Automatica 51 (2015) 1–8

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Coarsest quantization for networked control of uncertain linear systems ☆

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ARTICLE INFO

Article history:
Received 15 October 2013
Received in revised form
19 May 2014
Accepted 10 October 2014

Keywords:
Communication constraints
Networked control
Quadratic stability
Quantization
System uncertainties

ABSTRACT

In the design of networked control systems, one must take account of communication constraints in the form of data rate. In this paper, we consider a quantized control problem for stabilizing uncertain linear systems in the sense of quadratic stability. For a class of finite-order (possibly time-varying) uncertain autoregressive plants, we show that the coarsest quantizer for achieving quadratic stabilization is of logarithmic type. In particular, for a given quadratic Lyapunov function, the largest coarseness is derived in an analytic form. The result explicitly shows that plants with more uncertainties require more precise information in the quantized signals to achieve quadratic stabilization. We also provide a numerical method based on a linear matrix inequality to search for a Lyapunov function along with a quantizer of a given level of coarseness.

1. Introduction

In the context of networked control systems, issues related to quantization have recently been studied extensively. Such research is motivated by the fact that in networked systems, analog signals must be quantized into digital ones to be transferred over networks in the form of packets. The quantization process certainly introduces some loss in data, and thus a fundamental interest lies in finding how much information is necessary for the purpose of feedback control. For stabilization of linear systems, limitations in the communication of control signals have been found under several problem formulations including the well-known minimum data rate theorem (see, e.g., Bemporad, Heemels, & Johansson, 2010; Matveev & Savkin, 2008; Nair, Fagnani, Zampieri, & Evans, 2007 and the references therein).

The work of Elia and Mitter (2001) initiated a line of research on characterizing the coarseness in quantization in networked control when static (deterministic) quantizers are employed. Such quantizers are useful in applications for their simplicity in implementation. In particular, the work exhibited that for achieving quadratic stability of the closed-loop system, the so-called logarithmic quantizers are the coarsest. Such quantizers have an interesting property that quantization is fine around the origin, but becomes coarser as the input becomes larger. This seems a reasonable structure for the objective of controlling the states to the origin. Moreover, the coarseness has a limitation depending on the plant properties and specifically on the unstable poles. This indicates that more unstable systems require finer quantization.

This problem setting has further been studied by various researchers. In Fu and Xie (2005) and Gao and Chen (2008), control performance such as quadratic costs and the H^∞ norm is considered while Tsumura, Ishii, and Hoshina (2009) extend the coarsest quantization results to the random packet loss scenario. Sampled-data control strategies are developed in Ishii and Francis (2002) and Ishii, Başar, and Tempo (2004), where quadratic stability is guaranteed in the continuous-time domain. In Ceragioli and De Persis (2007) and Liu, Jiang, and Hill (2012), nonlinear control problems with the use of logarithmic type quantizers are addressed. The works Chen and Qiu (2013) and Qiu, Gu, and Wan (2013) derive limitations for resource allocation in the multiple channel case.

On the other hand, in the area of quantized control, the effect of plant uncertainty has also received some attention. For data-rate limited control, in Phat, Jiang, Savkin, and Petersen (2004), a sufficient condition on the data rate to attain quantized feedback stabilization has been developed. The paper by Vu and Liberzon

☆ This work was supported in part by the Ministry of Education, Culture, Sports, Science and Technology, Japan, under Grant-in-Aid for Scientific Research Grant No. 23760385 and by the Aihara Project, the FIRST program from JSPS, initiated by CSTP. The material in this paper was partially presented at the 4th IFAC Workshop on Distributed Estimation and Control in Networked Systems, September 14–15, 2012, Santa Barbara, California, USA. This paper was recommended for publication in revised form by Associate Editor Constantino M. Lagoa under the direction of Editor Roberto Tempo.

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http://dx.doi.org/10.1016/j.automatica.2014.10.113
0005-1098/

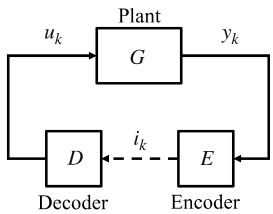

Fig. 1. Networked control system.

(2012) examines the use of a switching control scheme which determines the suitable controller by estimating the plant parameters at the time. Also, the work of Martins, Dahleh, and Elia (2006) studies the stabilization of a stochastically time-varying system. For control with coarse quantization, the work of Fu and Xie (2010) considers robust control problems for both static and dynamic logarithmic quantizers. Adaptive control strategies can also be found in Hayakawa, Ishii, and Tsumura (2009), where the knowledge of the bound on uncertainties is not needed.

More recently, in Okano and Ishii (in press), the minimum data rate is obtained for a class of finite-order uncertain autoregressive plants. It is expressed by the unstable poles of the nominal plant and a bound on an uncertain parameter. An interesting implication there is that an optimal nonuniform quantizer can be designed for reducing the necessary communication.

In this paper, we study the stabilization problem for a similar class of uncertain linear systems. Based on the approach of Elia and Mitter (2001), our goal is to characterize the coarsest quantization scheme for such systems. The strategy here is (i) to fix the desired quadratic Lyapunov function suitable for closed-loop stability when there is no network effect, and (ii) then to find the controller structure as well as the limitation on the quantization coarseness under the given Lyapunov function.

