quad i = 1, \dots, n-1 \\ \dot{x}_{0,n} = f_0(\bar{x}_0, t) \\ y_0 = x_{0,1} \end{cases} \tag{2}$$

where $\bar{x}_0 = [x_{0,1}, \dots, x_{0,n}]^T \in R^n$ and $y_0 \in R$ denote the state and output, respectively; $f_0(\cdot) \in R$ is unknown for all follower nodes.

The interaction topology among followers is usually modeled by a weighted undirected graph $G = (v, E, A)$, where the set of nodes $v = (v_1, \dots, v_N)$ is the nonempty set of nodes, set of edges $E \subseteq \{(v_i, v_j) : v_i, v_j \in v\}$ is the set of edges, $(v_i, v_j) \in E$ means that there is an edge from node i to node j, $A = [a_{ij}] \in R^{N\times N}$ is a weighted adjacency matrix. The node indices belong to a finite index set $I = \{1, \dots, N\}$. The set of neighbors of node i is denoted as $N_i = \{j|(v_j, v_i) \in E\}$. The adjacency matrix $A = [a_{ij}]$ of a weighted undirected graph is defined as $a_{ij} = a_{ji}$, $a_{ii} = 0$ and $a_{ij} > 0$ if $(v_j, v_i) \in E$ where $i \neq j$. The Laplacian matrix with graph G is $L = [l_{ij}] = D - A \in R^{N\times N}$, where $D = diag(d_1, \dots, d_N) \in R^{N\times N}$, $d_i = \sum_{j=1}^{N} a_{ij}$ for $\forall i \in I$. Another weighted graph $\bar{G}$ is used to describe the interconnection topology of the system consisting of n followers (node $v_i \in E$, $i \in I$) and one leader (node v_0). Define a diagonal matrix $B = diag\{b_1, \dots, b_N\} \in R^{N\times N}$ as a leader adjacency matrix, where $b_i > 0$ if and only if node v_i is connected to node v_0 across the communication (v_i, v_0) and $b_i = 0$ otherwise. Throughout this paper, it is assumed that the topology is undirected and fixed. In addition, it is also assumed that the graph $\bar{G}$ is connected.

*The objective of this paper*is to design a NNs-based distributed consensus control law for each follower such that the follower output y_k synchronizes to the dynamic leader output y_0 with the tracking error is CSUUB.

In this paper, as in Zhang and Lewis (2012), NNs are employed to approximate the unknown smooth functions $\vartheta_{f,k,i}$ and $\vartheta_{g,k,i}$ in the following form:

$$\begin{aligned} \vartheta_{f,k,i}(\bar{Z}_{f,k,i}) &= W^{*T}_{f,k,i}S_{f,k,i}(\bar{Z}_{f,k,i}) + \varepsilon_{f,k,i}(\bar{Z}_{f,k,i}) \\ \vartheta_{g,k,i}(\bar{Z}_{g,k,i}) &= W^{*T}_{g,k,i}S_{g,k,i}(\bar{Z}_{g,k,i}) + \varepsilon_{g,k,i}(\bar{Z}_{g,k,i}) \end{aligned} \tag{3}$$

where $k = 1, \dots, N$, $i = 1, \dots, n$, $W^*_{f,k,i}$ and $W^*_{g,k,i}$ are optimal weights, $S_{f,k,i}(\cdot)$ and $S_{g,k,i}(\cdot)$ are suitable basis functions, $\varepsilon_{f,k,i}$ and $\varepsilon_{g,k,i}$ are optimal approximation errors.

For notational simplicity, $\bullet$ is used to denote $\bullet(\cdot)$ in the following. In addition, notation $\tilde{\bullet} = \bullet^* - \hat{\bullet}$ is defined.

Define approximation error $v_{k,i}$ as follows:

$$v_{k,i} = \vartheta_{f,k,i} - W^{*T}_{f,k,i}S_{f,k,i} + (\vartheta_{g,k,i} - W^{*T}_{g,k,i}S_{g,k,i})u_{f,k,i}$$

where

$$u_{f,k,i} = -\hat{W}^T_{f,k,i}S_{f,k,i}/\hat{W}^T_{g,k,i}S_{g,k,i} \tag{4}$$

$\hat{W}_{f,k,i}$ and $\hat{W}_{g,k,i}$ are the estimates of $W^*_{f,k,i}$ and $W^*_{g,k,i}$, respectively.

