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Brief paper

Nonparametric time-variant frequency response function estimates using arbitrary excitations☆

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ABSTRACT

The time-variant frequency response function (TV-FRF) uniquely characterises the dynamic behaviour of a linear time-variant (LTV) system. This paper proposes a method for estimating nonparametrically the dynamic part of the TV-FRF from known input, noisy output observations. The arbitrary time-variation of the TV-FRF is modelled by Legendre polynomials. In opposition to existing solutions, the proposed method is applicable to arbitrary inputs.

1. Introduction

Time-variant dynamics are present in all kinds of engineering applications, and they can be classified according to the nature of the time-variation. Either the time-variation is due to a physical phenomenon or a (scheduling) parameter that varies smoothly as a function of time (class A), or it is induced by the switching between a finite number of linear time-invariant systems (class B). Examples of class A dynamics are, thermal drift in power electronics (Chen & Yuan, 2011); fatigue, ageing and mortification in biomedical measurements (Aerts & Dirckx, 2010); pit corrosion of metals (Van Ingelgem, Tourwé, Vereecken, & Hubin, 2008); control of crane dynamics (Abdel-Rahman, Nayfeh, & Masoud, 2003); aeroplane dynamics during take off and landing (Dimitriadis & Cooper, 2001); and impedance measurements for determining the state-of-charge of batteries (Pop, Bergveld, Notten, & Regtien, 2005; Rodrigues, Munichandraiah, & Shukia, 2000). Examples of class B dynamics are, regime switching in power electronics (Aguilera, Godoy, Agüero, Goodwin, & Yuz, 2014), econometrics (Hamilton, 1990), and control applications (Yin, Kan, Wang, & Xu, 2009); and more general, hybrid systems (see Paoletti, Juloski, Ferrari-Trecate, & Vidal, 2007 and the references therein).

In this paper we consider class A dynamics only. The time-variant frequency response function (TV-FRF) introduced in Zadeh (1950a,b) provides deep insight into the time-variant behaviour of class A dynamics. Hence, there is a need for methods that estimate the TV-FRF from input–output data. According to the parametrisation used, one can distinguish four model classes for describing the class A dynamics:

1. Parametric in both the dynamics and the time-variation: a lot of estimation algorithms are available, see Niedzwiecki (2000), Poulimenos and Fassois (2006) and Tóth, Laurain, Gilson, and Garnier (2012) and the references therein. The time- or parameter-variant system is modelled using a differential, difference, or state space equation where the (matrix) coefficients are affine functions of time- or parameter-dependent basis functions, for example, wavelets in Li and Billings (2011) and Tsatsanis and Giannakis (1993), polynomials in Lataire and Pintelon (2011), sines and cosines in Allen (2008) and Louarroudi, Lataire, Pintelon, Janssens, and Swevers (2013), or integrated white noise in Kitagawa and Gersch (1985).
2. Parametric in the dynamics and nonparametric in the (slow) time-variation: see Georgiev (1989), Liu (1997) and Niedzwiecki and Kaczmarek (2005).

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3. Nonparametric in the dynamics and parametric in the (slow) time-variation: periodic time-variation in Sams and Marmarelis (1988) and Louarroudi, Pintelon, and Lataire (2012) parametrised by sines and cosines; and arbitrary time-variation in Lataire, Pintelon, and Louarroudi (2012) parametrised by polynomials.
4. Nonparametric in both the dynamics and the (very) slow time-variation: the TV-FRF is estimated using the short-time Fourier transform (Allen & Rabiner, 1977). The basic assumption made is that the system is time-invariant within the short sliding time window: see, for example, Spiridonakos and Fassois (2009) for noise power spectra and Sanchez, Louarroudi, Bragos, and Pintelon (2013) for FRFs.

Model classes 1 and 2 require a parametric model for describing the system dynamics, which is not the case for model classes 3 and 4. The latter are natural extensions of the nonparametric FRF representation of linear time-invariant systems. The three disadvantages of parametrising the system dynamics w.r.t. to a nonparametric representation are the following: (i) the type of dynamic model must be chosen: differential equation (s-domain), difference equation (z-domain), fractional differential equation (e.g., $\sqrt{s}$-domain), or partial differential equation; (ii) the dynamic model order must be chosen (orders time-domain derivatives or time-domain shifts of the input and output signals); and (iii) estimating the model parameters mostly involves a nonlinear minimisation. The latter requires the generation of initial estimates and includes possible problems with local minima. The main advantages of parametric models are the compact description and the smaller estimation uncertainty. Nonparametric estimation techniques are very helpful to get an idea of the complexity of the parametric modelling step and to validate the estimated parametric system model. Compared with the algorithms for model class 3, the methods developed for model class 4 have the disadvantage that they require a trade-off between accurate tracking of the time-variation (the sliding time window should be as small as possible) and sufficiently large frequency resolution of the estimated dynamics (the sliding time window should be as large as possible). In addition, at the cost of a more complicated estimation algorithm, the methods for model class 3 result in TV-FRF estimates with a much larger frequency resolution.

