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Brief paper

# Adaptive partial-state feedback control for stochastic high-order nonlinear systems with stochastic input-to-state stable inverse dynamics<sup>☆</sup>

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## ABSTRACT

This paper aims to solve the adaptive stabilization problem for a class of stochastic high-order nonlinear systems with stochastic inverse dynamics and nonlinear parameterization. Under the weaker assumptions on stochastic inverse dynamic and nonlinear functions, by generalizing the adding a power integrator technique, using the idea of changing supply function and parameter separation technique, a smooth adaptive partial-state feedback controller is constructed to render the closed-loop system globally stable in probability and all states can be regulated to the origin almost surely. The effectiveness of the designed controller is demonstrated by a simulation example.

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## 1. Introduction

In recent years, the study of stabilization problems on stochastic nonlinear systems has achieved remarkable development, see, e.g., Deng and Krstić (1997), Deng, Krstić, and Williams (2001), Fan, Han, Wen, and Xu (2012), Krstić and Deng (1998), Liu and Zhang (2008), Shi, Luan, and Liu (2012), Wu, Cui, Shi, and Karimi (2013), Zhang, Boukas, and Lam (2008), Zhou, Shi, Liu, and Xu (2012), Zhou, Shi, Xu, and Li (2013) and the references therein. This paper will consider the following stochastic high-order nonlinear systems with stochastic inverse dynamics:

$$dz = f_0(z, x_1)dt + g_0^T(z, x_1)d\omega,$$

$$dx_i = (x_{i+1}^{p_i} + f_i(z, \bar{x}_i, \theta))dt + g_i^T(z, \bar{x}_i, \theta)d\omega,$$

$$i = 1, \dots, n-1,$$

$$dx_n = (u^{p_n} + f_n(z, x, \theta))dt + g_n^T(z, x, \theta)d\omega, \quad (1)$$

where  $z \in R^m$  is the unmeasurable stochastic inverse dynamics,  $x = (x_1, \dots, x_n)^T \in R^n$  is the system state,  $u \in R$  is the system control input.  $\bar{x}_i = (x_1, \dots, x_i)^T$ ,  $i = 1, \dots, n$ ,  $\bar{x}_n = x$ .  $\theta \in R^l$  is an unknown constant vector,  $p_i \in R^* =: \{q \in R : q \geq 1 \text{ is a ratio of odd integers}\}$  is called the high order of system,  $\omega$  is an  $r$ -dimensional standard Wiener process defined on a complete probability space  $(\Omega, \mathcal{F}, P)$  with  $\Omega$  being a sample space,  $\mathcal{F}$  being a filtration, and  $P$  being a probability measure.  $f_0 : R^m \times R \rightarrow R^m$ ,  $g_0 : R^m \times R \rightarrow R^r \times m$ ,  $f_i : R^m \times R^i \times R^l \rightarrow R$  and  $g_i : R^m \times R^i \times R^l \rightarrow R^r$  are assumed to be locally Lipschitz with  $f_i(0, 0, \theta) = 0$  and  $g_i(0, 0, \theta) = 0$ ,  $i = 1, \dots, n$ .

For system (1), when  $p_i = 1$ , the controller design problem has been studied by Liu and Zhang (2006), Pan, Liu, and Shi (2001), Wu, Xie, and Zhang (2006), Wu, Xie, and Zhang (2007), et al. But the above-mentioned controllers are only robust against the inverse dynamics with stringent stability margin. To weaken this condition, Tang and Basar (2001) firstly introduced the concept of stochastic input-to-state stability. Then, Liu, Zhang, and Jiang (2007) gave a sufficient condition on stochastic input-to-state

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stable (SISS) by using the Lyapunov function. Liu, Zhang, and Jiang (2008) considered the cascaded stochastic nonlinear systems with SISS inverse dynamics.

