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Brief paper

# Finite-time control of robotic manipulators☆

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ABSTRACT

This work offers the solution at the control feed-back level of the accurate trajectory tracking subject to finite-time convergence. Dynamic equations of a rigid robotic manipulator are assumed to be uncertain. Moreover, globally unbounded disturbances are allowed to act on the manipulator when tracking the trajectory. Based on the suitably defined non-singular terminal sliding vector variable and the Lyapunov stability theory, we propose a class of absolutely continuous robust controllers which seem to be effective in counteracting both uncertain dynamics and unbounded disturbances. The numerical simulation results carried out for a robotic manipulator consisting of two revolute kinematic pairs operating in a two-dimensional joint space illustrate performance of the proposed controllers.



## 1. Introduction

Present-day robotic tasks require high precision and stability of their performance. Trajectory tracking seems to be a fundamental task in robot control. In order to fulfil aforementioned requirements, control algorithms should take into account the following factors: model uncertainties, parameter variations and external disturbances. However, they are, in fact, never known exactly in practice. Therefore, it is particularly important to design control algorithms that ensure accurate and fast convergence to the stable equilibrium when trajectory tracking despite the existence of the aforementioned factors. In such a context, several control schemes for asymptotic tracking of manipulator trajectories can be found in the literature (Corless, 1993; Galicki, 2008, 2012; Hsu & Fu, 2006; Slotine & Li, 1991; Utkin, 1978; Zhang, Dawson, de Queiroz, & Dixon, 2000) which partially or fully take into account these factors. Sliding-mode control seems to be one of the most effective approaches to cope with uncertainties. As is well known, sliding mode is accurate and insensitive to disturbances (Edwards & Spurgeon, 1998; Utkin, 1992). However, the main drawback of the standard first-order sliding modes is mostly related to the undesirable chattering effect (Fridman, 2002). The second- and higher-order sliding techniques to eliminate the chattering have been proposed (Bartolini, Ferrara, & Punta, 2000; Bartolini, Ferrara, Usai, & Utkin, 2000; Bartolini, Pisano, Punta, & Usai, 2003; Bartolini & Pydynowski, 1996; Ferrara & Capisani, 2011; Levant, 1998, 2003, 2005, 2011; Levant & Michael, 2009; Mondal & Mahanta, 2014; Shtessel, Shkolnikov, & Brown, 2003; Sira-Ramírez, 1992). Nevertheless, the approaches from Bartolini, Ferrara, Punta (2000), Bartolini, Ferrara, Usai et al. (2000), Bartolini et al. (2003), Bartolini and Pydynowski (1996), Ferrara and Capisani (2011), Mondal and Mahanta (2014), Shtessel et al. (2003) and Sira-Ramírez (1992) are able to steer a tracking error to zero asymptotically and those from Levant (1998, 2003, 2005, 2011) and Levant and Michael (2009) are only applicable to single input dynamic systems. In order to both increase tracking accuracy and accelerate a convergence process to the stable equilibrium, terminal sliding mode (TSM) control techniques have been offered as a particularly useful tool for high precision control of robotic manipulators. In such a context, several approaches can be distinguished (Hong, Xu, & Huang, 2002; Su, 2009; Su & Zheng, 2011) that produce (non-smooth) continuous controls but require the full knowledge of robot dynamic equations. By using the regressor matrix technique, adaptive-discontinuous TSM controllers have been designed in works Barambones and Etxebarria (2002), Parra-Vega, Rodrigues-Angeles, and Hirzinger (2001) and Tang (1998). An alternative terminal sliding manifold has been proposed in Feng, Yu, and Man (2002), Jin, Lee, Chang, and Choi (2009) and Yu, Yu, Shirinzadeh, and Man (2005) to eliminate the singularity problem. Nevertheless, the common feature of the approaches from Feng et al. (2002), Jin et al. (2009) and Yu et al. (2005) is necessity of knowledge of the nominal robot dynamic equations whose construction may not be a trivial task. Recently, a robust discontinuous TSM control for robotic manipulators has been proposed in Zhao, Li, and Gao (2009). A similar approach with a singularity problem has also been presented in Man, Paplinski, and Wu (1994). From the

