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Brief paper

Adaptive consensus output regulation of a class of nonlinear systems with unknown high-frequency gain[☆]

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ABSTRACT

This paper deals with adaptive consensus output regulation of a class of network-connected nonlinear systems with completely unknown parameters, including the high frequency gains of the subsystems. The subsystems may have different dynamics, as long as the relative degrees are the same. A new type of Nussbaum gain is proposed to deal with adaptive consensus control of network-connected systems without any knowledge of the high frequency gains. Adaptive laws and internal models are designed for the subsystems to deal with unknown parameters for tracking trajectories and unknown system parameters. In the control design, only the relative information of subsystem outputs are used, provided that regulation error of one of the subsystems is available. The proposed control inputs and the adaptive laws are decentralized. If the relative degree is one, only the relative subsystem outputs are exchanged. For the case of higher relative degrees, the nonlinear model structure of the subsystems is exploited for backstepping control design, and some variables generated by the subsystem controllers are exchanged among the subsystems in the neighbourhood defined by the connection graph.

1. Introduction

Many dynamic systems are connected by networks to perform certain common or similar tasks that include formation control and cooperative control etc. Consensus control often refers to the situation where network-connected subsystems are controlled to achieve the same or very similar control objectives. A significant difference of consensus control to other control design is the use of the information collected from the subsystems in the neighbourhood, and the success of any proposed consensus control design depends on the structure of network connections which are described as connection graphs. A useful description of a connection graph is the Laplacian matrix that plays an important role in all the design methods on consensus control (Fax & Murray, 2004; Olfati-Saber & Murray, 2004; Qu, 2009). In particular, some important properties of Laplacian matrices in relation to consensus control are well described in Qu (2009).

Consensus control started from the single-integrator dynamics of subsystems as shown in the review paper (Olfati-Saber, Fax, & Murray, 2007) and then to subsystems with full linear system dynamics (Fax & Murray, 2004; Grip, Yang, Saberi, & Stoorvogel, 2012; Li, Duan, Chen, & Huang, 2010; Olfati-Saber & Murray, 2004; Seo, Shim, & Back, 2009; Su & Huang, 2012; Xiang, Wei, & Li, 2009; Yang, Roy, Wan, & Saberi, 2009). Results on consensus control of systems with nonlinear subsystem dynamics have appeared in various publications (Chopra & Spong, 2008; Ding, 2013a; Hong, Hu, & Gao, 2006; Li, Liu, Fu, & Xie, 2012; Li, Ren, Liu, & Fu, 2013; Münz, Papachristodoulou, & Allgöwer, 2011; Su & Huang, 2013; Zhao, Hill, & Liu, 2011). Most of the results on nonlinear dynamics are obtained for subsystems with relative degree one or the systems with Lipschitz nonlinearities. When there are uncertainties in the system, adaptive control strategies are naturally considered. One challenge is the implementation of adaptive laws in a decentralized manner (Yu & Xia, 2012). In the robust adaptive consensus control shown in Das and Lewis (2010), Zhang and Lewis (2012), the adaptive laws are decentralized, with the influence of the uncertainties of the adjacent subsystems being treated as bounded disturbances, and the resultant consensus control errors are kept bounded instead of the convergence to zero due to the robust adaptive control treatment. Decentralized adaptive laws have been proposed for first-order nonlinear systems in Yu and Xia (2012).

We consider consensus output regulation of a class of network-connected nonlinear dynamic systems whose subsystems have all the system parameters completely unknown, including the high

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frequency gains. It is well known that Nussbaum gains can be used to tackle adaptive control with unknown high-frequency gains for single-input single-output systems including the case of nonlinear output regulation (Ding, 2001; Nussbaum, 1983). However, for a network connected system with multiple subsystems, the existing Nussbaum gain designed for individual systems would not be able to establish the boundedness of all the variables in the adaptive consensus control system, as Nussbaum gain parameters for different subsystems could move in different directions, and a usual contradiction could not be obtained. A very recent result (Chen, Li, Ren, & Wen, 2014) proposes a Nussbaum gain for multi-agent systems with unknown control directions when the lower and upper bounds of the control coefficients are known. In this paper, we propose a new Nussbaum gain with a potentially faster rate such that the boundedness of the system parameters can be established by an argument of contradiction even if the Nussbaum gain parameter for only one of the subsystems goes unbounded. This new Nussbaum gain can be applied to the systems considered in Chen et al. (2014) to remove the assumption of known lower and upper bounds of the control coefficients.

Tracking and disturbance rejection can be unified as an output regulation problem (Isidori, 1995). A recent result for consensus output regulation of linear systems is shown in Grip et al. (2012) and for nonlinear systems in Ding (2013a), Su and Huang (2013). The results shown in the later two are for the nonlinear subsystems with relative degree one. In particular, the result in Su and Huang (2013) extends the result for nonlinear output regulation with unknown exosystems (Ding, 2003) to consensus output regulation. The key step in designing an adaptive scheme for systems with consensus control is to ensure that only the information available in the local neighbourhood can be used for the adaptive control design. This is relatively easier when the subsystems are of relative degree one, for which no exchange of adaptive parameters are needed in the consensus control. With higher relative degrees, filtered-transformation and backstepping, the tools to tackle high relative degrees, tend to propagate uncertainties in the network. In such a case, adaptive laws need to be designed with collaboration of the subsystems in the neighbourhood. We propose adaptive laws and control inputs with the information available from the subsystems in the neighbourhood, and therefore the adaptive laws and inputs are still viewed as decentralized, as no information from the subsystems outside the neighbourhood are needed. It is also noted that as for the subsystem outputs, the proposed design only uses the relative information between the subsystems. Lyapunov function based analysis is used to establish the stability of the adaptive output regulation design for consensus control using the proposed adaptive laws and the new Nussbaum gain design. The proposed control can deal with the subsystems with different dynamics as long as the subsystems with the same relative degree. An example is included to demonstrate the proposed control design with the simulation results shown.

