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A simple existence condition of one-degree-of-freedom block decoupling controllers☆

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ABSTRACT

A block decoupling problem in linear multivariable systems is treated for one-degree-of-freedom controller configuration with unity output feedback. The plant transfer matrix, which may be non-square, is assumed to have unstable simple poles and zeros that may coincide. A simple existence condition of a block decoupling controller is obtained by directional interpolation approaches.

1. Introduction

A number of researchers have been studying the design of diagonal decoupling controllers which amounts to eliminate coupling characteristics between the reference inputs and the plant outputs so that one input affects only a single output. A more general form of decoupling design is the block decoupling problem which includes the diagonal decoupling problem as a special case. The block decoupling problem has been studied in both of the state-space and frequency domains. Wonham and Morse (1970) obtain a solvability condition using geometric approach in the state-space domain. In the frequency domain, Hautus and Heymann (1983) show that the decoupling design and the stability problem can be treated independently by two-degree-of-freedom (2DOF) controller configuration. Lee and Bongirono (1993) show that a diagonal decoupling controller, hence a block decoupling controller, always exists when 2DOF controller configuration is adopted for a plant whose transfer matrix is rectangular with full row rank. Recently, Kučera (2013) treats a 2DOF block decoupling problem in the most general setting in which the measurement output may be different from the output to be decoupled. He obtains the parameterized form of block decoupling controllers and solves an H_2 optimal problem as well.

As stated previously, 2DOF controller configuration is particularly ideal to decoupling design since stability and decoupling problems are separate issues. In this regard, decoupling design with 1DOF controller configuration is more restrictive than 2DOF configuration and its solvability condition is usually hard to obtain. Howze and Bhattacharyya (1997) point out, however, that the asymptotic property of unity feedback 1DOF controllers is not fragile with respect to the controller parameters, which is not the case of 2DOF controllers. Since this asymptotic tracking property is inherent to the controller configurations, the similar results would be inferred in 1 and 2DOF block decoupling design and hence 1DOF block decoupling design has its own advantage (see also Introduction section of Park, 2012). The existence condition for 1DOF block decoupling controllers is presented by Linnemann and Wang (1993) and Lin and Wu (1998). Linnemann and Wang (1993) show that a block decoupling controller exists if a strict block-adjoint matrix and a stability-factor matrix are externally skew prime. Lin and Wu (1998) derive an existence condition by using partial fraction expansions of the plant transfer matrix and its inverse under the assumption that they have one coincident unstable pole of order of 1 or 2. Lin and Wu (1999) present all achievable input–output block maps as a parameterized form for the case that the plant does not have coincident unstable poles and zeros. The formula for the existence condition in Linnemann and Wang (1993) needs a stability factorization of the plant, which requires tedious derivation of a Smith–McMillan form of the plant. The formula in Lin and Wu

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(1998) requires checking of rank equality of two matrices consisting of Kronecker products, which usually causes a dimension inflation problem, of residue matrices. The purpose of this article is to present a simple existence condition of a block decoupling controller when the plant, which may be rectangular, has unstable simple poles and zeros that may be coincident. The only calculation needed is to obtain input and output zero direction vectors from the coprime fractional descriptions of the plant.

Throughout the article, we consider only real rational matrices. A rational matrix $G(s)$, not necessarily proper, is said to be stable if it is analytic in $\mathrm{Re}\, s \geq 0$. The notations C and C_+ denote the complex number plane and the closed right half plane, respectively. The notations G^T and σ^* denote the transpose of the matrix G and the conjugate transpose of the vector σ, respectively. The degree of a zero or a pole of a rational matrix is defined in the sense of the Smith–McMillan degree. We adopt coprime polynomial matrix fractional descriptions of rational matrices. Though all formulas are described in terms of coprime polynomial fractions, the results derived in this article hold with the coprime stable rational matrix fractions.

