ARTICLE IN PRESS

Automatica ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Technical communique

Some remarks on stability of stochastic singular systems with state-dependent noise☆

Weihai Zhang [a,1], Yong Zhao [a,b], Li Sheng [c]

[a] College of Electrical Engineering and Automation, Shandong University of Science and Technology, Qingdao, 266590, China

[b] College of Mathematics and Systems Science, Shandong University of Science and Technology, Qingdao, 266590, China

[c] College of Information and Control Engineering, China University of Petroleum (East China), Qingdao, 266580, China

ARTICLE INFO

Article history:
Received 16 March 2014
Received in revised form
10 August 2014
Accepted 8 September 2014
Available online xxxx

Keywords:
Stochastic singular systems
Stability
Linear matrix inequality

ABSTRACT

This paper discusses the stability for stochastic singular systems with state-dependent noise in both continuous-time and discrete-time cases. Firstly, the condition for the existence and uniqueness of the solution to stochastic singular systems is given. Based on this condition, the stochastic Itô singular system is transformed into a deterministic standard singular system by means of $\mathcal{H}$-representation method. Secondly, new sufficient conditions for the stability of systems considered are derived in terms of strict linear matrix inequalities. Finally, an example is given to illustrate the effectiveness of the obtained theoretical results.

1. Introduction

Singular systems have received considerable attention, which are able to describe a larger class of practical systems than the normal linear systems model. For example, mechanical systems, electric circuits, interconnected systems, and so on, can be modeled by descriptor systems. In the past years, the control of singular systems has been extensively studied and a lot of concepts and results in the normal linear systems such as stability (Dai, 1989; Ishihara & Terra, 2002; Takaba, Morihara, & Katayama, 1995; Xu & Lam, 2006; Xu & Yang, 1999), H_∞ control (Shi, Boukas, & Agarwal, 2000; Xu & Lam, 2006; Xu & Zou, 2003; Zhang & Huang, 2004; Zhang, Huang, & Lam, 2003), have been extended to singular systems. In recent years, the study of singular systems has made a rapid progress and a class of stochastic singular systems called the Markov jumping singular systems have also been investigated; see Boukas, Xu, and Lam (2005), Boukas and Yang (1999), Mao (1997), Wu, Shi, and Gao (2010); Wu, Su, and Shi (2012), Xia, Boukas, Shi, and Zhang (2009); Xia, Zhang, and Boukas (2008), Xu and Lam (2006) and the references therein. In particular, the references (Wu et al., 2010, 2012) studied the sliding mode control of Markov jump singular systems. However, there are few report on the study of stochastic singular systems with state-dependent noise, which is a more realistic mathematical model due to that in many branches of science and industry, the system is often perturbed by various types of environment noises; see Problem 1 in Oksendal (1998), Example 2.4 in Mao (1997). The difficulty lies in the following two aspects: one is about the condition for the existence and uniqueness of the solution to the system equation, the other one is how to select a suitable Lyapunov function in order to apply the Itô's formula to stochastic singular systems. Boukas (2006) dealt with the stability for stochastic singular nonlinear hybrid systems with Gaussian noise, and some sufficient conditions were established provided that the solution exists. However, the use of Itô's formula was incorrect, which led to inappropriate conclusions about stability. In Huang and Mao (2011), the stability for stochastic singular systems with Markov jumping and state-dependent noise were studied. By transforming a stochastic singular system into a deterministic singular one, a sufficient condition for the exponential stability was given in terms of linear matrix inequalities (LMIs). It is convinced that the stability for stochastic singular systems have not been fully investigated, many valuable topics deserve further study.

☆ This work was supported by National Natural Science Foundation of China (Nos. 61174078, 61403420), Research Fund for the Taishan Scholar Project of Shandong Province of China, SDUST Research Fund (No. 2011KYTD105), and the State Key Laboratory of Alternate Electrical Power System with Renewable Energy Sources (Grant No. LAPS13018). This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate Editor Fabrizio Dabbene under the direction of Editor André L. Tits.

