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Brief paper

Collective behavior of mobile agents with state-dependent interactions ☆

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ABSTRACT

In this paper, we develop a novel self-propelled particle model to describe the emergent behavior of a group of mobile agents. Each agent coordinates with its neighbors through a local force accounting for velocity alignment and collision avoidance. The interactions between agents are governed by path loss influence and state-dependent rules, which results in topology changes as well as discontinuities in the local forces. By using differential inclusion technique and algebraic graph theory, we show that collective behavior emerges while collisions between agents can be avoided, if the interaction topology is jointly connected. A trade-off between the path loss influence and connectivity condition to guarantee the collective behavior is discovered and discussed. Numerical simulations are given to validate the theoretical results.

1. Introduction

Collective motions of animals such as fish and birds are examples of large scale self-organization observed in nature. In many cases, cohesive groups are formed, where hundreds or thousands of agents move together in the same direction. In order to reveal the underlying mechanisms, several self-propelled particle models have been proposed and analyzed (Couzin, Krause, James, Ruxton, & Franks, 2002; Cucker & Smale, 2007; Olfati-Saber, 2006; Vicsek & Zafeiris, 2012). Three widely adopted rules behind these models include repulsion, attraction and alignment (Reynolds, 1987). More recently, collective behavior and self-organization have also attracted attention from engineers with the aim of controlling mobile robots in the context of cooperative control, formation control and so on (Haghighi & Cheah, 2012; Liu, Xie, & Zhang, 2014).

Besides considerable interests in the numerical or empirical modeling for collective behavior, much attention has been paid to rigorous mathematical analysis. In Jadbabaie, Lin, and Morse (2003) and Ren and Beard (2005), sufficient conditions were given for convergence of a simplified first-order Vicsek model. It was shown that under some joint connectivity conditions on the interaction topologies, all the agents eventually move in the same direction. For the second-order dynamics, the so called C–S model was proposed in Cucker and Smale (2007). Sufficient conditions were established to show that flocking can be achieved asymptotically. Extension to collision avoidance can be seen in Cucker and Dong (2011). One limitation of the C–S model is that each agent must interact with all the others during the motion. A theoretical framework for design and analysis of flocking algorithms with second-order dynamics was presented in Olfati-Saber (2006). Collective motions were obtained theoretically by applying artificial potentials embodying the three rules mentioned previously. In Zhang, Zhai, and Chen (2011), the authors proposed a self-propelled model with only repulsion and alignment forces. Under a joint connectivity condition, flocks would be assembled in finite time. Distributed coordination of mobile agents with nonlinear interactions was studied in Mei, Ren, and Ma (2013), where only relative positions are needed for each agent. But it is required that the connection

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pattern between agents be fixed all the time. To tackle the issue of dynamic topology, results of differential inclusions and switched systems were used to examine the stability analysis of time-varying flocking in Chen and Zhang (2011), Tanner, Jadbabaie, and Pappas (2007) and Zavlanos, Tanner, Jadbabaie, and Pappas (2009).

Most of the previous models rely on the aprioristic assumption that agents interact with all those within a fixed range. However, in a recent field study of flocks of starlings (Ballerini et al., 2008a), it is found that interaction is ruled by topological distance rather than metric distance. In other words, the relevant quantity is how many intermediate agents separate two birds, not how far apart they are. This means that the interactions are varying rather than fixed. In this case, whether the collective behavior of mobile agents can be achieved theoretically is still unclear.

As a first step towards this direction, in this paper, we introduce the state-dependent interaction for a self-propelled particle model in the topological sense. With the state-dependent interactions, the connection pattern is no longer fixed, but dynamic. Moreover, the path loss influence depending on relative distances between agents is considered. The main objective is to develop rigorous analysis in a general setting and explore how to achieve the collective motion with collision avoidance. The contributions of this paper can be summarized as follows:

- For the fixed-range interactions, a widely adopted method is to employ the invariance principle to determine the asymptotic stability. In this case, compactness of certain invariant sets follows from the connectivity directly (Chen & Zhang, 2011; Olfati-Saber, 2006; Tanner et al., 2007; Zhang et al., 2011). However, for the state-dependent interactions this is not a trivial task, since no *a priori* information about the boundedness of the state-dependent interactions can be inferred. We develop some novel techniques in terms of nonsmooth analysis coupled with algebraic theory to solve this problem.
- We investigate the impact of the path loss influence on the collective behaviors. Theoretically we show that there is a trade-off between the path loss influence and connectivity condition to guarantee the collective behavior. This is one unique feature of state-dependent interactions, which is not observed in the previous work on fixed-range interactions.

The paper is organized as follows. Section 2 presents the self-propelled particle model with state-dependent interactions. In Section 3, we give some preliminaries about the solution of the model, followed by the convergence analysis in Section 4. Simulation results are provided in Section 5. Finally, Section 6 concludes the paper.

2. Model formulation

Consider a group of N mobile agents with the dynamics of each agent described by a double integrator

$$\dot{\hat{x}}_i(t) = \hat{v}_i(t), \qquad \dot{\hat{v}}_i(t) = \hat{u}_i(t), \quad i = 1, 2, \ldots, N, \tag{1}$$

where $\hat{x}_i, \hat{v}_i \in \mathbb{R}^n$ are the position and velocity of agent *i*, respectively, and $\hat{u}_i \in \mathbb{R}^n$ is the acceleration to be designed.

Numerical and empirical investigations support the idea that the behavior of agents results from local coordination based upon the relative positions and velocities with each other. Motivated by the findings in Ballerini et al. (2008a), we consider that the interaction range is state-dependent by incorporating the topological distance.