This paper considers the third model class with nonparametric dynamics and arbitrary time-variation parametrised by Legendre polynomials. The approach presented in Lataire et al. (2012) – called the direct method in the sequel of this paper – has the disadvantages that a lot of signal periods are needed and that it is not applicable to random excitations. In this paper an indirect method is proposed that is applicable to arbitrary excitations and a few (less than one) period(s) of periodic inputs.

The paper is organised as follows. First, the class of linear time-variant systems considered and the stochastic framework are defined (Sections 2 and 3). Next, an indirect method for estimating nonparametrically the TV-FRF of this class of systems is developed and analysed in detail (Section 4). Further, the proposed indirect method is compared with the direct approach (Section 5). Finally, the whole procedure is illustrated via measurements on a time-variant electronic circuit (Section 6).

2. The time-variant frequency response function

First, we recall the definition and the properties of the time-variant frequency response function (TV-FRF). Next, a nonparametric-in-the-dynamics and parametric-in-the-time-variation representation for a class of (slowly) time-varying systems is given.

2.1. Definition and properties of the TV-FRF

The dynamic behaviour of a linear time-variant (LTV) system is uniquely characterised by its response $g(t, \tau)$ to a Dirac impulse applied at time instant $t = \tau$ (Zadeh, 1950a,b). Taking the Fourier transform of the shifted time-variant impulse response $g(t, t - \tau)$ defines the TV-FRF, called the system function in Zadeh (1950a,b),

$$G(j\omega, t) = \int_{-\infty}^{+\infty} g(t, t - \tau)\, e^{-j\omega\tau} d\tau. \tag{1}$$

For causal systems ($g(t, \tau) = 0$ for $t < \tau$) the lower integration boundary in (1) is replaced by zero. The time-variant FRF (1) has the following properties:

1. The steady state response to $\sin(\omega_0 t)$ equals
$$|G(j\omega_0, t)| \sin(\omega_0 t + \angle G(j\omega_0, t)) \tag{2}$$
which is an amplitude and phase modulated sine wave. Note that the Fourier spectrum of (2) is non-zero in the close neighbourhood of ω_0, resulting in a skirt-like spectrum around ω_0 (see Lataire et al., 2012).
2. Assuming zero initial conditions, the transient response $y_0(t)$ to an input $u(t)$ is found as
$$y_0(t) = L^{-1}\{G(s, t)\, U(s)\} \tag{3}$$
with $U(s)$ the Laplace transform of $u(t)$, and $L^{-1}\{\}$ the inverse Laplace transform.

Note that properties (2) and (3) are natural extensions of the linear time-invariant (LTI) case.

2.2. Nonparametric representation of the TV-FRF

The nonparametric representation of the dynamics of the TV-FRF is obtained in two steps.

First, the TV-FRF (1) is expanded in series w.r.t. time

$$G(j\omega, t) = \sum_{r=0}^{\infty} G_r(j\omega) f_r(t) \quad , t \in [0, T] \tag{4}$$

with $f_r(t)$, $r = 0, 1, \ldots,$ a complete set of basis functions, and T the experiment time. $G_r(j\omega)$, $r = 0, 1, \ldots,$ are the complex coefficients of the series expansion which can be interpreted as FRFs of LTI systems. Note that the basis functions can always be chosen such that the constraints

$$f_0(t) = 1 \quad \text{and} \quad \frac{1}{T}\int_0^T f_r(t)\, dt = 0 \quad \text{for } r > 0 \tag{5}$$

are satisfied.

In a second step, the infinite sum (4) is approximated by a finite sum

$$G(j\omega, t) = \sum_{r=0}^{N_b} G_r(j\omega) f_r(t) \quad t \in [0, T]. \tag{6}$$

Representation (6) is parametric in the time-variation (the basis functions $f_r(t)$ are known), and nonparametric in the unknown FRFs $G_r(j\omega)$, $r = 0, 1, \ldots, N_b$. Note that in practise N_b is unknown and, hence, should also be estimated from the data.

Eq. (6) motivates the following assumption:

Assumption 1 (*Slow Time-variation*)**.** The TV-FRF (1) of the linear time-variant system can be written as (6), where $f_r(t)$, $r = 0, 1, \ldots, N_b$, are polynomials of order r satisfying (5).

The term "slow time-variation" in Assumption 1 is justified as follows. For the Legendre polynomials used in the indirect method (see Section 3), the spectral content of $f_r(t)$ is concentrated around DC. However, this does not exclude that the time-variation can be