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Brief paper

Adaptive fault-tolerant stabilization for nonlinear systems with Markovian jumping actuator failures and stochastic noises[☆]

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ABSTRACT

In this paper we consider the fault-tolerant stabilization problem for a class of nonlinear systems with uncertain parameters. Uncertainties caused by Markovian jumping actuator failures and stochastic noises are also taken into consideration. Different from most existing results, the number of actuator failures may be infinite and stochastic functions related to multi-Markovian variables have been introduced to denote the failure scaling factors for the actuators, which is practical, but challenging. Three main difficulties arise: first is how to establish fundamentals for systems involving multi-Markovian variables and stochastic noises, including the joint transition probability, the infinitesimal generator, the existence and uniqueness of the solution and so on; second is how to handle the extra transition rate related terms appearing in the infinitesimal generator of the Lyapunov function; last is how to cope with the involved higher order Hessian term in the Itô stochastic differentiation. By proposing a new adaptive fault tolerant control scheme, the boundedness in probability of all the closed-loop signals has been ensured. An example of altitude fault-tolerant control for a generic hypersonic air vehicle is presented to show the effectiveness of the proposed scheme.

1. Introduction

Actuator failures can be commonly encountered in practical systems. They may degrade system performance, render instability of the closed-loop system, or even worse, lead to catastrophic accidents. To increase system reliability and security, a bank of fault tolerant control (FTC) schemes, which compensate for the actuator failures and maintain the performance of the closed-loop system, have been proposed for deterministic systems, for instance in Boskovic, Jackson, Mehra, and Nguyen (2009), Corradini and Orlando (2007), Jiang (1994) and Zhang, Parisini, and Polycarpou (2004). Recently, adaptive FTC strategy has received much attention for its capability to deal with system uncertainties as well as variations caused by actuator failures simultaneously (Tang, Tao, & Joshi, 2003, 2007; Tao, Chen, & Joshi, 2002; Tao, Joshi, & Ma, 2001; Wang & Wen, 2010, 2011). It is noted that most existing studies using adaptive control approach focus on the case of finite number of actuator failures, that is, once an actuator fails, it will stay at the faulty mode during its rest operation. However, in practice, the failed actuators may recover itself and the actuators may fail more than once; see for example: the failures caused by the electromagnetic wave interference from the outer space in spacecraft control systems, or the intermittent failures occurred in computers or electronic equipments embedded into the actuators, or the failures caused by the communication network in a control loop. To be specific, in the spacecraft system, the control torque is generated by four reaction wheels (Hu & Xiao, 2013), which are activated by four respective control loops embedding usually with electronic parts for a high accuracy purpose. Note that the control loop is sensitive to electromagnetic wave interference from the outer space. That is, at some time intervals, the control loops may be partial/total loss of effectiveness due to the appearance of the electromagnetic wave interference, and hence actuator failures happen during these time intervals. While at other time intervals, the electromagnetic wave interference disappears, then the failed reaction

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wheel returns to normality again. Recently, a new adaptive failure compensation scheme has been proposed (Wang & Wen, 2011), in which infinite number of actuator failure cases has been taken into account and the boundedness of all closed-loop signals has been guaranteed.

Due to the fact that the patterns, times and modes of actuator failures are practically undetermined beforehand and are essentially stochastic, the FTC with random failures is of great interests to be studied. Meanwhile, the studies on nonlinear systems with stochastic noises have attracted considerable attentions; see for example (Deng & Krstić, 1997; Fan, Han, Wen, & Xu, 2012; Ji & Xi, 2006; Liu, Zhang, & Jiang, 2007; Pan & Basar, 1999; Wu, Xie, Shi, & Xia, 2009; Wu, Yang, & Shi, 2010; Xie & Xie, 2000) and the references therein. However, compared with the deterministic counterpart, the FTC problem for stochastic nonlinear systems has seldom been discussed. In Fan, Liu, Shen, and Wang (2014), the adaptive failure compensation problem has been studied for a class of nonlinear uncertain systems subjected to stochastic actuator failures and unknown parameters, in which the stuck case and stochastic noises are not involved. In Chen and Shen (2012), the robust reliable H_∞ control problem has been studied for a class of nonlinear stochastic Markovian jump systems. The considered statuses of different actuators comply with an identical Markov process, which means that all the actuators experience changes simultaneously and the statuses of different actuators are dependent. Such an assumption is rather strict because the status of each actuator is mostly likely to be governed by an independent random variable in practice, that is, each actuator may fail at any sampling time independently of the others.

In this paper, adaptive FTC problem for a class of nonlinear systems subjected to Markovian jumping actuator failures and stochastic noises is studied. The characteristics of the considered system can be featured by two aspects: Firstly, the total number of actuator failures is not restricted to finite and Markovian jumping actuator failures including the stuck case are taken into consideration. Secondly, stochastic functions related to Markovian variables are adopted to denote the failure scaling factors for each actuators and multi-Markovian variables are involved because of the different actuators. Three main challenges arise due to the existing multi-Markovian variables and stochastic noises: (1) establishing the properties for nonlinear systems involving multi-Markovian variables and stochastic noises; (2) dealing with the extra transition rate related terms appearing in the infinitesimal generator of the Lyapunov function; (3) coping with the higher order Hessian term involved in the Itô stochastic differentiation.

The rest part of the paper is organized as follows. Section 2 establishes some necessary preliminaries for nonlinear systems with multi-Markovian variables and stochastic noises. Section 3 describes the stochastic actuator fault model in detail and presents the considered system model. By employing backstepping technique, adaptive FTC scheme and stability analysis are given in Section 4. Section 5 presents a simulation to illustrate the effectiveness of the proposed control scheme. Finally, the paper is concluded in Section 6.