Inspired by the deterministic results (Lin, 2013; Qian, 2001) and the related papers, system (1) has been widely studied in recent years. When  $p_i \geq 1$ , some intrinsic features of system (1), such as its Jacobian linearization is neither controllable nor feedback linearizable, lead to the existing design tools being hardly applicable to this kind of systems. For  $z = 0$ , Liu and Duan (2010), Xie and Li (2009), Xie and Tian (2009) solved respectively the state-feedback and output-feedback stabilization problems for such systems with different structures. For the case of  $z \neq 0$ , Xie and Tian (2007) addressed the full-state feedback stabilization of system (1), but the designed controller is robust against the stochastic inverse dynamics. Furthermore, Liu and Xie (2012) and Xie, Duan, and Yu (2011) studied the state-feedback control for system (1) with SISS and SISS inverse dynamics. However, all of them are not considered the adaptive stabilization problem of system (1).

The purpose of this paper is to design the following adaptive partial-state feedback controller for system (1)

$$u = u(x, \hat{\theta}), \quad u(0, \hat{\theta}) = 0, \\ \dot{\hat{\theta}} = \psi(x, \hat{\theta}), \quad \psi(0, \hat{\theta}) = 0,$$

such that the equilibrium  $(z, x, \tilde{\theta}) = (0, 0, 0)$  of the closed-loop system is globally stable in probability and all states can be regulated to the origin almost surely, where  $\hat{\theta}$  and  $\tilde{\theta}$  are defined in Section 3. And the contributions of this paper are characterized by the following novel features:

- (1) System (1) is more general than some existing systems, such as Deng and Krstić (1997), Deng et al. (2001) in which  $z = 0$  and  $p_i = 1$ ; Liu and Duan (2010), Xie and Li (2009), Xie and Tian (2009) in which  $z = 0$  and  $p_i \geq 1$ ; Liu and Zhang (2008), Liu et al. (2007), Liu et al. (2008), Liu and Zhang (2006), Pan et al. (2001), Wu et al. (2006), Wu et al. (2007) in which  $z \neq 0$  and  $p_i = 1$ ; Liu and Xie (2012), Xie and Tian (2007) in which  $z \neq 0$  and  $p_i \geq 1$ .
- (2) By generalizing the adding a power integrator technique into stochastic case, using the idea of changing supply function and parameter separation technique, constructing an appropriate Lyapunov function, a smooth adaptive partial-state feedback controller is designed to ensure that the equilibrium at the origin of the closed-loop system is globally stable in probability.
- (3) The appearance of stochastic inverse dynamics, high-order, unknown parameter, nonlinear assumption and Hessian term will unavoidably produce too many nonlinear terms, how to deal with them is not an easy job.
- (4) By skillfully choosing an appropriate adaptive parameter, the over-parameterization phenomenon which often encounter in the backstepping design is avoided. It should be emphasized that how to choose the adaptive parameter is not a simple work.

This paper is organized as follows. Section 2 provides some preliminary results and system assumption conditions. Adaptive partial-state feedback controller design and stability analysis are given in Section 3, following a simulation example in Section 4. Section 5 concludes this paper.

## 2. Preliminary results and assumption conditions

### 2.1. Preliminary results

The following notations, definitions and lemmas are to be used throughout the paper.

$R_+$  denotes the set of all nonnegative real numbers and  $R^n$  denotes the real  $n$ -dimensional space. For a given vector or matrix  $X$ ,  $X^T$  denotes its transpose,  $\text{Tr}\{X\}$  is its trace when  $X$  is square,  $\|X\|$  denotes the Euclidean norm of a vector  $X$ ,  $\|X\| = (\text{Tr}\{X^T X\})^{\frac{1}{2}}$  is the norm of a matrix  $X$ .  $C^i$  denotes the set of all functions with continuous  $i$ th partial derivatives.  $\mathcal{K}$  denotes the set of all functions  $\mathcal{K} : R_+ \rightarrow R_+$ , which are continuous, strictly increasing and vanishing at zero;  $\mathcal{K}_\infty$  denotes the set of all functions which are of class  $\mathcal{K}$  and unbounded.