2. Problem formulation

In this paper, we consider a set of N nonlinear subsystems, of which the subsystems are described by

$$\begin{aligned}\dot{x}_i &= A_{ci}x_i + \phi_i(y_i, w, \mu_i) + b_i u_i,\\ y_i &= C_i x_i,\end{aligned} \tag{1}$$

with $b_i, C_i^T \in \mathbb{R}^{n_i}$ and

$$A_{ci} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0\\ 0 & 0 & 1 & \dots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & 0 & \dots & 1\\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}, \qquad b_i = \begin{bmatrix} 0\\ \vdots\\ 0\\ b_{i,\rho}\\ \vdots\\ b_{i,n} \end{bmatrix},$$

$$C_i^T = \begin{bmatrix} 1\\ 0\\ \vdots\\ 0 \end{bmatrix}$$

for $i = 1, \dots, N$, where $x_i \in \mathbb{R}^{n_i}$ is the state vector, with n_i known positive constant integers denoting the order of the subsystems, $y_i, u_i \in \mathbb{R}$ are the output and input respectively of the ith subsystem, $\mu_i \in \mathbb{R}^{q_i}$ and $b_i \in \mathbb{R}^{n_i}$ are vectors of unknown parameters, with b_i being a Hurwitz vector with $b_{i,\rho} \neq 0$, which implies the relative degree of the system is ρ, $\phi_i : \mathbb{R} \times \mathbb{R}^m \to \mathbb{R}^{n_i}$ contains nonlinear functions with each element as polynomials of its variables and satisfies $\phi_i(0, w, \mu_i) = 0$, and $w \in \mathbb{R}^m$ are disturbances, and they are generated from an unknown exosystem

$$\dot{w} = S(\sigma)w \tag{2}$$

with unknown $\sigma \in \mathbb{R}^s$, of which, $S \in \mathbb{R}^{m\times m}$ is a constant matrix with distinct eigenvalues of zero real parts.

The connections between the subsystems are specified by an undirected graph $\mathcal{G}$ that consists of a set of vertices denoted by $\mathcal{V}$ and a set of edges denoted by $\mathcal{E}$. A vertex represents a subsystem, and each edge represents a connection. Associated with the graph, its adjacency matrix A with elements a_{ij} denotes the connections such that $a_{ij} = 1$ if there is a path from subsystem j to subsystem i, and $a_{ij} = 0$ otherwise. Since the connection is undirected, we have $A = A^T$. We define the Laplacian matrix L is a normal way as $l_{ii} = \sum_{j=1}^{N} a_{ij}$ and $l_{ij} = -a_{ij}$ when $i \neq j$.

We define the output regulation errors as

$$e_i = y_i - g(w) \tag{3}$$

with $g : \mathbb{R}^m \to \mathbb{R}$ being polynomials of its variables, for $i = 1, \dots, N$. In our set up, not every subsystem has access to $g(w)$, and the consensus output regulation will be achieved through the network connections among the subsystems.

The adaptive consensus output regulation problem considered in this paper is to design an adaptive control strategy using the relative output information $y_i - y_j$, $i \neq j$, provided by the network connection to each subsystem to ensure the convergence to zero of output regulation errors e_i for $i = 1, \dots, N$ under any initial condition of the system in the state space, i.e., the convergence of the subsystem outputs y_i to the common function $g(w)$.

Not all the subsystems have the access to the function value of $g(w)$. We use a diagonal matrix Δ to denote the access to $g(w)$ in the way that if $\delta_{ii} = 1$, the ith subsystem has access to the value of $g(w)$ for the control design, and $\delta_{ii} = 0$ otherwise. At least one subsystem has the access. The subsystems which do not have access to the tracking signal rely on the network connections to achieve the consensus tracking.

We make several assumptions about the dynamics of the subsystems, the exosystem and the connections between the subsystems.

Assumption 1. The invariant zeros of $\{A_{ci}, b_i, C_i\}$ are stable, for $i = 1, \dots, N$, and all the subsystems have the same sign of the high-frequency gains.

Assumption 2. The eigenvalues of S are distinct and on the imaginary axis.

Assumption 3. The adjacency matrix A is irreducible.

Remark 1. When the disturbance term w disappears, each subsystem is in the standard output feedback form to which the geometric conditions for a nonlinear system to be transformed are specified in Krstic, Kanellakopoulos, and Kokotovic (1995). A_{ci} and C_i are parts of the standard form, and therefore as long as the systems are in the output feedback form even with different dynamics, we can always write the corresponding system matrices in these formats. ◁

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