Lemma 1 (Polycarpou & Ioannou, 1995). *For $\forall x \in R$, $|x| - \tanh(x/\delta)x \leq 0.2785\delta$, where $\delta > 0 \in R$.*

Assumption 1. There exists an unknown real constant $M_{k,i,\omega} > 0$ such that $|\omega_{k,i}(\bar{x}_{k,i}, t)| \leq M_{k,i,\omega}$, $\forall k \in \{1, \dots, N\}$, $\forall i \in \{1, \dots, n\}$.

Assumption 2. $g_{k,i}(\bar{x}_{k,i}) > 0$ is unknown and bounded, i.e., there exists an unknown real constant $g_{m,k,i} > 0$ such that $g_{k,i}(\bar{x}_{k,i}) \geq g_{m,k,i}$, $\forall k \in \{1, \dots, N\}$, $\forall i \in \{1, \dots, n\}$. Let $g_m = \min_{k=1,\dots,N, i=1,\dots,n}\{g_{m,k,i}\}$.

Assumption 3. The output of the leader and its first derivative, i.e., y_0 and $\dot{y}_0$, are bounded and available.

Assumption 4. There exists an unknown constant $\bar{v}_{k,i} > 0 \in R$ such that $|v_{k,i}| \leq \bar{v}_{k,i}$, $\forall k \in \{1, \dots, N\}$, $\forall i \in \{1, \dots, n\}$.

3. Design of controller and stability analysis

The recursive design procedure contains n steps. For convenience, virtual control $\alpha_{k,i}$, adaptive laws $\dot{\hat{W}}_{f,k,i}$, $\dot{\hat{W}}_{g,k,i}$ and $\dot{\hat{\bar{v}}}_{k,i}$, $i = 1, \dots, n$, are first designed as follows:

$$\alpha_{k,i} = u_{f,k,i} + u_{v,k,i} \tag{5}$$

$$\dot{\hat{W}}_{f,k,i} = \eta_f z_{k,i} S_{f,k,i} \tag{6}$$

$$\dot{\hat{W}}_{g,k,i} = \eta_g z_{k,i} S_{g,k,i} u_{f,k,i} \tag{7}$$

$$\dot{\hat{\bar{v}}}_{k,i} = \eta_{\bar{v}} z_{k,i} \tanh(z_{k,i}/\delta), \qquad \hat{\bar{v}}(0) > 0 \tag{8}$$

where $z_{k,i}$ will be defined by (12), $\eta_f > 0$, $\eta_g > 0$ and $\eta_{\bar{v}} > 0$ are design parameters,

$$u_{v,k,i} = -(\tanh(z_{k,i}/\delta)\hat{\bar{v}}_{k,i} + z_{k,i})/(g_m \Delta_i)$$

$$\Delta_1 = (d_k + b_k) + \sum_{j\in N_k} \frac{a_{jk}z_{j,1}}{z_{k,1}}, \qquad \Delta_l = 1, l = 2, \dots, n$$

$\hat{\bar{v}}_{k,i}$ is the estimate of $\bar{v}_{k,i}$, and $u_{f,k,i}$ is defined by (4).

In order to avoid the "explosion of complexity", the following command filter is designed to estimate $\alpha_{k,i-1}$,

$$\dot{\theta}_{k,i} = -\eta_\alpha e_{\theta,k,i} - sgn(e_{\theta,k,i}\dot{\alpha}_{k,i-1})\dot{\alpha}_{k,i-1} \tag{9}$$

where $e_{\theta,k,i} = \theta_{k,i} - \alpha_{k,i-1}$, $\eta_\alpha > 0$ is a design parameter, $k = 1, \dots, N$, $i = 1, \dots, n$ and $\alpha_{k,0} = y_0$. Define

$$V_{k,\alpha i} = (\theta_{k,i} - \alpha_{k,i-1})^2/2.$$

Its time derivative is

$$\dot{V}_{k,\alpha i} \leq -\eta_\alpha(\theta_{k,i} - \alpha_{k,i-1})^2 \leq 0. \tag{10}$$

According to Lyapunov stability theorem, it can be easily seen that $\theta_{k,i}$ can asymptotically converge to $\alpha_{k,i-1}$.

The neighborhood synchronization error is defined as:

$$\begin{aligned} e_k &= \sum_{j\in N_k} a_{kj}(y_j - y_k) + b_k(y_0 - y_k) \\ &= -(d_k + b_k)y_k + \sum_{j\in N_k} a_{kj}y_j + b_k y_0. \end{aligned} \tag{11}$$

For $k = 1, \dots, N$, $i = 2, \dots, n$, define

$$z_{k,1} = e_k, \ z_{k,i} = x_{k,i} - \alpha_{k,i-1}. \tag{12}$$