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Brief paper

Fault tolerant control allocation via continuous integral sliding-modes: A HOSM-Observer approach☆

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ABSTRACT

In this paper a *continuous* fault tolerant control allocation is proposed. This approach is based on a *uniform* High-Order Sliding-Mode Observer where only measurable outputs are used. The fault tolerant control scheme is developed using, for the *first time*, a *continuous* integral sliding-mode and a fixed control allocation technique which provides an approximate estimation of matched faults. The conditions for stability are found by ensuring the stability of the closed loop system in the presence of possible faults in the components, and actuator faults or failures. The effectiveness of the proposed approach is verified through simulation of a linear version of the benchmark $B747-100/200$ civil aircraft model.

1. Introduction

Fault Tolerant Control (FTC) is used to avoid a potentially hazardous, out-of-tolerance or dangerous behavior of the controlled system. In general, FTC strategies are classified into two types, namely passive and active approaches. In the passive approach, a single control law is designed to maintain the stability and the satisfactory performance in both fault-free situation, i.e. when all components are operating faithfully, and in the faulty environment. However, the disadvantage of this approach is that the nominal and fault-free performance of the system is deteriorated. An active FTC strategy requires fault information for control reconfiguration, in contrast to passive strategy. The fault information can be obtained from a dedicated FDI system (see, for instance Blanke, Kinnaert, Lunze, and Staroswiecki (2003), Zolghadri, Castang, and Henry (2006) and the references therein). Obviously, the feasibility of both FTC approaches is dependent on the recoverability/compensability of faults (Chen & Saif, 2011; Djeghali, Ghanes, Djennoune, & Barbot, 2013; Efimov, Cieslak, & Henry, 2013; Zolghadri, 2012). In overactuated systems, one appealing solution to fault tolerance is the control allocation (CA) where the desired total control effort is distributed among the redundant set of actuators (Härkegård & Glad, 2005). In this case, the controller is designed based on a "virtual control" signal and the CA element will map the virtual control to the actual control demand to the actuators. One of the main benefits is that the controller design is independent of the CA unit and hence it can be used in conjunction with any other control design paradigm. The other benefits of CA are that the controller structure does not have to be reconfigured in fault situations and it can also deal directly with actuator failures without any controller reconfiguration. The CA scheme automatically redistributes the control effort. As in sliding-mode control (SMC), another benefit of CA is that it can reject disturbance from the initial time (Hamayun, Edwards, & Alwi, 2013). The SMC (Shtessel, Edwards, Fridman, & Levant, 2013; Utkin, Guldner, & Shi, 1999) has many attractive features such as invariance to matched uncertainties, order reduction, simplicity in design, robustness against perturbations and some others. The characteristic feature of continuous-time SMC system is that, sliding-mode occurs on a prescribed manifold (sliding surface), where switching control is employed to maintain the states on the surface. Nevertheless, there is no guarantee of robustness during the reaching phase. The integral sliding-mode (Cao & Xu, 2004; Castaños & Fridman, 2006; Utkin & Shi, 1996) though is a variant, it eliminates the reaching phase by enforcing the sliding-mode throughout the entire system

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response. Integral sliding-mode control (ISMC) is a combination of two controls, one nominal control which will decide the performance of the system without disturbance and second is a discontinuous control which will take care of the disturbance and model uncertainty. In this approach one gets better performance because sliding-mode starts from the initial time but, here the control input is discontinuous. This control is not desirable from the implementation point of view because the oscillations caused by the high-frequency switching of sliding-mode/discontinuous controller will excite the unmodeled dynamics in the closed-loop, known as chattering (Utkin et al., 1999). Among all the different kinds of strategies of the sliding-modes, ISMC can be easily combined with other existing control strategies such as linear feedback control, PID control, optimal control, model predictive control etc., and also retains their properties (Utkin & Shi, 1996). So, one can explain ISMC as a bridge between other control strategies and sliding-mode which has a better robustness property than the other existing control strategies. However, the conventional ISMC suffers from the drawback that its second part of the control is *discontinuous*. Here, the ISMC law is modified for the CA by replacing the *discontinuous* part of the feedback control by a uniform super-twisting control (USTC) (Cruz-Zavala, Moreno, & Fridman, 2011). The USTC is a *continuous* controller ensuring all the main properties of the first order SMC for the system with smooth matched bounded uncertainties/disturbances. The replacement of the ISMC *discontinuous* term with the USTC is possible due to the disturbance observation property (Utkin et al., 1999). Moreover, the proposed controller is *continuous* due to the combination of two continuous controls.

