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Brief paper

Globally exponentially stable attitude and gyro bias estimation with application to GNSS/INS integration ☆

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ABSTRACT

This paper deals with the construction of nonlinear observers for navigation purposes. We first present a globally exponentially stable observer for attitude and gyro bias, based on gyro measurements and two or more pairs of vector measurements. We avoid the well-known topological obstructions to global stability by not confining the attitude estimate to SO(3), but rather estimating a full rotation matrix with nine degrees of freedom. We also show how the attitude can be estimated under a relaxed persistency-of-excitation condition, with a single vector measurement as a special case. Next, we use the attitude observer to construct a globally exponentially stable observer for GNSS/INS integration, based on accelerometer, gyro, and magnetometer measurements, as well as GNSS measurements of position and (optionally) velocity. We verify the stability properties of the design using experimental data from a light fixed-wing aircraft.

1. Introduction

Navigation is the task of determining an object's position, velocity, or attitude based on various types of information. For decades the Kalman filter, and nonlinear extensions thereof, have been used to provide integrated navigation solutions based on different types of measurements. There is nevertheless a current interest in the design of nonlinear navigation observers, which can provide explicit stability guarantees and reduced computational complexity.

Attitude is typically estimated by comparing a set of vectors measured in the body-fixed coordinate frame with a set of reference vectors in a reference frame. The attitude can be algebraically resolved using two or more pairs of non-parallel vectors (see Shuster & Oh, 1981), but it is beneficial to integrate vector measurements with gyro measurements to improve the estimator bandwidth and to mitigate the effect of noise. An overview of early attitude estimators based on the extended Kalman filter (EKF) is provided by Lefferts, Markley, and Shuster (1982). Crassidis, Markley, and Cheng (2007) survey more recent results using the EKF as well as other estimation techniques. In the domain of nonlinear attitude observers with stability guarantees, the work of Salcudean (1991) is important, and it has been built upon by Thienel and Sanner (2003) and Vik and Fossen (2001). The observers based on Salcudean's work assume that the attitude is available as an explicit measurement. Observers that make direct use of vector measurements have been presented by Mahony, Hamel, and Pflimlin (2008), Hamel and Mahony (2006) and Vasconcelos, Silvestre, and Oliveira (2008).

Attitude can be represented by Euler angles, but more commonly a unit quaternion or a rotation matrix is used. For a continuous observer with estimates on the unit sphere or SO(3), topological obstructions prevent global asymptotic stability (see Bhat & Bernstein, 2000). These topological obstructions can be avoided by allowing the attitude estimate to evolve on a larger state space; this strategy has recently been employed by Batista, Silvestre, and Oliveira (2011a,b, 2012b) and by the authors (Grip, Saberi, & Johansen, 2011, 2012b), by estimating a matrix with nine degrees of freedom that converges to a rotation matrix on SO(3).

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Batista et al. (2012b) and Batista, Silvestre, and Oliveira (2012c) also considered the use of a separate observer to produce globally exponentially stable gyro bias estimates, although their approach requires stationary reference vectors. Attitude estimation using a single vector measurement has been considered by several authors (Batista et al., 2011b; Batista, Silvestre, & Oliveira, 2012a; Kinsey & Whitcomb, 2007; Lee, Leok, McClamroch, & Sanyal, 2007; Mahony, Hamel, Trumpf, & Lageman, 2009) and is possible under a persistency-of-excitation (PE) condition.

In many applications, a combination of inertial instruments and a satellite navigation system is available, often together with additional sensors such as altimeters and magnetometers. The integration of satellite and inertial measurements, referred to as GNSS/INS integration, has been studied for several decades (Maybeck, 1979; Phillips & Schmidt, 1996), typically based on the EKF. Vik and Fossen (2001) designed a nonlinear GNSS/INS integration observer with exponential stability results, based on the assumption that an independent attitude measurement was available. When an independent attitude measurement is not available, one may look for vector measurements that can be used for attitude estimation. The accelerometer offers one such measurement, but the corresponding reference vector (the sum of the gravity vector and the vehicle acceleration in the reference frame) is not explicitly available. Hua (2010) exploited the implicit availability of this reference vector in the derivatives of the GNSS measurements, by making use of GPS velocity information in combination with inertial measurements and a magnetometer.

1.1. Contributions of this paper

In this paper we begin by considering the attitude estimation problem based on gyro and vector measurements. We employ the same strategy as Batista et al. (2011a,b, 2012b) and Grip et al. (2011, 2012b) of letting the attitude be estimated as a full 3×3 matrix, the main contribution being the additional estimation of gyro bias. The bias estimation is of lower order than that of Batista et al. (2012b,c) and is capable of handling time-varying reference vectors, which is a prerequisite in the latter part of our paper. We nominally assume availability of at least two pairs of non-parallel vectors; however, as a fault-tolerance strategy we prove that the observer can be employed without gyro bias estimation using a single vector pair, under a persistency-of-excitation (PE) condition.

We continue by constructing a GNSS/INS integration observer based on the newly developed attitude observer. To do so, we leverage a general design framework for cascaded nonlinear and linear systems previously presented by Grip et al. (2012b), where a simplified version of the observer, without gyro bias estimation, was used as an application example. The design framework provides flexibility in handling different GNSS measurement setups, including partial or no velocity information. We prove global exponential stability of the observer error, and verify the stability results using an experimental platform consisting of a GNSS receiver and an inertial measurement unit (IMU) mounted in a light fixed-wing aircraft. A preliminary version of this paper was presented at the 2012 *American Control Conference* (Grip, Fossen, Johansen, & Saberi, 2012a).