We provide an analytic solution to this problem, which generalizes those for plants without uncertainties. In particular, we clarify how the level of uncertainty affects the tolerable coarseness in quantization. It will be observed that for more uncertain systems, finer quantization is required for stabilization. We will also demonstrate that unlike the non-uncertain case of Elia and Mitter (2001), it is difficult to find the coarsest quantizer in a closed form over *all* quadratic Lyapunov functions. The results are first developed for the special case of global asymptotic stability with infinite output values in quantization. We also provide a numerical method based on a linear matrix inequality (LMI) that searches for a Lyapunov function along with a quantizer of a given level of coarseness. It can be used to find the coarsest quantizer over all quadratic Lyapunov functions.

This paper is organized as follows. In Section 2, we formulate the quantized control problem for uncertain networked systems. In Section 3, we discuss the coarsest quantizer for a given Lyapunov function and provide the main result. In Section 4, we derive local stability results using finite quantizers. A numerical example is presented in Section 5 to illustrate the proposed approach. Finally, concluding remarks are given in Section 6. This paper is based on the earlier version (Kang & Ishii, 2013), but contains the full proofs with enhanced results.

2. Problem formulation

In this section, we formulate the quantized control problem studied in this paper.

Consider the networked control system depicted in Fig. 1. We first describe the system setup briefly. The plant G is a single-input single-output discrete-time linear system and has uncertain parameters. The control signal $u_k \in \mathcal{R}$ is generated by the controller formed by the encoder E and the decoder D. Between them, the discrete signal i_k is sent over the network. In the channel, we assume that there is no network latency and the data rate is high enough to transfer all the data within the sampling period. To simplify the problem, we initially assume that the word length is infinite. That is, the quantized signal takes discrete values which may form an infinite set. In Section 4, we will discuss the more realistic case with a finite number of values.

The plant G is an n-dimensional autoregressive system with possibly time-varying uncertain parameters as

$$y_{k+1} = a_{1,k}y_k + a_{2,k}y_{k-1} + \cdots + a_{n,k}y_{k-n+1} + u_k, \tag{1}$$

where $u_k \in \mathcal{R}$ is the input and $y_k \in \mathcal{R}$ is the output. The parameters $a_{i,k}$ are uncertain and take the form

$$a_{i,k} = a_i^* + \Delta_{i,k}, \quad i = 1, \ldots, n, \ k \in \mathcal{Z}_+,$$

where a_i^* is the nominal part and $\Delta_{i,k}$ is the uncertain part. Let $\Delta_k \in \mathcal{R}^{1\times n}$ be the uncertainty vector given by $\Delta_k := \begin{bmatrix}\Delta_{n,k} & \Delta_{n-1,k} & \cdots & \Delta_{1,k}\end{bmatrix}$. For a given $\delta \in [0, 1)$, the uncertainty is bounded as

$$\|\Delta_k\| \le \delta, \quad \forall k \in \mathcal{Z}_+. \tag{2}$$

The uncertain plant (1) can be expressed in the controllable canonical form as

$$x_{k+1} = A(\Delta_k)x_k + bu_k, \qquad y_k = cx_k, \tag{3}$$

where $x_k := [y_{k-n+1} \quad y_{k-n+2} \quad \cdots \quad y_k]^T \in \mathcal{R}^n$ is the state, and the system matrices $A(\Delta_k) \in \mathcal{R}^{n\times n}$, $b \in \mathcal{R}^n$, and $c \in \mathcal{R}^{1\times n}$ are given by

$$A(\Delta_k) := \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ a_{n,k} & a_{n-1,k} & \cdots & a_{1,k} \end{bmatrix},$$

$$b := \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \qquad c := b^T.$$

We denote the nominal A-matrix by $A^* := A(0)$.

The encoder E maps the output y_k to the discrete signal $i_k \in \mathcal{Z}$, and more specifically, it is a static function of the state x_k as $i_k = E(y_k, y_{k-1}, \ldots, y_{k-n+1}) = E(x_k)$. The decoder D generates the control input taking discrete values $\{\tilde{u}_i\}_{i\in\mathcal{Z}}$ based on the current i_k as $u_k = D(i_k) = \tilde{u}_{i_k}$. The maps E and D are assumed to be static, and thus we may write the overall controller as

$$u_k = K(x_k), \tag{4}$$

where $K : \mathcal{R}^n \to \{\tilde{u}_i\}_{i\in\mathcal{Z}}$ is called the quantized controller, or simply the quantizer.

In our networked control problem, the objective is to achieve stabilization of the closed-loop system. In particular, we would like to guarantee quadratic stability as defined below.

Definition 1. The uncertain networked control system in Fig. 1 is quadratically stable if there exists a positive-definite function $V(x) := x^T Px$ with $P > 0$ such that each trajectory $\{x_k\}$ of the closed-loop system satisfies

$$V(x_{k+1}) - V(x_k) < 0, \quad \text{if } x_k \neq 0, \ k \in \mathcal{Z}_+. \tag{5}$$

For the quantized controller K, we employ the notion of coarseness introduced in Elia and Mitter (2001) given by

$$d_K := \limsup_{\epsilon\to 0} \frac{\sharp u[\epsilon]}{-\ln \epsilon}, \tag{6}$$