Main Contribution: A *Continuous* Fault Tolerant Control Allocation that does not need a fault detection and isolation scheme. For fulfilling the above mentioned goal, for the *first time*, a *Continuous* ISMC based fixed control allocation scheme is developed using estimated states information. A uniform HOSM-Observer is used to obtain the estimated state. Moreover, the conditions for the stability are derived to ensure the closed-loop stability of the system in the presence of component faults, and actuator faults or failures. This particular approach presents two interesting properties: the chattering effect is avoided, and it provides an estimation of faults, that manifest themselves as matched disturbances to the system; without need of filtration, in contrast to classical SMC approaches.[1]

Structure of the paper: The problem statement is given in Section 2. Section 3 deals with the HOSM-Observer design. In Section 4 the closed-loop stability and continuous ISMC are described. Section 5 shows simulation results on a civil aircraft model, and the conclusions are given in Section 6.

Notation: The pseudo-inverse of $F \in \mathbb{R}^{n\times m}$ is defined as $F^{\dagger} = \left(F^T F\right)^{-1} F^T \in \mathbb{R}^{m\times n}$. For $J \in \mathbb{R}^{n\times m}$ with $n \geq m$ and $\text{rank}(J) = r$, $J^{\perp} \in \mathbb{R}^{(n-r)\times n}$ is defined as a matrix such that $\text{rank}(J^{\perp}) = n - r$ and $J^{\perp}J = 0$. I_n denotes an $n\times n$ identity matrix. $\|\cdot\|$ represents the Euclidean norm, and $\lceil \xi \rfloor^r$ is defined as $\lceil \xi \rfloor^r = |\xi|^r \,\text{sign}(\xi)$.

2. Problem statement

Consider the following system with uncertainties and subjected to actuator faults

$$\dot{x}(t) = \left(A + A^{\delta}\right) x(t) + (B - BK(t))\, u(t), \tag{1}$$

$$y(t) = Cx(t), \tag{2}$$

where $x \in \mathbb{R}^n$ is the state, $y \in \mathbb{R}^p$ is the output, $u \in \mathbb{R}^m$ is the input,[2] and the matrices $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, and $C \in \mathbb{R}^{p\times n}$ have corresponding dimensions. The term $A^{\delta}x$, with $A^{\delta} \in \mathbb{R}^{n\times n}$, represents the possible component faults or uncertainties. The diagonal weighting matrix $K(t) = diag\,\{k_1(t), \ldots, k_m(t)\}$, models the effectiveness level of the actuators, i.e. if $k_j(t) = 1$, the *jth* actuator has completely failed, whilst if $1 > k_j(t) > 0$, an actuator fault is active, and if $k_j(t) = 0$, the corresponding *jth* actuator is fault free. It is assumed that actuator faults are bounded, i.e. $\|K(t)u(t)\| \leq K^{+} < \infty, \forall t \geq 0$, and that the case $K(t) = I_m$ (all actuators have completely failed simultaneously) is not possible since there does not exist fault compensability. The controlled outputs for the system are given by $y_c = C_c x(t)$, where $C_c \in \mathbb{R}^{l\times n}$ with $l < m$. Therefore, there exists redundancy in the system in terms of the number of the control inputs. To solve this redundancy it is assumed the system (1)–(2) is transformed, according to $\bar{x}(t) = T_0 x(t)$, in the following form