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literature survey, it follows that all the aforementioned algorithms are not able to generate continuous controls resulting in finite-time stability of the equilibrium when both dynamic equations are uncertain and (unbounded) disturbances act on the robotic manipulators. Hence, there is a need to provide additional information (including the joint position and velocity or its estimation) for a control scheme to be designed further on. From the robotic point of view, joint acceleration is such additional quantity. In general, there are two approaches for the joint acceleration acquisition. The first is based on the direct measurement of joint acceleration (De Luca, Schroder, & Thummel, 2007; Godler, Akahane, Maruyama, & Yamashita, 1995). The second approach uses a class of uniform robust differentiators (Levant, 2003; Levant & Livne, 2012). Based on the available joint acceleration or its estimation, a new non-singular TSM manifold is introduced in this study. The proposed TSM manifold makes it possible to simultaneously join the first-order sliding mode approach possessing the finite-time control capabilities with the second-order sliding mode techniques generating the (absolutely) continuous controls. It is worth to emphasise that the finite-time control of robotic manipulators subject to uncertain dynamic equations, absolute continuity control requirement and globally unbounded disturbances, is still a non-trivial problem whose solution is based in this work on introducing a dynamic version of a static computed torque approach presented in e.g. works Siciliano, Sciavicco, Villani, and Oriolo (2009) and Spong and Vidyasagar (1989). The remainder of the paper is organised as follows. Section 2 formulates the finite-time trajectory tracking task. Section 3 sets up a class of robust absolutely continuous controllers solving the trajectory tracking task in a finite-time subject to uncertain robot dynamic equations and unbounded disturbances. Section 4 presents computer examples of trajectory tracking by a robotic manipulator consisting of two revolute kinematic pairs. Finally, some concluding remarks are drawn in Section 5. Throughout this paper, $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimal and maximal, respectively, eigenvalues of the symmetric matrix $(\cdot)$. Moreover, the real branch of $x^{\frac{a}{b}}$, where $x \in \mathbb{R}$; $a, b$ are positive odd numbers, and $a < b < 2a$, is taken here into account.

## 2. Problem formulation

The dynamics of a rigid robotic manipulator of $n$-DoF is given by the following general equations (Spong & Vidyasagar, 1989):

$$M(q)\ddot{q} + H(q,\ \dot{q}) + G(q) + D(t,\ q,\ \dot{q}) = v, \tag{1}$$

where $q = (q_1, \ldots, q_n)^T$, $\dot{q}$ and $\ddot{q}$ represent the position, velocity and acceleration, respectively. The $n \times n$ inertia matrix $M(q)$ is positive definite and symmetric. The term $H$ in (1) equals $H = B(q)(\dot{q} \cdot \dot{q}) + C(q)(\dot{q}^2)$, where $B$ and $C$ are the $n \times \frac{n(n-1)}{2}$ and $n \times n$ matrices of coefficients of the Coriolis and centrifugal forces, respectively. Quantities $(\dot{q} \cdot \dot{q})$ and $(\dot{q}^2)$ are the symbolic notations for the $\frac{n(n-1)}{2}$-dimensional and $n$-dimensional vectors $(\dot{q} \cdot \dot{q}) = (\dot{q}_1\dot{q}_2, \ldots, \dot{q}_{n-1}\dot{q}_n)^T$ and $(\dot{q}^2) = (\dot{q}_1^2, \ldots, \dot{q}_n^2)^T$, respectively. Term $v = (v_1, \ldots, v_n)^T$ stands for the $n$-dimensional vector of controls (torques/forces). Term $G(q)$ is the $n$-dimensional vector of generalised gravity forces. Vector $D(t,\ q,\ \dot{q})$ means the $n$-dimensional external disturbance signal which is (by assumption) at least absolutely continuous mapping with $\dot{D}(t,\ q,\ \dot{q})$ as being a locally bounded Lebesgue measurable mapping (this implies existence of control $v$). Moreover, $\|D\|$ and $\|\dot{D}\|$ are (by assumption) upper estimated as follows

$$\|D\| \le \alpha_0(t), \qquad \|\dot{D}\| \le \alpha_1(t), \tag{2}$$

where $\alpha_0, \alpha_1$ stand for the known, non-negative functions. In the sequel, useful properties of (1) are summarised which will be utilised while designing the controller. The following inequalities are satisfied (Spong & Vidyasagar, 1989):

$$\begin{aligned} &0 < \lambda_{\min}(M^{-1}) \le \|M^{-1}\| \le \lambda_{\max}(M^{-1}), \\ &\|B + C\| \le c_1, \qquad \|G\| \le c_2, \end{aligned} \tag{3}$$

where $c_1, c_2$ are known positive scalar coefficients. In order to obtain at least absolutely continuous control $v$, let us differentiate the dynamic equations (1) with respect to time

