Receding Horizon Optimal Control of Wiener Systems by Application of an Asymmetric Cost Function

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Abstract: Wiener models are an important class of nonlinear systems which well approximate many applications. Real time optimal control of Wiener models, for instance in the form of receding horizon optimal control, can be done using the nonlinear setup and corresponding nonlinear optimization tools. However, as this paper shows, under rather mild conditions on the static nonlinearity, it is possible to re-formulate the optimal control problem as a linear problem with an asymmetric cost function, whose solution can be computed using a slack variable extension of the initial quadratic problem with a small additional computational cost. This paper shows the approach and the achievable performance at the example of the emission control of a large gas engine used in the U.S. pipeline network.

Keywords: Receding horizon optimal control, Wiener model, asymmetric cost function, slack variables

1. INTRODUCTION

Wiener systems are a well known class of nonlinear systems consisting of a linear dynamics followed by a static nonlinear map. Due to their simple form, they have attracted much interest especially as a way to approximate more complex systems, see *e.g.* (Norquay*, et al.*, 1999) and references therein. Accordingly, the identification of Wiener and Hammerstein models has also attracted much attention, see *e.g.* (Guo, 2003; Pearson and Pottmann, 2000; Vandersteen and Schoukens, 1997).

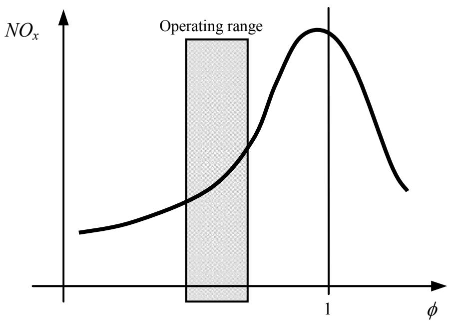

Fig. 1: NO_x as a function of ϕ

From a practical point of view, there are two different interpretations of Wiener models. The nonlinear static map can be interpreted as a measurement characteristic, implying the output of the linear part to be the real target value and the nonlinear element to be the deformation due to the sensor. In such a case, the nonlinear map can usually be inverted and the system treated to almost all purposes as a linear one. However, there are other cases in which the real target value is the output of the nonlinearity, so any linearization would induce substantial errors.

Consider, for example, the application behind this paper, *i.e.* the NO_x control of stationary operating gas engines used in pipeline compressor stations: the fuel/air ratio ϕ of these engines can be described rather well by linear models, but the real target quantity is the value of the exhaust NO_x concentration. Due to the limited speed and temperature range of these engines and the lean operation, it turns out that the relation between NO_x and ϕ can be approximated by a static map as shown in Fig. 1. It is evident that optimal control of ϕ is not equivalent to the optimal control of the NO_x concentration.

Receding horizon optimal control, in particular in the form of model predictive control, has been the topic of a huge number of publications and of many years of successful experience (Camacho and Bordons, 2004; Rawlings, 1999; Scokaert and Mayne, 1998). These methods have also been applied to the automotive field, see *e.g.* (Falcone*, et al.*, 2008; Ortner and del Re, 2007).

Still, very few works have been centered on the optimal control of Wiener models, some exceptions being a work on its application to a pH neutralization experiment (Norquay*, et al.*, 1999), or (Pérez*, et al.*, 2006) who consider the application of Wiener and Hammerstein models to predictive control of a Diesel engine airpath. Both have in common, that they only take into account the linear part in the optimal controller. (Bloemen*, et al.*, 2001) determine bounds for the nonlinearities and then apply a robust MPC approach. Recapitulatory, in all these cases a linear problem is solved instead of the original one.

To better understand the problem, recall that a discrete-time optimal control problem comprising linear system dynamics, a quadratic cost function and (possibly) linear constraints is well-known to be a quadratic programming problem (QP). Moreover, this QP is strictly convex – and thus has a unique solution – whenever the cost function is strictly convex. Convex QPs can be solved very efficiently and reliably using dedicated QP solvers.

Unfortunately, when system dynamics are given in form of a Wiener system, this valuable structure is lost. In fact, without additional assumptions on the nonlinear static output map, the optimal control problem will result in a general, and usually non-convex, nonlinear programming problem (NLP). This loss of structure increases computational load for solving these problems significantly (easily by one or more orders of magnitude) and, due to the lack of convexity, the solver might even get stuck in a local minimum. While keeping convexity for systems with nonlinear dynamics is almost hopeless (see (Azhmyakov and Raisch, 2008) for a few limited exceptions), this property remains for optimal control problems comprising a Wiener system if the static map satisfies certain conditions, as we will point out in the next section. If they are met, the resulting optimal control problem is equivalent to a convex NLP, where every local solution is also a global one. In such a situation, we propose substituting this convex NLP by a convex QP using an asymmetric cost function. In doing so, computational effort is greatly reduced while the nonlinearities of the Wiener systems are still taken into account, thus improving performance over a purely linear model.