E-mail addresses: w_hzhang@163.com (W. Zhang), zhaoyong_er@126.com (Y. Zhao), victory8209@163.com (L. Sheng).

[1] Tel.: +86 532 80691786; fax: +86 532 86057187.

http://dx.doi.org/10.1016/j.automatica.2014.10.044
0005-1098/

Please cite this article in press as: Zhang, W., et al., Some remarks on stability of stochastic singular systems with state-dependent noise. Automatica (2014), http://dx.doi.org/10.1016/j.automatica.2014.10.044

The study of this paper improves the results of Boukas (2006) and Huang and Mao (2011). Firstly, for the stability of stochastic singular systems with state-dependent noise, a new condition for the existence and uniqueness of the solution is given. Based on this condition, by $\mathcal{H}$-representation method developed in Zhang and Chen (2012), the stochastic Itô singular system is changed into a deterministic standard singular system. It is concluded that the mean square stability of the stochastic singular system is equivalent to the asymptotic stability of the deterministic standard singular system on condition that both of them have solutions. Secondly, a sufficient condition for the stability is presented in terms of a strict LMI. Compared with Huang and Mao (2011), our results have the computational advantage. The reason why Theorem 3.1 of Boukas (2006) is improper is explained. Finally, an illustrative example is given to show the effectiveness of the proposed results.

Notations: S_n: the set of all $n \times n$ symmetric matrices; $A > 0$ (resp. $A < 0$): A is a real symmetric positive definite (resp. negative definite) matrix; A^T: the transpose of A; vec$(\cdot)$: the row stacking operator; $A \otimes B$: the Kronecker product of two matrices A and B; $\|\cdot\|$: the Euclidean norm of a vector; $\mathcal{E}(\cdot)$: the expectation operator; I_n: the $n \times n$ identity matrix.

2. Preliminaries

Consider the following n-dimensional linear stochastic Itô singular system

$$\begin{cases} Edx(t) = Ax(t)dt + Fx(t)dw(t), \\ x(0) = x_0 \in R^n, \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the system state vector, $x_0 \in R^n$ is the initial condition which is deterministic, E, A, F are constant $n \times n$ matrices and rank$(E) = r \leq n$. $w(t)$ is one-dimensional standard Wiener process that is defined on the complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$.

In order to guarantee the existence and uniqueness of the solution to system (1), we give the following lemma.

Lemma 1. *If there are a pair of nonsingular matrices $M \in R^{n\times n}$ and $N \in R^{n\times n}$ for the triplet (E, A, F) such that one of the following conditions is satisfied, then (1) has a unique solution.*

(i) $MEN = \begin{bmatrix} I_{n_1} & 0 \\ 0 & N \end{bmatrix}, \qquad MAN = \begin{bmatrix} A_1 & 0 \\ 0 & I_{n_2} \end{bmatrix},$

$$MFN = \begin{bmatrix} F_1 & F_2 \\ 0 & 0 \end{bmatrix}, \tag{2}$$

where $N \in R^{n_2\times n_2}$ is a nilpotent, $F_1 \in R^{n_1\times n_1}$, $F_2 \in R^{n_1\times n_2}$, $n_1+n_2 = n$.

(ii) $MEN = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \qquad MAN = \begin{bmatrix} A_1 & 0 \\ 0 & I_{n-r} \end{bmatrix},$

$$MFN = \begin{bmatrix} F_1 & F_2 \\ 0 & F_3 \end{bmatrix}, \tag{3}$$

where $A_1, F_1 \in R^{r\times r}$, $F_2 \in R^{r\times(n-r)}$ and $F_3 \in R^{(n-r)\times(n-r)}$.

