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Dynamic coupling design for nonlinear output agreement and time-varying flow control☆

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ABSTRACT

This paper studies the problem of output agreement in networks of nonlinear dynamical systems under time-varying disturbances, using dynamic diffusive couplings. Necessary conditions are derived for general networks of nonlinear systems, and these conditions are explicitly interpreted as conditions relating the node dynamics and the network topology. For the class of incrementally passive systems, necessary and sufficient conditions for output agreement are derived. The approach proposed in the paper lends itself to solve flow control problems in distribution networks. As a first case study, the internal model approach is used for designing a controller that achieves an optimal routing and inventory balancing in a dynamic transportation network with storage and time-varying supply and demand. It is in particular shown that the time-varying optimal routing problem can be solved by applying an internal model controller to the dual variables of a certain convex network optimization problem. As a second case study, we show that droop-controllers in microgrids have also an interpretation as internal model controllers.

1. Introduction

Output agreement has evolved as one of the most important control objectives in cooperative control. It appears in various contexts, ranging from distributed optimization (Tsitsiklis, Bertsekas, & Athans, 1986), formation control (Olfati-Saber, Fax, & Murray, 2007) up to oscillator synchronization (Stan & Sepulchre, 2007). Over the past years, it has become evident that the *internal model principle* takes a central role in output agreement problems, see e.g. Bai, Arcak, and Wen (2011), De Persis (2013), De Persis and Jayawardhana (2014),Wieland, Sepulchre, and Allgöwer (2011). Independently and in parallel, it was shown that *passivity* and related systems theoretic concepts have outstanding relevance in the analysis and synthesis of synchronizing networks and output agreement problems, see e.g, Arcak (2007), Bai et al. (2011), Bürger, Zelazo, and Allgöwer (2014), De Persis and Jayawardhana (2012), Scardovi, Arcak, and Sontag (2010), Stan and Sepulchre (2007), van der Schaft and Maschke (2013).

The present paper studies output agreement in networks of heterogeneous nonlinear dynamical systems affected by external disturbances and presents an approach that combines elements from internal model control with those known in passivity-based cooperative control. We follow here the trail opened in Pavlov and Marconi (2008) for centralized output regulation and provide necessary and sufficient conditions for the solution of the output agreement problem for the class of *incrementally passive* systems. Our results provide a bridge connecting the two complementary approaches for output agreement problems, namely the internal model approach on the one hand, and the passivity-based approach on the other hand.

The proposed approach is inherently different from other internal model approaches such as Isidori, Marconi, and Casadei (2013), Wieland et al. (2011), and Wieland, Wu, and Allgöwer (2013). In Wieland et al. (2011) each node is augmented with a local controller that contains a reference system, identical for all nodes, and the local controllers track the reference system. The local (''virtual'') copies of the reference system are then synchronized with static diffusive couplings. The approach considered in the present paper differs in central points. Most obviously, dynamic couplings, rather than local controllers, are investigated. Furthermore, external signals are assumed to affect the node dynamics, a case that is not covered in Wieland et al. (2011). Incrementally passive systems and disturbance rejection are also dealt with in De Persis and Jayawardhana (2014). However, the framework we propose here,

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inspired by Pavlov and Marconi (2008), is completely different and leads to a family of new distinct results that have not been considered in De Persis and Jayawardhana (2014).

The main contribution of this paper is the development of a control framework for network systems that integrates the ideas of internal model and passivity-based cooperative control. The proposed framework leads to both, constructive methods for the design of distributed controllers and a novel understanding of existing control approaches. In particular, we consider networks of heterogeneous nonlinear systems, interacting according to an undirected network topology. The objective is to design dynamic controllers placed on the edges of the network such that output agreement is achieved. Necessary conditions for the feasibility of the problem are presented for the general case of heterogeneous nonlinear systems with external disturbances. Sufficient conditions for convergence to output agreement are derived for *incrementally passive* systems. Following this, we present a relevant class of heterogeneous nonlinear systems for which all assumptions are met and the coupling controllers can be found following a constructive design procedure. The constructiveness of the result is further demonstrated via the design of *optimal routing controllers* for distribution systems with time-varying demand. To explain the relation to existing approaches the special situations where output agreement can be reached with static diffusive couplings or where the disturbances are constant are discussed. Based on these results, it is shown that *droop-controllers* in microgrids, as, e.g., studied in Simpson-Porco, Dörfler, and Bullo (2013), are designed exactly in accordance with the internal model control approach. Early results on the internal model approach to output agreement have been presented in Bürger and De Persis (2013).

The remainder of the paper is organized as follows. The problem formulation and necessary conditions for output agreement are presented in Section 2. Sufficient conditions for output agreement in networks of incrementally passive systems are discussed in Section 3. A constructive procedure for the design of such controllers for a class of nonlinear systems is presented in Section 4. The design procedure is applied to a time-varying optimal distribution problem in Section 5. In Section 6, the relation to known methods in the literature is formally discussed, and an interpretation of droop-controllers as internal model controllers is provided in Section 7.