2. MOTIVATION OF AN ASYMMETRIC COST FUNCTION

2.1 MPC with Symmetric Cost Function

In receding horizon control the objective is formulated at every time instant by use of the system description. Frequently the objective is to minimize the squared deviations from a given reference and to keep the control action small. The determination of the control signal Δu can then be formulated as the following optimization problem (with positive definite matrices Q_y and R):

$$\begin{aligned}
&\arg\min_{\Delta u} \frac{1}{2}\sum_{i=0}^{n_{PH}} \left(y_i - y_{ref,i}\right)^T Q_y \left(y_i - y_{ref,i}\right) + \Delta u_i^T R \Delta u_i \\
&\textit{subject to} \\
&\qquad u_i = u_{i-1} + \Delta u_i \\
&\qquad x_{i+1} = f(x_i, u_i) \\
&\qquad y_i = h(x_i) \\
&\qquad \underline{y} \le y_i \le \overline{y} \qquad i = 0 \ldots n_{PH} \\
&\qquad \Delta\underline{u} \le \Delta u_i \le \Delta\overline{u} \quad i = 0 \ldots n_{CH} - 1 \\
&\qquad \Delta u_i = 0 \qquad\qquad i = n_{CH} \ldots n_{PH}
\end{aligned} \tag{1}$$

In (1) the predicted system output y (y_i is actually equal to $y(t_0 + t_i \mid t_0)$) is used to determine the optimal input Δu over the control horizon with length n_{CH} that minimizes the objective over the prediction horizon with length n_{PH}. In case of a linear state space representation

$$\begin{aligned} x_{i+1} &= A \cdot x_i + B \cdot u_i \\ y_i &= C \cdot x_i \end{aligned} \quad , \tag{2}$$

formulation (1) simplifies to the standard linear MPC setup. In this case the optimization problem can be stated in the form of a quadratic problem

$$\begin{aligned} \min_{\Delta u} \quad & \frac{1}{2}\Delta u^T H \Delta u + \Delta u^T g \\ \text{s.t.} \quad & lbG \le G\Delta u \le ubG \\ & lb \le \Delta u \le ub \end{aligned} \quad . \tag{3}$$

The matrices H and G are constant, whereas the gradient g and the constraint vectors need to be updated every time instant. As mentioned before, problems in the form of (3) can be solved efficiently by numerical QP solvers, *e.g.* using the online active set strategy as proposed in (Ferreau*, et al.*, 2008) for the MPC context. If the system dynamics $f(x_i, u_i)$ or the output function $h(x_i)$ is nonlinear, optimization problem (1) cannot be stated as a QP anymore and thus a nonlinear solver is required to solve the resulting generic NLP. This does not only increase the online computational load significantly, also the implementation and software maintenance effort of an NLP solver is typically higher than that of a simpler QP solver. In order to circumvent these undesirable complications, the next section presents a class of nonlinear systems where the optimization problem (1) can be reformulated to make it suitable for QP solvers.

2.2 Asymmetric Cost Function for a Class of Wiener Systems

In general, Wiener systems consist of a linear dynamic block followed by a static nonlinearity.

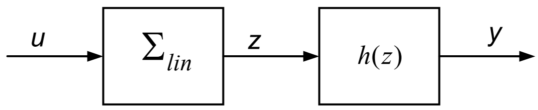

Fig. 2: Wiener system

In a state-space representation the system can be described by

$$\begin{aligned} x_{i+1} &= A \cdot x_i + B \cdot u_i \\ z_i &= C \cdot x_i \\ y_i &= h(z_i) \end{aligned} \quad , \tag{4}$$

where we make the standing assumption that h is invertible. Still, due to the nonlinearity $y_i = h(z_i)$ a nonlinear solver is required for the solution of problem (1) for Wiener systems. In theory, regulating the system to a constant setpoint $z_{ref} = h^{-1}(y_{ref})$ is equal to regulating the nonlinear system to y_{ref} for steady state. However, in case of actuator and/or system limits as well as model-plant mismatches, a perfect tracking might not be possible and thus minimizing the deviations to z_{ref} is not equal to minimizing the deviations to y_{ref}. In other words, if a tracking of the linear system part is done, than the objective needs to be changed to maintain the initial performance requirements.