$$\dot{\bar{x}}(t) = \left(\bar{A} + \bar{A}^{\delta}\right) \bar{x}(t) + \left(\bar{B} - \bar{B}K(t)\right) u(t), \tag{3}$$

$$y(t) = \bar{C}\bar{x}(t), \tag{4}$$

where $\bar{A} = T_0 A T_0^{-1}$, $\bar{A}^{\delta} = T_0 A^{\delta} T_0^{-1}$, $\bar{B} = T_0 B = \begin{bmatrix} \bar{B}_1^T & \bar{B}_2^T \end{bmatrix}^T$, and $\bar{C} = CT_0^{-1}$. The nonsingular transformation matrix T_0 is designed such that, by an appropriate scaling of the last l and a change in the state space coordinates, by means of elementary matrices, the input distribution matrix can be partitioned such that $\bar{B}_2\bar{B}_2^T = I_l$, which implies that $\left\|\bar{B}_2\right\| = 1$. As argued in Alwi and Edwards (2008), it is assumed that $\left\|\bar{B}_1\right\| \ll \left\|\bar{B}_2\right\|$ with $\bar{B}_1 \in \mathbb{R}^{(n-l)\times m}$ and $\bar{B}_2 \in \mathbb{R}^{l\times m}$, so that the control action predominantly acts in the last l channels of the system. It is assumed that the last l channels can be only faulty, i.e. $1 > k_j(t) > 0$, hence, fault compensability is possible. The system (3) can be written as

$$\dot{\bar{x}}(t) = \left(\bar{A} + \bar{A}^{\delta}\right) \bar{x}(t) + \begin{bmatrix} \bar{B}_1 \\ \bar{B}_2 \end{bmatrix} (I_m - K(t))\, u(t). \tag{5}$$

The physical control law $u(t)$ is realized by a fixed CA scheme of the form

$$u(t) = \bar{B}_2^T \left(\nu_0(t) + \nu_1(t)\right), \tag{6}$$

where $\nu_0(t) \in \mathbb{R}^l$ is the "*nominal*" virtual control designed for the system (5) in the absence of faults ($K(t) = 0$), and $\nu_1(t) \in \mathbb{R}^l$ is designed to compensate the actuator faults. In the fault-free case, i.e., $K(t) = 0$, the system (5) is rewritten as

$$\dot{\bar{x}}(t) = \bar{A}\bar{x}(t) + \underbrace{\begin{bmatrix} \bar{B}_1\bar{B}_2^T \\ I_l \end{bmatrix}}_{\bar{B}_{\nu_0}} \nu_0(t) + \underbrace{\bar{A}^{\delta}\bar{x}(t)}_{g(t,\bar{x})}. \tag{7}$$

Note that the term $g\left(t, \bar{x}\right)$ is a function that vanishes at the origin, i.e., $g\left(t, 0\right) = 0$, and satisfies $\|g\left(t, \bar{x}\right)\| \leq \bar{\gamma}\,\|\bar{x}\|$, i.e. it is locally Lipschitz in x, uniformly in t, $\forall t \geq 0$. The following assumption ensures the possibility for designing the "*nominal*" virtual control $\nu_0(t)$.

Assumption 1. The pair $\left(\bar{A}, \bar{B}_{\nu_0}\right)$ is controllable.

The aim is to design a continuous FTC, based on the output information only, that can maintain closed-loop stability in spite of a certain class of actuator faults and failures. To achieve this goal a continuous ISMC based on uniform HOSM-Observer is proposed.

[1] Note that a FTC system, based on ISMC and fixed CA, is proposed in Hamayun et al. (2013). The scheme ensures closed-loop stability in the presence of certain actuator faults. However, this controller presents a main drawback: it is *discontinuous*; and it does not provide an estimation of the fault.

[2] Note that condition $p \geq m$ is not required.

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