Throughout this paper, $\mathbb{R}$ represents the set of real numbers, $\mathbb{R}_+$ denotes the set of nonnegative real numbers, $\mathbb{R}^n$ and $\mathbb{R}^{n\times r}$ denote, respectively, n-dimensional real space and $n \times r$-dimensional real matrix space. $\|\cdot\|$ is the Euclidean norm, $(\cdot)^T$ and $Tr(\cdot)$ stand for, respectively, the transpose and the matrix trace. Let (Ω, F, P) be a complete probability space. $P(\cdot)$ means the probability, and $E(\cdot)$ denotes the expectation. C^i stands for the set of all functions with continuous ith-order partial derivative. $C^{2,1}(\mathbb{R}^n \times \mathbb{R}_+ \times S^m; \mathbb{R}_+)$ represents the family of all nonnegative functions $V(x(t), t, \mathbf{r}(t))$ on $(\mathbb{R}^n \times \mathbb{R}_+ \times S^m)$ which are C^2 in x and C^1 in t, and $S = \{1, 2, \ldots, N\}$ is a finite set, t^- is the left limit of time instant t, $a \wedge b \triangleq \min\{a, b\}$.

2. Preliminaries

In this section, some necessary preliminaries including the joint transition probability, the infinitesimal generator and so on w.r.t. multi-Markovian variables and stochastic noises (i.e. diffusion terms) involved, are to be established.

Consider the following stochastic nonlinear system

$$dx(t) = f\big(x(t), t, \mathbf{r}(t)\big)dt + g\big(x(t), t\big)dw, \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state of the system. For every $p_i \in S$, $i = 1, 2, \ldots, m$, when $r_i = p_i$, $f(x, t, p_1, p_2, \ldots, p_m)$ is locally Lipschitz in x for all $t \geq 0$, and $g(x, t)$ is locally Lipschitz in x which plays an essential role in ensuring the existence and uniqueness of the solution. w is a standard Wiener process, defined on the probability space (Ω, F, P), $\mathbf{r}(t) = (r_1(t), r_2(t), \ldots, r_m(t))$ is a Markovian vector which is independent of the Wiener process w. $r_i(t)$ is a right-continuous homogeneous and irreducible Markovian process on the probability space (Ω, F, P) and independent of $r_j(t)$, for $i \neq j$, taking values in a finite set $S = \{1, 2, \ldots, N\}$ with generator matrix $\Gamma = (\gamma_{pq})_{(N\times N)}$, where

$$\begin{aligned} P_{pq}(t) &= P\big\{r_i(s+t) = q | r_i(s) = p\big\} \\ &= \begin{cases} \gamma_{pq}t + o(t), & \text{if} \quad p \neq q; \\ 1 + \gamma_{pp}t + o(t), & \text{if} \quad p = q, \end{cases} \end{aligned}$$

for any $s, t \geq 0$, $p, q \in S$, $t \geq 0$ is the infinitesimal transition time interval, $o(t)$ is composed of infinitesimal terms of order higher than that of t. Note that $\gamma_{pq} > 0$ is the transition rate from mode p to mode q if $p \neq q$, and $\gamma_{pp} = -\sum_{q=1,q\neq p}^{N} \gamma_{pq}$.

2.1. Infinitesimal generator related to a Markovian vector

In Mao and Yuan (2006) an infinitesimal generator operator is defined and related properties for systems involved with a single Markovian variable are also presented. Here, taking account of the multi-Markovian variables and stochastic noises, a joint transition probability and an infinitesimal generator w.r.t. (1) are deduced and presented as follows.

The joint transition probability $P_{(p_1p_2\cdots p_m)(q_1q_2\cdots q_m)}$ of m Markovian variables shifting from modes $(p_1, p_2, \ldots, p_m)$ to modes $(q_1, q_2, \ldots, q_m)$ is derived as follows.

$$\begin{aligned} &P_{(p_1p_2\cdots p_m)(q_1q_2\cdots q_m)} \\ &\quad = P\big\{r_1(s+t) = q_1, \ldots, r_m(s+t) = q_m \\ &\qquad | r_1(s) = p_1, \ldots, r_m(s) = p_m\big\} \\ &\quad = \begin{cases} 1 + (\gamma_{p_1p_1} + \cdots + \gamma_{p_mp_m})t + o(t), & \text{for Case 1} \\ \gamma_{p_kq_k}t + o(t), & \text{for Case 2} \\ o(t), & \text{for Case 3} \end{cases} \end{aligned}$$

Case 1. $(p_1, p_2, \ldots, p_m) = (q_1, q_2, \ldots, q_m)$;
Case 2. only one mode, say p_k with $k \in \{1, 2, \ldots, m\}$, changes to q_k $(q_k \neq p_k)$, the rest $m-1$ modes remain the same;
Case 3. at least two modes, say p_i and p_j with $i, j \in \{1, 2, \ldots, m\}$, change to $q_i (\neq p_i)$ and $q_j (\neq p_j)$ respectively, the rest modes remain the same,

where $p_i, q_i \in S$ for $i = 1, 2, \ldots, m$, represent the modes of the ith Markovian variable at time instant s and $s + t$, respectively.

Then a useful lemma about the infinitesimal generator is presented as below, which plays an essential role in the controller design and stability analysis.

Lemma 1. *For any $l > 0$, define the first explosion time η_l as $\eta_l = \inf\{t : t > t_0, \|x(t)\| \geq l\}$. For a function $V(x, t, \mathbf{r}(t)) \in C^{2,1}(\mathbb{R}^n \times$*