1.2. Notation and preliminaries

For a vector or matrix X, X' denotes its transpose. The operator $\|\cdot\|$ denotes the Euclidean norm for vectors and the Frobenius norm for matrices. For a symmetric positive-semidefinite matrix X, the minimum eigenvalue is denoted by $\lambda_{\min}(X)$. The skew-symmetric part of a square matrix X is denoted by $\mathbb{P}_a(X) = \frac{1}{2}(X - X')$. For a vector $x \in \mathbb{R}^3$, $S(x)$ denotes the skew-symmetric matrix such that for any $y \in \mathbb{R}^3$, $S(x)y = x \times y$. The linear function $\mathrm{vex}(X)$ such that $S(\mathrm{vex}(X)) = X$ and $\mathrm{vex}(S(x)) = x$ is well-defined for all 3×3 skew-symmetric matrix arguments. The function $\mathrm{sat}_L(\cdot)$ denotes a component-wise saturation of its vector or matrix argument to the interval $[-L, L]$.

When referring to the notion of *global exponential stability*, we apply the definition of Michel, Hou, and Liu (2008), specialized to our circumstances. In particular, for the *motion* $x(t)$, originating from an initial condition $x(0)$ at time $t = 0$, the origin is globally exponentially stable (or exponentially stable *in the large*) if there exist an $\alpha > 0$, $\gamma > 0$, and for each $\beta > 0$, there exists a $k(\beta) > 0$ such that $\|x(t)\| \leq k(\beta)\|x(0)\|^{\gamma} \mathrm{e}^{-\alpha t}$, whenever $\|x(0)\| < \beta$.

2. Attitude estimation

We operate with a body-fixed frame (BODY), indicated by the superscript $^{\mathrm{b}}$, and an inertial coordinate frame. For the sake of continuity with later sections, we make the simplifying assumption that the inertial coordinate frame is the local North-East-Down (NED) coordinate frame and use the superscript $^{\mathrm{n}}$. The attitude is represented by $R \in \mathrm{SO}(3)$, which rotates any vector w^{b} from BODY to NED according to the relationship $w^{\mathrm{n}} = Rw^{\mathrm{b}}$. The kinematics of the rotation matrix satisfies

$$\dot{R} = RS(\omega^{\mathrm{b}}), \tag{1}$$

where ω^{b} is the angular velocity of the BODY frame relative to the NED frame, decomposed in BODY coordinates.

We assume availability of a gyro measurement $\omega^{\mathrm{b}}_{\mathrm{m}} = \omega^{\mathrm{b}} + b$, where b is an unknown gyro bias. We furthermore assume availability of k body-fixed vector measurements $w^{\mathrm{b}}_1, \ldots, w^{\mathrm{b}}_k$, as well as corresponding reference vectors $w^{\mathrm{n}}_1, \ldots, w^{\mathrm{n}}_k = Rw^{\mathrm{b}}_1, \ldots, Rw^{\mathrm{b}}_k$, which are allowed to be time-varying.

Assumption 1. There exists a $c_{\mathrm{obs}} > 0$ such that, for each $t \geq 0$, there are $i, j \in 1, \ldots, k$ such that $\|w^{\mathrm{n}}_i \times w^{\mathrm{n}}_j\| \geq c_{\mathrm{obs}}$.

Assumption 2. The gyro bias b is constant, and there exists a known constant $M_b > 0$ such that $\|b\| \leq M_b$.

We also make the physically reasonable assumption that ω^{b} and $w^{\mathrm{n}}_1, \ldots, w^{\mathrm{n}}_k$ are continuous in t and uniformly bounded.

2.1. Observer

We introduce an observer for R and b given by

$$\dot{\hat{R}} = \hat{R}S(\omega^{\mathrm{b}}_{\mathrm{m}} - \hat{b}) + \sigma K_P J(t, \hat{R}), \tag{2a}$$

$$\dot{\hat{b}} = \mathrm{Proj}(\hat{b}, -k_I\, \mathrm{vex}(\mathbb{P}_a(\hat{R}_s' K_P J(t, \hat{R})))), \tag{2b}$$

where K_P is a symmetric positive-definite gain matrix, $k_I > 0$ is a scalar gain, $\sigma \geq 1$ is a scaling factor that will be tuned to achieve stability, and $\hat{R}_s = \mathrm{sat}_1(\hat{R})$. The function $\mathrm{Proj}(\cdot, \cdot)$ represents a parameter projection (see Appendix), which ensures that $\|\hat{b}\|$ remains smaller than some design constant $M_{\hat{b}} > M_b$. The function $J(t, \hat{R})$ is a stabilizing injection term on the form

$$J(t, \hat{R}) = \sum_{j=1}^{q} (A^{\mathrm{n}}_j(t) - \hat{R}A^{\mathrm{b}}_j(t))A^{\mathrm{b}}_j(t)', \tag{3}$$

where the time-varying matrices $A^{\mathrm{n}}_j(t) \in \mathbb{R}^{3\times r_j}$ and $A^{\mathrm{b}}_j(t) \in \mathbb{R}^{3\times r_j}$, $j \in 1, \ldots, q$, satisfy the following property.

Property 1. *For each $j \in 1, \ldots, q$, the matrices $A^{\mathrm{n}}_j(t)$ and $A^{\mathrm{b}}_j(t)$ are piecewise continuous in t and uniformly bounded by $\|A^{\mathrm{n}}_j(t)\| = \|A^{\mathrm{b}}_j(t)\| \leq M_A$. Furthermore, they satisfy the relationship $A^{\mathrm{n}}_j(t) = RA^{\mathrm{b}}_j(t)$, and there exists a constant $\varepsilon > 0$ such that for all $t \geq 0$, $Q^{\mathrm{n}}(t) := \sum_{j=1}^{q} A^{\mathrm{n}}_j(t)A^{\mathrm{n}}_j(t)' \geq \varepsilon I_3$.*