Automatica 51 (2015) 149–157

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Dimensionality reduction for point feature SLAM problems with spherical covariance matrices[☆]

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ARTICLE INFO

Article history:
Received 22 December 2013
Received in revised form
19 August 2014
Accepted 12 October 2014

Keywords:
Multiple-step SLAM
Least squares
Dimensionality reduction

ABSTRACT

The main contribution of this paper is the dimensionality reduction for multiple-step 2D point feature based Simultaneous Localization and Mapping (SLAM), which is an extension of our previous work on one-step SLAM (Wang et al., 2013). It has been proved that SLAM with multiple robot poses and a number of point feature positions as variables is equivalent to an optimization problem with only the robot orientations as variables, when the associated uncertainties can be described using spherical covariance matrices. This reduces the dimension of original problem from $3m+2n$ to m only (where m is the number of poses and n is the number of features). The optimization problem after dimensionality reduction can be solved numerically using the unconstrained optimization algorithms. While dimensionality reduction may not provide computational saving for all nonlinear optimization problems, for some SLAM problems we can achieve benefits such as improvement on time consumption and convergence. For the special case of two-step SLAM when the orientation information from odometry is not incorporated, an algorithm that can guarantee to obtain the globally optimal solution (in the maximum likelihood sense) is derived. Simulation and experimental datasets are used to verify the equivalence between the reduced nonlinear optimization problem and the original full optimization problem, as well as the proposed new algorithm for obtaining the globally optimal solution for two-step SLAM.

1. Introduction

For more than 15 years, Simultaneous Localization and Mapping (SLAM) has been a key problem in robotics (Bailey & Durrant-Whyte, 2006). As a result, many algorithms have been proposed to solve SLAM in various forms. In the state-of-the-art approaches, under the assumption of (independent) Gaussian noise, the SLAM problem is formulated as a sparse non-linear least squares (NLLS) problem over m robot poses and n features' positions ($3m + 2n$ variables in 2D) (Dellaert & Kaess, 2006). Newton-based iterative solvers such as Gauss–Newton and Levenberg–Marquardt are among the most popular algorithms for solving this NLLS. The sparseness of this NLLS is a consequence of (i) conditional independence of features given the robot poses, (ii) limited range of sensors, and finally (iii) uncorrelated measurement noise. Exploiting this inherent property of SLAM problems is a key characteristic of many of the modern solvers (Huang, Wang, & Dissanayake, 2008; Kaess et al., 2012; Kummerle, Grisetti, Strasdat, Konolige, & Burgard, 2011).

It is now well-known that the SLAM problem becomes considerably easier to analyse when the noise covariance matrices are *spherical* (Wang, Huang, Frese, & Dissanayake, 2013). In Huang, Lai, Frese, and Dissanayake (2010), the authors reported an unexpected convergence of vanilla Gauss–Newton algorithm to the optimal solution from random initial guesses in high-dimensional SLAM problems when the noise covariance matrices are spherical.

☆ This work has been supported in part by the Funds of National Science of China (Grant Nos. 61004061, 61210306067), Australian Research Council Discovery project (DP120102786), and the German Research Foundation (DFG) under grant SFB/TR 8 Spatial Cognition. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Andrey V. Savkin under the direction of Editor Ian R. Petersen.

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http://dx.doi.org/10.1016/j.automatica.2014.10.114
0005-1098/

Similarly, in the tree-based network optimizer (TORO) (Grisetti, Stachniss, Grzonka, & Burgard, 2007), good convergence results from bad initial values are reported for large pose-graphs when the noise covariance matrices are spherical. More recently, Carlone (Carlone, 2013) addressed the convergence of Gauss–Newton algorithm in pose-graph SLAM. Under the assumption of spherical noise covariance matrices, a conservative estimate for the basin of attraction of the ML estimate in pose-graph SLAM was derived. In Huang, Wang, Frese, and Dissanayake (2012) and Wang, Hu, Huang, and Dissanayake (2012); Wang et al. (2013), authors studied the ML objective function in "one-step SLAM" when the noise covariance matrices are spherical. They show that minimizing the ML objective function (in one-step SLAM) can be reduced to a one-dimensional optimization problem over the robot orientation. Furthermore, it is shown that the reduced problem has a unique minimizer unless the noise is extremely large.

In this paper we extend the results of Huang et al. (2012) and Wang et al. (2012, 2013) along two directions: (i) we show that the ML estimate in general (m-step) feature-based SLAM problems can be obtained by solving a NLLS problem over only m variables (i.e., the robot orientations in m poses), in particular, the structure of the problem is closely related to the incidence matrix of the directed graph of the SLAM problem, and (ii) based on this result, we develop an algorithm that can be guaranteed to find the globally optimal solution to the 2-step SLAM when the orientation information in odometry is not used.

This paper is organized as follows. In Section 2, the ML estimate in SLAM is formulated as the solution to a NLLS problem. Section 3 introduces the definition of spherical matrices and derives an alternative SLAM formulation when covariance matrices are spherical. In Section 4, it is shown that the m-step ($m \geq 2$) SLAM problem is equivalent to another NLLS over only the robot orientations. In Section 5, we show that the globally optimal solution to a special case of two-step SLAM problem can be obtained by finding the roots of a polynomial with degree 6. In Section 6, examples are presented to illustrate the benefits of dimensionality reduction. Finally Section 7 concludes the paper.