Notation: The set of (positive) real numbers is denoted by $\mathbb{R}$ ($\mathbb{R}_{\geq}$). Given two matrices A and B, the *Kronecker product* is denoted by $A \otimes B$. The Moore–Penrose inverse (or pseudo-inverse) of a non-invertible matrix A is denoted by $A^\dagger$. The *range-space* and *null-space* of a matrix B are denoted by $\mathcal{R}(B)$ and $\mathcal{N}(B)$, respectively. A graph $\mathcal{G} = (V, E)$ is an object consisting of a finite set of nodes, $|V| = n$, and edges, $|E| = m$. The incidence matrix $B \in \mathbb{R}^{n\times m}$ of the graph $\mathcal{G}$ with arbitrary orientation, is a $\{0, \pm 1\}$ matrix with $[B]_{ik}$ having value '+1' if node i is the initial node of edge k, '−1' if it is the terminal node, and '0' otherwise.

2. Problem formulation and necessary conditions

We consider a network of dynamical systems defined on a connected, undirected graph $\mathcal{G} = (V, E)$. Each node represents a nonlinear system

$$\begin{aligned} \dot{x}_i &= f_i(x_i, u_i, w_i) \\ y_i &= h_i(x_i, w_i), \quad i = 1, 2, \dots, n, \end{aligned} \tag{1}$$

where $x_i \in \mathbb{R}^{r_i}$ is the state, and $u_i, y_i \in \mathbb{R}^p$ are the input and output, respectively. Each system (1) is driven by the time-varying signal $w_i \in \mathbb{R}^{q_i}$, representing, e.g., a disturbance or reference. We assume that the exogenous signals w_i are generated by systems of the form

$$\dot{w}_i = s_i(w_i), \qquad w_i(0) \in \mathcal{W}_i, \tag{2}$$

where $\mathcal{W}_i$ is a set whose properties are specified below. A common assumption in nonlinear output regulation theory is neutral stability[1] of the exo-systems (Isidori & Byrnes, 2008). For the purpose of this paper, it is advantageous to restrict the discussion to a slightly smaller class of exo-systems.

Assumption 1. The vector field $s_i(w_i)$ satisfies for all w_i, w_i' the inequality

$$(w_i - w_i')^T (s_i(w_i) - s_i(w_i')) \leq 0. \tag{3}$$

Remarkably, Assumption 1 includes in particular neutrally stable linear exo-systems, i.e., a linear function $s_i(w_i) = S_i w_i$ with skew-symmetric matrix S, i.e., $S_i^T + S_i = \mathbf{0}$ satisfies the requirements. These linear exosystems can generate signals that are combinations of constant and periodic modes.

In addition, our assumption includes various nonlinear dynamical systems. For example, it has been shown in DeLellis, di Bernardo, and Garofalo (2009, Sec. 4.3) that nonlinear *Chua's oscillators* satisfy Assumption 1. We stack together the signals w_i, for $i = 1, 2, \dots, n$, and obtain the vector $w \in \mathbb{R}^q$, which satisfies the equation $\dot{w} = s(w)$. In what follows, whenever we refer to the solutions of $\dot{w} = s(w)$, we assume that the initial condition is chosen in a compact set $\mathcal{W} = \mathcal{W}_1 \times \dots \times \mathcal{W}_n$. The set $\mathcal{W}$ is assumed to be forward invariant for the system $\dot{w} = s(w)$. Similarly, let x, u, and y be the stacked vectors of x_i, u_i, and y_i, respectively. Using this notation, the totality of all systems is

$$\begin{aligned} \dot{w} &= s(w) \\ \dot{x} &= f(x, u, w) \\ y &= h(x, w) \end{aligned} \tag{4}$$

with state space $\mathcal{W} \times \mathcal{X}$ and $\mathcal{X}$ a compact subset of $\mathbb{R}^{r_1} \times \dots \times \mathbb{R}^{r_n}$.

The control objective is to reach output agreement of all nodes in the network, independent of the exact representation of the time-varying external signals. We aim to achieve this control objective by a suitable design of dynamic couplings between the systems. This means, between any pair of neighboring nodes, i.e., on any edge of $\mathcal{G}$, a dynamical system (in the following called "*controller*") is placed, taking the form

$$\begin{aligned} \dot{\xi}_k &= F_k(\xi_k, v_k) \\ \lambda_k &= H_k(\xi_k, v_k), \quad k = 1, 2, \dots, m, \end{aligned} \tag{5}$$

with state $\xi_k \in \mathbb{R}^{l_k}$, input $v_k \in \mathbb{R}^p$ and output $\lambda_k \in \mathbb{R}^p$. Using the same notational convention as before, we define ξ and λ as the stacked state and output vector. Together, the controllers (5) give raise to the overall controller

$$\begin{aligned} \dot{\xi} &= F(\xi, v) \\ \lambda &= H(\xi, v), \end{aligned} \tag{6}$$

where $\xi \in \Xi$, a compact subset of $\mathbb{R}^{l_1} \times \dots \times \mathbb{R}^{l_m}$. The collection (6) of dynamical systems (5) generates the overall output λ that determines the control input u applied to the network system (4) via the interconnection (9) below. This motivates the choice of referring to the dynamical systems (5) as controllers.

Throughout the paper the following interconnection structure between the plants, placed on the nodes of $\mathcal{G}$, and the controllers, placed on the edges of $\mathcal{G}$, is considered. A controller (5), associated with edge k connecting nodes i, j, has access to the relative outputs $y_i - y_j$. In vector notation, the relative outputs of the systems are

$$z = (B^T \otimes I_p) y, \tag{7}$$

[1] The dynamics (2) is said to be neutrally stable if it admits a Lyapunov stable equilibrium point for $w = 0$ and there exists a neighborhood of Poisson stable points around $w = 0$.