Definition 1. The state-dependent interaction is defined as follows: (i) each agent *i* interacts with the $N^* \le N$ closest neighbors $\mathcal{N}^*(\hat{x}_i)$; (ii) if agent *j* interacts with agent *i*, then agent *i* also interacts with agent *j*.

Remark 2. Note that if $N^* = N$, then each agent interacts with all the others and the state-dependent interaction coincides with the all-to-all interaction (Cucker & Dong, 2011; Cucker & Smale, 2007; Gazi & Passino, 2003; Vicsek & Zafeiris, 2012).

We can model the interaction topology between agents as a dynamic bidirected graph $\mathcal{G}(\hat{x}(t)) = (\mathcal{V}, \mathcal{E}(\hat{x}(t)))$, where $\mathcal{V} = \{1, 2, \ldots, N\}$, and $\mathcal{E}(\hat{x}(t)) \subset \mathcal{V} \times \mathcal{V}$ is the set of edges at t. At each time, each agent assesses the position and/or velocity of its neighbors $\mathcal{N}_i(\hat{x}) = \mathcal{N}^*(\hat{x}_i) \bigcup \{j \notin \mathcal{N}^*(\hat{x}_i) : i \in \mathcal{N}^*(\hat{x}_j)\}$ within two non-overlapping behavioral zones: zone of repulsion and zone of alignment.

In this paper, we propose the following control law for each agent:

$$\hat{u}_i = \underbrace{\phi_{i,rep}}_{\text{repulsion force}} + \underbrace{\psi_{i,al}}_{\text{alignment force}},$$
$$\phi_{i,rep} = \sum_{j \in \mathcal{N}_i(\hat{x})} \phi(\|\hat{x}_{ij}\|^2)\hat{x}_{ij},$$
$$\psi_{i,al} = \sum_{j \in \mathcal{N}_i(\hat{x})} \psi(\|\hat{x}_{ij}\|)\mathrm{SGN}(\hat{v}_{ji}), \tag{2}$$

where $\hat{x}_{ij} = \hat{x}_i - \hat{x}_j$, $\hat{v}_{ij} = \hat{v}_i - \hat{v}_j$, $\mathrm{SGN}(\hat{v}_{ji}) = \frac{\hat{v}_{ji}}{\|\hat{v}_{ji}\|}$, if $\hat{v}_{ji} \neq 0$ and 0 otherwise; $\phi_{i,rep}$ is the repulsion force in the zone of repulsion with diameter $r > 0$ corresponding to the hard sphere of agent i (Ballerini et al., 2008b), in which $\phi(s) \ge 0$ is nonincreasing, $\phi(s) = 0, \forall s \in [r^2, \infty)$ and $\int_0^\infty \phi(s)\mathrm{d}s = \infty$; $\psi_{i,al}$ is the alignment force in a larger zone outside the hard sphere with $\psi(s) \ge 0$ being continuous and nonincreasing. The introduction of $\psi(s)$ can capture the influence or the path loss of communication between neighboring agents, e.g., $\psi(\|\hat{x}_{ij}\|) \propto \frac{1}{\sqrt{1+\|\hat{x}_{ij}(t)\|^\eta}}$,[2] where $\eta \ge 0$ denotes the path loss exponent.

Remark 3. A general form of $\phi(\|\hat{x}_{ij}\|^2)$ and $\psi(\|\hat{x}_{ij}\|)$ is adopted in (2). We note that some specific forms of the controller have been introduced for applications in Vicsek and Zafeiris (2012) and references therein.

In the following sections, we will examine the conditions under which collective behaviors would emerge and the impact of the path loss influence on the eventual motion.

3. Filippov solution and its properties

Let $\hat{x} = [\hat{x}_1^T, \hat{x}_2^T, \ldots, \hat{x}_N^T]^T$, $\hat{v} = [\hat{v}_1^T, \hat{v}_2^T, \ldots, \hat{v}_N^T]^T$, $\hat{u} = [\hat{u}_1^T, \hat{u}_2^T, \ldots, \hat{u}_N^T]^T$, and the set of discontinuous points of $\hat{u}$ be $\mathcal{S}$. Then $\mathcal{S} \subseteq \mathcal{S}_1 \bigcup \mathcal{S}_2$, where $\mathcal{S}_1$ is the set of discontinuous points due to switching from one connection pattern to another, $\mathcal{S}_2 = \Big\{[\hat{x}^T, \hat{v}^T]^T \in \mathbb{R}^{2Nn}$: there exist i, j such that $\hat{v}_i = \hat{v}_j$ or $\phi(s)$ is discontinuous at $\|\hat{x}_{ij}\|^2\Big\}$. It is clear that $\mathcal{S}_1 \bigcup \mathcal{S}_2$ has measure zero, and so does $\mathcal{S}$. In this case, the Filippov solution is an appropriate choice for such dynamical system (1) with discontinuous right-hand side (2).

Define the center of mass of the group as $x_c = \frac{1}{N}\sum_{i=1}^N \hat{x}_i$ and $v_c = \frac{1}{N}\sum_{i=1}^N \hat{v}_i$. Let $x_i = \hat{x}_i - x_c$ and $v_i = \hat{v}_i - v_c$, then

$$\dot{x}_i(t) = v_i(t), \qquad \dot{v}_i(t) = u_i(t), \quad i \in \mathcal{V}, \tag{3}$$

[2] This form is inspired by the path loss effect for wireless communications (Goldsmith, 2005). A similar function called communication strength is used in the C–S model (Cucker & Smale, 2007; Vicsek & Zafeiris, 2012).