2. Block decoupling problem and solvability condition

We consider a block decoupling problem for one-degree-of-freedom controller configuration with unity output feedback. For a given plant $G(s)$ whose size is $n \times q$, $n \leq q$, the problem can be described as finding stabilizing controllers $C(s)$ that make the input–output transfer matrix $T(s)$ block-diagonal and invertible, where $T(s)$ is given by

$$T(s) = G(s)C(s)(I + G(s)C(s))^{-1}. \tag{1}$$

Here we assume that the plant $G(s)$ is free of unstable hidden poles. For ease of presentation, we consider the case that the block diagonal $T(s)$ has two blocks of $T_1(s)$ and $T_2(s)$ whose sizes are $n_1 \times n_1$ and $n_2 \times n_2$, respectively, with $n_1 + n_2 = n$. In this case, the input–output transfer matrix $T(s)$ is said to be block-decoupled with the partition (n_1, n_2). Since $T(s)$ is required to be block-diagonal and invertible, $G(s)$ is assumed to have full row rank. Let $G(s) = A^{-1}(s)B(s) = B_1(s)A_1^{-1}(s)$ denote coprime polynomial matrix fractional descriptions. There always exist polynomial matrices $X(s)$, $Y(s)$, $X_1(s)$ and $Y_1(s)$ such that

$$\begin{bmatrix} X_1 & Y_1 \\ -B & A \end{bmatrix}\begin{bmatrix} A_1 & -Y \\ B_1 & X \end{bmatrix} = \begin{bmatrix} A_1 & -Y \\ B_1 & X \end{bmatrix}\begin{bmatrix} X_1 & Y_1 \\ -B & A \end{bmatrix} = I. \tag{2}$$

It is known that the class of all stabilizing controllers is given by $C(s) = (Y + A_1K)(X - B_1K)^{-1}$, where $K(s)$ is a stable rational matrix. Therefore, the transfer matrix $T(s)$ of block-decoupled systems with internal stability must be of the form

$$diag\{T_1(s), T_2(s)\} = B_1(s)Y_1(s) + B_1(s)K(s)A(s), \tag{3}$$

and a block decoupling controller exists if there exists a stable $K(s)$ that makes $B_1Y_1 + B_1KA$ block-diagonal. The purpose of this article is to present a simple existence condition of a block decoupling controller when the plant $G(s)$ has simple poles and zeros in C_+. The following lemmas are useful to prove Theorem 1. Lemma 1 is a special case of Lemma 3.3 in Lin and Wu (1998). (It is noted here that Lemma 3.3 in Lin and Wu (1998) is valid without the inequality condition $p \geq q$.)

Lemma 1. *Let σ and μ be given $n \times 1$ column vectors. If $\sigma^*\mu = 0$, then there exists an $n \times n$ constant matrix P satisfying the equalities*

$$\sigma^* P = 0 \quad and \quad P\mu = \mu. \tag{4}$$

Lemma 2 (Park, 2009)**.** *Suppose that $G(s)$ is a square stable rational matrix with full normal rank and it has distinct simple zeros $z_i \in C_+$, $i = 1 \to m$. Let σ_i^* be an output zero direction vector of $G(z_i)$ so that $\sigma_i^* G(z_i) = 0$ and $G^{-1}(s)$ be denoted by the partial fractional expression*

$$G^{-1}(s) = \sum_{i=1}^{m} \frac{M_i}{s - z_i} + F(s), \tag{5}$$

where M_i is the residue matrix at z_i and $F(s)$ is a stable matrix. Then the jth row of M_i is either zero or proportional to σ_i^*. That is, the jth row of M_i is of the form $k_{ij}\sigma_i^*$, $k_{ij} \in C$.

In this article we assume that the plant $G(s)$ has simple zeros and poles in C_+. It is well known that when the plant has no coincident poles and zeros in C_+, there exists a diagonal decoupling controller. So it is assumed here that the plant has common poles and zeros in C_+.