Proof. See Lemma 2.2 of Huang and Mao (2011) for the proof of item (ii). As to (i), if we let $\xi(t) = N^{-1}x(t) = [\xi_1(t)^T \; \xi_2(t)^T]^T$, $\xi_1(t) \in R^{n_1}$, $\xi_2(t) \in R^{n_2}$, then under the condition (2), (1), is equivalent to

$$d\xi_1(t) = A_1\xi_1(t)dt + (F_1\xi_1(t) + F_2\xi_2(t))dw(t) \tag{4}$$

and

$$Nd\xi_2(t) = \xi_2(t)dt. \tag{5}$$

Taking the Laplace transform on both sides of (5), we have

$$(sN - I)\xi_2(s) = N\xi_2(0). \tag{6}$$

From (6), we obtain

$$\xi_2(s) = (sN - I)^{-1}N\xi_2(0). \tag{7}$$

The inverse Laplace transform of $\xi_2(s)$ yields

$$\xi_2(t) = -\sum_{i=1}^{h-1} \delta^{i-1}(t)N^i\xi_2(0), \tag{8}$$

where the Dirac function $\delta(t)$ has the Laplace transformation of $L[\delta^i(t)] = s^i$.

On the other hand, substituting (8) into (4), we obtain an ordinary stochastic differential equation. By Theorem 5.2.1 of Oksendal (1998), the solution of (4) exists and is unique, so does (1).

Remark 1. The response of system (1) may contain an impulse term, it is necessary to eliminate it to guarantee the solution of (1) to be well-posed.

In the following, we introduce three concepts which play important roles in this paper.

Definition 1. The stochastic Itô singular system (1) is said to be impulse free if deg$|sE - A|$ = rank(E).

Definition 2. The stochastic Itô singular system (1) is said to be asymptotically mean square stable if for any initial condition $x_0 \in R^n$, we have $\lim_{t\to\infty} \mathcal{E}\|x(t)\|^2 = 0$.

Definition 3. The stochastic Itô singular system (1) is said to be asymptotically mean square admissible if it has a unique solution and is impulse free and asymptotically mean square stable.

The following lemmas will be used in the sequel.

Lemma 2 (Bellman, 1995). *For any three matrices A, B and C of suitable dimensions,* vec$(ABC) = (A \otimes C^T)$vec(B).

Lemma 3 (Zhang & Chen, 2012). (i) *For any $X \in S_n$, there is a $n^2 \times \frac{n(n+1)}{2}$ matrix H_n independent of X such that* vec$(X) = H_n\tilde{X}$, $\tilde{X} = [x_{11}, x_{12}, \ldots, x_{1n}, x_{22}, \ldots, x_{2n}, \ldots, x_{nn}]^T$, *where $\tilde{X}$ is an $\frac{n(n+1)}{2}$-dimensional vector that is derived by deleting the repeated elements of* vec(X). *Conversely, for any $\zeta \in C^{n(n+1)/2}$, there is $X \in S_n$ such that* vec$(X) = H_n\zeta$, (ii) $H_n^T H_n$ *is nonsingular, i.e., H_n has full column rank.*

Lemma 4. *Let $Z_1, Z_2 \in S_n$, If H_n^Tvec$(Z_1) = H_n^T$vec(Z_2), then* vec(Z_1) = vec(Z_2).

Proof. By Lemma 3(i), vec$(Z_1) = H_n\tilde{Z}_1$, vec$(Z_2) = H_n\tilde{Z}_2$. From H_n^Tvec$(Z_1) = H_n^T$vec(Z_2), we have

$$H_n^T H_n\tilde{Z}_1 = H_n^T H_n\tilde{Z}_2.$$

By Lemma 3(ii), the above formula yields $\tilde{Z}_1 = \tilde{Z}_2$, which is equivalent to vec(Z_1) = vec(Z_2).

3. Stability of continuous-time systems

This section will be devoted to the study of stability for continuous-time stochastic Itô singular systems and Lemma 1 is assumed to be established.

Please cite this article in press as: Zhang, W., et al., Some remarks on stability of stochastic singular systems with state-dependent noise. Automatica (2014), http://dx.doi.org/10.1016/j.automatica.2014.10.044