$$M(q)\frac{d^3q}{dt^3} + F(q,\ \dot{q},\ \ddot{q},\ t) = \dot{v}, \tag{4}$$

where $F = \dot{M}\ddot{q} + \dot{B}(\dot{q}\cdot\dot{q}) + \dot{C}(\dot{q}^2) + B\frac{d}{dt}(\dot{q}\cdot\dot{q}) + C\frac{d}{dt}(\dot{q}^2) + \dot{G} + \dot{D}$. Based on the properties of (1), one obtains the following upper estimation of $\|F\|$:

$$\|F\| \le \mathcal{E}(q,\ \dot{q},\ \ddot{q},\ t), \tag{5}$$

where $\mathcal{E} = c_3\|\dot{q}\|\|\ddot{q}\| + c_4\|\dot{q}\|^3 + c_5\|\dot{q}\| + \alpha_1(t)$; $c_3, c_4, c_5$ are (known by assumption) positive scalar coefficients for which the following inequalities hold true: $\|\frac{\partial M}{\partial q}\| + \|B\| + \|C\| \le c_3$; $\|\frac{\partial C}{\partial q}\| \le c_4$; $\|\frac{\partial G}{\partial q}\| \le c_5$. Motivated in part by the static computed torque methodology (Siciliano et al., 2009; Spong & Vidyasagar, 1989), we propose now a dynamically computed torque vector $\dot{v}$ of the form

$$\dot{v} = \hat{M}(q)u + \hat{F}(q,\ \dot{q},\ \ddot{q},\ t), \tag{6}$$

where $\hat{M}$ and $\hat{F}$ denote known estimates of the corresponding unknown terms $M$ and $F$, respectively, in dynamic equations (4); $u \in \mathbb{R}^n$ is a new control to be found. The use of (6) as a dynamic non-linear control law gives $M\frac{d^3q}{dt^3} + F = \hat{M}u + \hat{F} = \dot{v}$. Since $M$ is invertible, we obtain

$$\frac{d^3q}{dt^3} = u + (\mathcal{R} - \mathbb{I}_n)u + Q, \tag{7}$$

where $\mathcal{R} = M^{-1}\hat{M}$; $Q = M^{-1}(\hat{F} - F)$; $\mathbb{I}_n$ stands for the $n \times n$ identity matrix. A task accomplished by the robotic manipulator consists in tracking a desired trajectory $q_d(t) \in \mathbb{R}^n, t \in [0,\ \infty)$ which is assumed to be at least triply continuously differentiable, i.e., $q_d(\cdot) \in C^3[0,\ \infty)$. By introducing the tracking error $e = (e_1, \ldots, e_n)^T = q - q_d(t)$, we may formally express the finite-time trajectory tracking control by means of the following equations:

$$\lim_{t\to T} e(t) = \lim_{t\to T} \dot{e}(t) = \lim_{t\to T} \ddot{e}(t) = 0, \tag{8}$$

where $0 \le T$ denotes a finite time of convergence of $q$ to $q_d$. The objective is to find an input signal $u(t)$ and consequently a control vector $v(t)$ by solving the differential equations (6) such that position vector $q$ follows $q_d$. The next section will present an approach to the solution of the control problem (6)–(8) making use of the Lyapunov stability theory.

## 3. Control of the robotic manipulator

In the sequel, we start the analysis of a controller design by the assumption that joint positions, velocities and accelerations are available from measurements. Let us note that the right-hand side of (7) requires the knowledge of joint acceleration $\ddot{q}$. Recently, a lot of techniques appeared in the literature which directly measure $\ddot{q}$ (De Luca et al., 2007; Godler et al., 1995). Based on (3), we can make the following remark:

$$(\exists \hat{M} > 0)(\exists \rho > 0)(\|\mathcal{R} - \mathbb{I}_n\| \le \rho < 1). \tag{9}$$

Let us note that it is not difficult to find matrix $\hat{M}$ fulfilling relations (9). If we set $\hat{M} = \frac{2}{\lambda_{\min}(M^{-1}) + \lambda_{\max}(M^{-1})}\mathbb{I}_n$ (see e.g. Spong & Vidyasagar, 1989) then $\rho = \frac{\lambda_{\max}(M^{-1}) - \lambda_{\min}(M^{-1})}{\lambda_{\max}(M^{-1}) + \lambda_{\min}(M^{-1})}$ satisfies inequality