Assumption 1. $B_1(s)$ has the simple zeros $v_i \in C_+$, $i = 1 \to m_1$ and $A(s)$ has the simple zeros $w_j \in C_+$, $j = 1 \to m_2$, with $v_i \neq w_j$ for any i and j. $B_1(s)$ and $A(s)$ have common simple zeros $z_k \in C_+$, $k = 1 \to m_3$.

Since v_i, w_j, and z_k are simple zeros, there exist nonzero vectors β_i, α_j, σ_k and μ_k such that

$$\beta_i^* B_1(v_i) = 0, \qquad A(w_j)\alpha_j = 0 \tag{6}$$

$$\sigma_k^* B_1(z_k) = 0 \quad \text{and} \quad A(z_k)\mu_k = 0, \tag{7}$$

for $i = 1 \to m_1$, $j = 1 \to m_2$ and $k = 1 \to m_3$. For a given vector $x = [x_1\, x_2\, \cdots\, x_n]^T$, let us define its two sub-vectors x_a and x_b as

$$x_a = [x_1\, x_2\, \cdots\, x_{n_1}]^T, \qquad x_b = [x_{n_1+1}\, x_{n_1+2}\, \cdots\, x_{n_1+n_2}]^T. \tag{8}$$

When there exist stable rational matrices $T_1(s)$ and $T_2(s)$ satisfying the equality in (3), it is suggestive to observe that the matrices T_1 and T_2 satisfy some directional interpolation conditions. In fact, pre-multiplying $\beta_i^* = [\beta_{ia}^*\, \beta_{ib}^*]$ and $\sigma_k^* = [\sigma_{ka}^*\, \sigma_{kb}^*]$ to (3) with $s = v_i$ and $s = z_k$, respectively, and post-multiplying $\alpha_j = [\alpha_{ja}^T\, \alpha_{jb}^T]^T$ and $\mu_k = [\mu_{ka}^T\, \mu_{kb}^T]^T$ to (3) with $s = w_j$ and $s = z_k$, respectively, yields, with the aid of the equality $B_1(s)Y_1(s) + X(s)A(s) = I$ which is obtained from (2), that

$$\beta_{ia}^* T_1(v_i) = 0, \qquad T_1(w_j)\alpha_{ja} = \alpha_{ja}, \tag{9}$$

$$\sigma_{ka}^* T_1(z_k) = 0, \qquad T_1(z_k)\mu_{ka} = \mu_{ka}, \tag{10}$$

$$\beta_{ib}^* T_2(v_i) = 0, \qquad T_2(w_j)\alpha_{jb} = \alpha_{jb}, \tag{11}$$

$$\sigma_{kb}^* T_2(z_k) = 0, \qquad T_2(z_k)\mu_{kb} = \mu_{kb}, \tag{12}$$

for $i = 1 \to m_1$, $j = 1 \to m_2$ and $k = 1 \to m_3$. Also differentiating (3) yields that

$$diag\{T_1'(s), T_2'(s)\} = B_1'Y_1 + B_1Y_1' + B_1'KA + B_1K'A + B_1KA'. \tag{13}$$

Now pre-multiplying σ_k^* and post-multiplying μ_k to this equation with $s = z_k$, we obtain an interpolation condition

$$\sigma_{ka}^* T_1'(z_k)\mu_{ka} + \sigma_{kb}^* T_2'(z_k)\mu_{kb} = \rho_k, \quad k = 1 \to m_3, \tag{14}$$

where $\rho_k = \sigma_k^* B_1'(z_k)Y_1(z_k)\mu_k$. Now we present the main result.

Theorem 1. *Under Assumption 1, a block decoupling controller with the partition (n_1, n_2) exists if and only if*

$$\sigma_{ka}^*\mu_{ka} = 0 \quad and \quad \sigma_{kb}^*\mu_{kb} = 0, \quad k = 1 \to m_3. \tag{15}$$