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Parametric identification of parallel Wiener–Hammerstein systems ☆

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ABSTRACT

Block-oriented nonlinear models are popular in nonlinear modeling because of their advantages to be quite simple to understand and easy to use. To increase the flexibility of single branch block-oriented models, such as Hammerstein, Wiener, and Wiener–Hammerstein models, parallel block-oriented models can be considered. This paper presents a method to identify parallel Wiener–Hammerstein systems starting from input–output data only. In the first step, the best linear approximation is estimated for different input excitation levels. In the second step, the dynamics are decomposed over a number of parallel orthogonal branches. Next, the dynamics of each branch are partitioned into a linear time invariant subsystem at the input and a linear time invariant subsystem at the output. This is repeated for each branch of the model. The static nonlinear part of the model is also estimated during this step. The consistency of the proposed initialization procedure is proven. The method is validated on real-world measurements using a custom built parallel Wiener–Hammerstein test system.

1. Introduction

Nonlinear models are much needed these days to improve plant control performance, to gain better insight in the behavior of the system under test, or to compensate for a potential nonlinear behavior. Due to the separation of the nonlinear dynamic behavior into linear time invariant (LTI) dynamics and the static nonlinearities (SNL), block-oriented nonlinear models are quite simple to understand and easy to use.

A wide variety of block-oriented models has been studied over the last years including Hammerstein (Nonlinear static—Linear dynamic or N–L connection) and Wiener models (L–N) (Giri & Bai, 2010). This type of single branch models can be extended to Hammerstein–Wiener models (N–L–N) (Bai, 1998; Crama & Schoukens, 2004; Schoukens, Bai, & Rolain, 2012), or Wiener–Hammerstein models (L–N–L) (Billings & Fakhouri, 1978; Schoukens, Pintelon, & Rolain, 2014; Sjöberg & Schoukens, 2012; Vandersteen, Rolain, & Schoukens, 1997; Westwick & Schoukens, 2012). To increase the flexibility of the single branch block-oriented models even more, parallel block-oriented models can be considered such as parallel Hammerstein (Gallman, 1975; Schoukens, Pintelon, & Rolain, 2011), and parallel Wiener models (Lyzell, Andersen, & Enqvist, 2012; Schoukens, Lyzell, & Enqvist, 2013; Schoukens & Rolain, 2012).

This paper presents a method to identify parallel Wiener–Hammerstein systems, whose structure is shown in Fig. 1. Previously published methods (Baumgartner & Rugh, 1975; Billings & Fakhouri, 1979; Wysocki & Rugh, 1976) studied a subclass of the parallel Wiener–Hammerstein structure that is called the S_M model structure. Identification methods based on repeated sine measurements (Baumgartner & Rugh, 1975; Wysocki & Rugh, 1976), or white Gaussian inputs (Billings & Fakhouri, 1979) are available for this model structure. In Palm (1978, 1979) it is shown that a wide class of Volterra systems can be approximated arbitrary well using a parallel Wiener–Hammerstein model structure. However, no method is presented there to identify such models.

The S_M identification method presented in Billings and Fakhouri (1979) uses Gaussian excitation signals, like the method presented in this paper. However, the S_M method is a generalization of a Wiener–Hammerstein identification algorithm based on a parameterized version of higher order correlation functions between input and output (Billings & Fakhouri, 1978). This approach has been compared in Schoukens et al. (2014) with two other approaches (Schoukens et al., 2014; Westwick & Schoukens, 2012), and it was outperformed by these alternatives. The main problem of the method seems to be the noise sensitivity.

The parallel Wiener–Hammerstein identification approach proposed here combines the parallel Hammerstein and parallel

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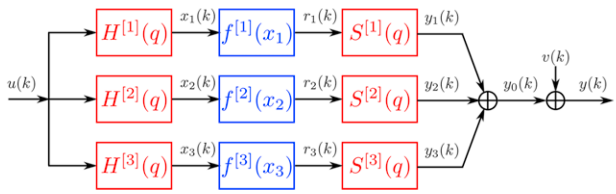

Fig. 1. A 3-branch parallel Wiener–Hammerstein system: a parallel connection of Wiener–Hammerstein systems. The static nonlinear block $f^{[i]}$ of the ith branch is sandwiched in between the LTI blocks $H^{[i]}(q)$ and $S^{[i]}(q)$. The noise source $v(k)$ is additive colored noise.

Wiener identification methods presented in Schoukens et al. (2011) and Schoukens and Rolain (2012) with a specific initialization approach for Wiener–Hammerstein systems presented in Sjöberg and Schoukens (2012). This paper hereby extends the results of Schoukens, Vandersteen, and Rolain (2013). In the paper presented here, the consistency of the proposed initialization procedure is proven, the computational aspects of the proposed method are discussed, the positive effect of the initialization method is shown, and the method is applied to a real-world measurement example.

The outline of the paper is as follows. Section 2 introduces the system and signal classes, and the stochastic framework used. Section 3 discusses the identifiability of a parallel Wiener–Hammerstein system. Next, the best linear approximation (BLA) of a parallel Wiener–Hammerstein system is studied in Section 4. The identification algorithm for parallel Wiener–Hammerstein systems is explained in Section 5. Section 6 discusses the persistence of excitation, Section 7 proves the consistency of the proposed identification method. A final, jointly nonlinear least squares optimization with respect to all the parameters of all the blocks is performed in Section 8. Some computational aspects of the method are discussed in Section 9. Finally, the good performance of the proposed method is illustrated in Section 10 on real-world measurements using a custom built parallel Wiener–Hammerstein test system. The positive effect of the proposed initialization method on the performance of the optimized model is also shown in this section.