Consider the following stochastic nonlinear system

$$dx = f(x, \theta)dt + g^T(x, \theta)d\omega, \quad x(0) = x_0 \in R^n, \quad (2)$$

where  $x \in R^n$  is the system state,  $\theta \in R^r$  is an unknown constant vector,  $\omega$  is an  $r$ -dimensional standard Wiener process defined on the complete probability space  $(\Omega, \mathcal{F}, P)$ . The Borel measurable functions  $f : R^n \times R^r \rightarrow R^n$  and  $g : R^n \times R^r \rightarrow R^{n \times r}$  are locally Lipschitz in  $x$  with  $f(0, \theta) = 0, g(0, \theta) = 0$ .

**Definition 1** (Krstić & Deng, 1998). For any given  $V(x) \in C^2$ , associated with system (2), the differential operator  $\mathcal{L}$  is defined as

$$\mathcal{L}V = \frac{\partial V}{\partial x} f(x, \theta) + \frac{1}{2} \text{Tr} \left\{ g(x, \theta) \frac{\partial^2 V}{\partial x^2} g^T(x, \theta) \right\}. \quad (3)$$

**Definition 2** (Krstić & Deng, 1998). The equilibrium  $x(t) = 0$  of system (2) is globally stable in probability if for  $\forall \varepsilon > 0$ , there exists a class  $\mathcal{K}$  function  $\gamma(\cdot)$  such that  $P\{|x(t)| < \gamma(|x_0|)\} \geq 1 - \varepsilon$  for any  $t \geq 0$  and  $x_0 \in R^n \setminus \{0\}$ .

**Lemma 1** (Krstić & Deng, 1998). Consider the stochastic system (2), if there exist a  $C^2$  function  $V(x)$ , class  $\mathcal{K}_\infty$  functions  $\beta_1(\cdot), \beta_2(\cdot)$ , constants  $c_1 > 0, c_2 \geq 0$ , and a nonnegative function  $W(x)$  such that

$$\beta_1(|x|) \leq V(x) \leq \beta_2(|x|), \quad \mathcal{L}V(x) \leq -c_1 W(x) + c_2,$$

then

- (1) There exists an almost surely unique solution on  $[0, \infty)$  for system (2);
- (2) When  $c_2 = 0$  and  $W(x)$  is continuous, then the equilibrium  $x = 0$  is globally stable in probability and  $P\{\lim_{t \rightarrow \infty} W(x(t)) = 0\} = 1$ .

**Lemma 2** (Liu & Xie, 2012). Let  $x_1, \dots, x_n, p$  be positive real numbers, then

$$(x_1 + \dots + x_n)^p \leq \max\{n^{p-1}, 1\}(x_1^p + \dots + x_n^p).$$

**Lemma 3** (Liu & Duan, 2010). Let  $c, d$  be positive constants. For any positive number  $\bar{\gamma}$ ,  $|x|^c |y|^d \leq \frac{c}{c+d} \bar{\gamma} |x|^{c+d} + \frac{d}{c+d} \bar{\gamma}^{-\frac{c}{d}} |y|^{c+d}$ .

**Lemma 4** (Liu & Duan, 2010). Let  $p \in R^*$  and  $x, y$  be real-valued functions. For a constant  $c > 0$ , one has

$$|x^p - y^p| \leq p|x - y|(x^{p-1} + y^{p-1}) \\ \leq c|x - y|(|x - y|)^{p-1} + y^{p-1}|x - y|.$$

**Lemma 5** (Lin & Qian, 2002). For any vector-valued continuous function  $h(x, y)$ , where  $x \in R^m, y \in R^n$ , there are smooth scalar functions  $a(x) \geq 1$  and  $b(y) \geq 1$ , such that

$$|h(x, y)| \leq a(x)b(y).$$

### 2.2. Assumption conditions

To achieve the objective of this paper, we need the following assumptions on system (1).

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