Notation. Throughout the paper, $\otimes$ denotes the Kronecker product, superscript T and -1 stand for, respectively, the transposition and the inverse of a matrix; $C \succ D$ means that matrix $C - D$ is positive definite; I and I_n denote the identity matrix with compatible dimension and dimension n, 0 represents the zero matrix with compatible dimension, and $\|e\|_C^2 = e^T C e$, where $C \succ 0$ and e is a vector. wrap$(\cdot)$ is the function that maps an angle to its equivalent angle in $(-\pi, \pi]$. The symbol diag$(C_1, \ldots, C_n)$ denotes a block-diagonal matrix whose diagonal blocks are $C_1, \ldots, C_n$.

2. Problem formulation

Suppose n 2D point features $\{f_i\}_{i=1}^n$ are observed from a sequence of $m+1$ 2D robot poses $\{r_i\}_{i=0}^m$. We use Z_k^i to denote the observation made from pose r_i to feature f_k. We use O_i ($1 \leq i \leq m$) to denote the odometry measurement between pose r_{i-1} and pose r_i. Both the odometry and observations are corrupted by zero-mean Gaussian noises with covariance matrices $P_{Z_k^i}$ and P_{O_i}, respectively. $X_{f_k} = (x_{f_k}, y_{f_k})^T$ denotes the position of feature f_k. $X_{r_i} = (x_{r_i}, y_{r_i})^T$ denotes the position of robot pose r_i while ϕ_{r_i} denotes the orientation of robot pose r_i. The coordinate frame is defined by the robot pose r_0. That is, $X_{r_0} = (0, 0)^T$ and $\phi_{r_0} = 0$. $R(\phi_{r_i})$ is the rotation matrix corresponding to ϕ_{r_i} defined by:

$$R(\phi_{r_i}) \triangleq \begin{bmatrix} \cos\phi_{r_i} & -\sin\phi_{r_i} \\ \sin\phi_{r_i} & \cos\phi_{r_i} \end{bmatrix}. \quad (1)$$

The non-linear least squares (NLLS) SLAM formulation (Dellaert & Kaess, 2006) uses the odometry and observation information to estimate the state vector containing all the robot poses and all the feature positions

$$X \triangleq (X_{f_1}^T, \ldots, X_{f_n}^T, X_{r_1}^T, \phi_{r_1}, \ldots, X_{r_m}^T, \phi_{r_m})^T \quad (2)$$

and minimizes the negative log-likelihood function

$$F(X) = \sum_{i=0}^{m} \sum_{j=1}^{n_i} \|Z_{k_{ij}}^i - H^{Z_{k_{ij}}^i}(X)\|^2_{P^{-1}_{Z^i_{k_{ij}}}} + \sum_{i=1}^{m} \|O_i - H^{O_i}(X)\|^2_{P^{-1}_{O_i}} \quad (3)$$

where O_i ($1 \leq i \leq m$) are odometries, $Z_{k_{ij}}^i$ are observations (assume n_i features are observed from robot pose r_i and k_{ij} is the global index of the jth feature observed from pose r_i), and P_{O_i} and $P_{Z_{k_{ij}}^i}$ are the corresponding covariance matrices.

In the above least squares SLAM formulation, $H^{Z_k^i}(X)$ and $H^{O_i}(X)$ are the corresponding functions relating Z_k^i and O_i to the state X. An odometry measurement is a function of two poses $(X_{r_{i-1}}^T, \phi_{r_{i-1}})^T$ and $(X_{r_i}^T, \phi_{r_i})^T$ and is given by

$$H^{O_i}(X) = \begin{bmatrix} R(\phi_{r_{i-1}})^T (X_{r_i} - X_{r_{i-1}}) \\ \text{wrap}(\phi_{r_i} - \phi_{r_{i-1}}) \end{bmatrix}. \quad (4)$$

A single observation is a function of one pose $(X_{r_i}^T, \phi_{r_i})^T$ and one feature position X_{f_k} which is given by

$$H^{Z_k^i}(X) = R(\phi_{r_i})^T (X_{f_k} - X_{r_i}). \quad (5)$$

In particular, since $\phi_{r_0} = 0$ and $X_{r_0} = (0, 0)^T$, the odometry function from robot r_0 to r_1 is given by

$$H^{O_1}(X) = \begin{bmatrix} X_{r_1} \\ \phi_{r_1} \end{bmatrix} \quad (6)$$

and the observation function from robot r_0 to f_k is given by

$$H^{Z_k^0}(X) = X_{f_k}. \quad (7)$$

3. Alternative formulation when covariance matrices are spherical

The NLLS in (3) can be simplified when matrices $P_{Z_{k_{ij}}^i}$ and P_{O_i} are spherical for every i and j.

3.1. Definition of spherical matrices

We first state the definitions of spherical matrices which were defined in Wang et al. (2013).

Definition 1. $A \in \mathbb{R}^{2\times 2}$ is called spherical if it commutes with $R(\phi)$ (defined in (1)) for every ϕ. i.e. $AR(\phi) = R(\phi)A$ for every ϕ. $B \in \mathbb{R}^{3\times 3}$ is called spherical if it has the format of $B = \text{diag}(A, a)$ where $A \in \mathbb{R}^{2\times 2}$ is spherical and a is a real number.

Remark 1. Every positive definite spherical matrix $A \in \mathbb{R}^{2\times 2}$ can be written as $A = a^2 I_2$ for some $a \neq 0$. Furthermore, for every positive definite spherical matrix $B \in \mathbb{R}^{3\times 3}$ we have $B = \text{diag}(a^2 I_2, b^2)$ for some non-zero a and b.

A more general definition of spherical matrices was also introduced in Wang et al. (2013), which will also be used in the following of this paper.