2. System, signals and stochastic framework

This section describes the system and signal classes, and introduces the stochastic framework considered in this paper.

Definition 1 (*Riemann Equivalence Class of Asymptotically Normally Distributed Excitation Signals*)**.** Consider a signal u with a power spectrum $S_U(j\omega)$, which is piecewise continuous, with a finite number of discontinuities. A random signal belongs to the Riemann equivalence class of u if it obeys by any of the following statements:

(1) It is a Gaussian noise excitation with power spectrum $S_U(j\omega)$.

(2) It is a random multisine or random phase multisine (Pintelon & Schoukens, 2012) such that:

$$\frac{1}{N}\sum_{k=k_1}^{k_2} E\left\{|U\left(j\omega_k\right)|^2\right\} = \frac{1}{2\pi}\int_{\omega_{k_1}}^{\omega_{k_2}} S_U\left(\nu\right)d\nu + O\left(N^{-1}\right),$$

with $\omega_k = k\frac{2\pi f_s}{N}, k \in \mathbb{N}, 0 < \omega_{k_1} < \omega_{k_2} < \pi f_s$, and f_s the sample frequency.

Assumption 1. The excitation signal $u(k)$ is stationary and belongs to the Riemann equivalence class of asymptotically normally distributed excitation signals.

Assumption 2. An additive, colored zero-mean noise source $v(k)$ with a finite variance is present at the output of the system only:

$$y(k) = y_0(k) + v(k), \tag{1}$$

where $y(k), y_0(k)$ and $v(k)$ are scalars. The noise $v(k)$ is assumed to be independent from the known input $u(k)$.

Assumption 2 excludes that the system operates in closed loop.

The class of parallel Wiener–Hammerstein systems is considered. A parallel Wiener–Hammerstein system consists of a parallel connection of different Wiener–Hammerstein systems that share the same input signal. The output of the total system is obtained as the sum of the outputs of the different branches. A parallel Wiener–Hammerstein system with three parallel branches is shown in Fig. 1.

The noiseless output $y_0(k)$ of a parallel Wiener–Hammerstein system is given by:

$$y_0(k) = \sum_{i=1}^{n_{br}} y_i(k), \tag{2}$$

$$y_i(k) = S^{[i]}(q)r_i(k), \tag{3}$$

$$r_i(k) = f^{[i]}(x_i(k)), \tag{4}$$

$$x_i(k) = H^{[i]}(q)u(k), \tag{5}$$

where n_{br} is the number of parallel branches in the parallel Wiener–Hammerstein system, $H^{[i]}(q)$ and $S^{[i]}(q)$ are the front and back discrete time representations of the LTI blocks present in branch i, $f^{[i]}(x_i(k))$ is the static nonlinear block present in branch i, and the signals are as shown in Fig. 1.

All the LTI blocks are considered to be modeled by stable infinite impulse response (IIR) filters, parameterized by a rational function in the backwards shift operator q^{-1}:

$$H^{[i]}(q) = \frac{B_h^{[i]}(q)}{A_h^{[i]}(q)} = \frac{b_{h,0}^{[i]} + b_{h,1}^{[i]}q^{-1} + \cdots + b_{h,n_{b_h,i}}^{[i]}q^{-n_{b_h,i}}}{a_{h,0}^{[i]} + a_{h,1}^{[i]}q^{-1} + \cdots + a_{h,n_{a_h,i}}^{[i]}q^{-n_{a_h,i}}}, \tag{6}$$

$$S^{[i]}(q) = \frac{B_s^{[i]}(q)}{A_s^{[i]}(q)} = \frac{b_{s,0}^{[i]} + b_{s,1}^{[i]}q^{-1} + \cdots + b_{s,n_{b_s,i}}^{[i]}q^{-n_{b_s,i}}}{a_{s,0}^{[i]} + a_{s,1}^{[i]}q^{-1} + \cdots + a_{s,n_{a_s,i}}^{[i]}q^{-n_{a_s,i}}}, \tag{7}$$

where $n_{b_h,i}$ and $n_{a_h,i}$ are respectively the finite orders of the numerator and denominator of the front dynamics of the ith parallel branch, and $n_{b_s,i}$ and $n_{a_s,i}$ are the orders of the numerator and denominator of the back dynamics of the ith parallel branch.

The static nonlinear function $f^{[i]}(x_i(k))$ contained in the ith branch is described by a linear combination of n_f nonlinear basis functions:

$$f^{[i]}(x_i(k)) = \sum_{j=1}^{n_f} \beta_f^{[i]} f_j^{[i]}(x_i(k)). \tag{8}$$

Each basis function $f_j^{[i]}(x)$ is assumed to have a finite output for any finite input x. Examples of such nonlinear functions are polynomial functions, piecewise linear functions or radial basis function networks.

Assumption 3. The true system is a discrete time parallel Wiener–Hammerstein system, as described by Eqs. (1)–(8).