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Resonant–repetitive controller with phase correction applied to uninterruptible power supplies

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ABSTRACT

This paper proposes a new control structure based on the parallel interconnection of a filtered repetitive controller and a resonant structure applied to uninterruptible power supplies — UPS. In particular, the filter in series with the repetitive controller adjusts the phase angle between resonant and repetitive loops, improving in this way the tracking performance. A particular filter structure is proposed, and an augmented state formulation is derived. Controller design is then carried out by the solution of an optimization problem with linear matrix inequalities constraints. Experimental results on a commercial 3.5kVA UPS illustrate the closed-loop performance.

1. Introduction

Uninterruptible power supplies are designed to deliver controlled voltage to critical loads with high quality and in a reliable way. International standards, such as *IEEE944* and *IEC62040-3*, state basic requirements for the output voltage in steady-state and also under transient conditions (IEC62040, 2011). These standards require a sinusoidal output voltage with fixed amplitude and frequency; in addition, it must have low *total harmonic distortion* (THD) and low *individual harmonic distortion* (IHD) rates when subjected to periodic load disturbances, usually caused by nonlinear loads. On the other hand, small variations in the amplitude and frequency of the output voltage are allowed. These standards also require a fast transient response to perturbations caused by load connections/disconnections.

Due to its simple structure and easy tuning, the proportional–integral–derivative (PID) controller has been extensively used by manufactures of UPS, although this type of controller is not appropriate to track a sinusoidal reference (Willmann, Coutinho, Pereira, & Libano, 2007). This drawback has motivated the development of new control techniques which make the controller more robust, enabling the closed-loop system not only to follow sinusoidal references but also to reject periodic disturbances, such as sliding-mode controllers (Liu, Vazquez, Wu, Marquez, Gao, & Franquelo, 2017; Liu, Yin, Luo, Vazquez, Franquelo, & Wu, 2017), which attempt to minimize the harmonic distortion caused by nonlinear loads. In this context, controllers based on the *internal model principle* (IMP) stand out, e.g. resonant (Karttunen, Kallio, Honkanen, Peltoniemi, & Silventoinen, 2017; Maccari, Pinheiro, Oliveira, & Montagner, 2017) and repetitive controllers (Nazir, 2017; Ramos, Costa-Castelló, & Olm, 2012; Yao, Tsai, & Yamamoto, 2013).

Resonant controllers with one resonant mode tuned to the fundamental frequency, when applied to a UPS, can track a sinusoidal reference with zero error. Furthermore, it provides a fast dynamic response when subjected to load transients (Fukuda & Yoda, 2001). In contrast, it is not able to completely reject disturbances containing harmonic components different from the fundamental frequency. An alternative in this case is a structure with multiple resonant controllers as showed in Pereira, Flores, Bonan, Coutinho, and Gomes da Silva (2014), which also contains resonant modes at the harmonic frequencies that mostly contribute to the disturbance signal.

On the other side, repetitive controllers satisfy the *internal model principle* (IMP) by means of a delay element corresponding to the fundamental period in a positive feedback loop, as described in Inoue, Nakano, and Iwa (1981). However, to achieve a stable operation and at the same time to avoid noise amplification, a low-pass first-order filter is connected in series with the delay element (Hara, Yamamoto, Omata, & Nakano, 1988). One disadvantage of this solution is a loss of tracking performance when following the reference signal caused by the reduction in magnitude and the displacement of resonance peaks in the controller frequency response. As a result, when applied to a UPS, this kind of controller can, in fact, provide rejection of perturbations with harmonic frequency components different from the fundamental frequency, but it shows a tracking error associated with the low-pass filter in series with the delay element. To mitigate this problem, a

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correction in the delay element is proposed in Hornik and Zhong (2011) and Weiss and Häfele (1999) to precisely locate the resonance peaks where they are required, whereas in Lorenzini, Flores, Pereira, Salton, and Castro (2015) this concept is used together with a correction in the magnitude of the filter response, thus restoring the infinite gain at the fundamental frequency of the repetitive controller.

The concept of a hybrid *resonant–repetitive* (RR) controller is relatively new in both control theory and power electronics literature as can be seen in Lidozzi, Ji, Solero, Zanchetta, and Crescimbini (2015), Salton, Flores, Pereira, and Coutinho (2013) and Yang, Zhou, and Blaabjerg (2016). The main idea behind this controller is to take full advantage of sinusoidal reference tracking with zero error, provided by the resonant structure, allied to harmonic rejection, yielded by the repetitive controller. In Salton et al. (2013), the authors proposed a parallel structure for the RR controller and robust synthesis conditions in the form of *linear matrix inequalities* (LMI). In addition, they considered the use of complementary filters to decouple resonant and repetitive effects at the fundamental frequency; nevertheless, this case cannot be treated with the proposed LMI conditions and only simulation results were provided. In Lidozzi et al. (2015) a series implementation of the RR controller is applied to a three-phase four-leg inverter, whereas Yang et al. (2016) employed a parallel structure to improve frequency adaptability of grid-connected power converters. In both works, the controller design is carried out by using frequency-domain techniques.

Based on the RR controller presented in Salton et al. (2013), this paper proposes the robust synthesis of a new RR controller with an alternative filter structure. In this new controller, termed here *filtered resonant–repetitive* (FRR) controller, two distinct corrections are considered: (i) the repetitive controller delay element is corrected as in Lorenzini et al. (2015) and (ii) an additional filter in series with the repetitive structure is used to adjust the phase angle between the resonant and repetitive loops. The closed-loop system is described based on an augmented state-space representation which is then used to design the feedback gains through the solution of a convex optimization problem under constraints in the form of LMI.

The main contributions of this paper are: (i) proposal of a filter in series with the repetitive structure to correct phase mismatch; (ii) tuning of controller parameters based on robust control techniques; (iii) experimental validation of the proposed method on a commercial UPS of 3.5 kVA and comparison with the RR structure from Salton et al. (2013) in the light of *IEC62040-3* performance requirements.

Notation: $\mathbb{R}$ is the set of the real numbers; the derivative of a function $r(t)$ is represented by $\dot{r}(t)$, while $\mathbf{I}_m$ stands for the identity matrix of order m. For two real matrices, say $\mathbf{A}$ and $\mathbf{X}$, $\mathbf{A}'$ indicates the transposed of $\mathbf{A}$, and $\mathbf{A} > 0$ ($\mathbf{A} < 0$) means that $\mathbf{A}$ is a symmetric, positive definite (negative definite) matrix and that $\mathrm{He}\{\mathbf{AX}\} = \mathbf{AX} + \mathbf{X}'\mathbf{A}'$.

2. Mathematical model of UPS

For the UPS considered here, sinusoidal output voltage is obtained through a half-bridge, single-phase inverter, whose output is connected to a second-order low-pass LC-filter, as schematically depicted in Fig. 1. The IGBTs (*Insulated Gate Bipolar Transistor*) S_1 e S_2 are driven by the control signal $u(t)$ according to a PWM technique (*Pulse-Width Modulation*).

Adopting a model that considers the mean value of the inverter voltage, the effects of switching S_1 and S_2 on and off can be represented by the gain $K_{\mathrm{PWM}} = V_{cc}/(2 \cdot \hat{V}_{tri})$ which multiplies the control signal $u(t)$ (Chen, Lai, Tan, & Tse, 2007). In this case, V_{cc} is the DC link voltage and $\hat{V}_{tri}$ is the amplitude of PWM triangular signal.

Loads at the inverter output are represented by the connection in parallel of an admittance $Y_0(t)$ and a current source $i_d(t)$ (Pereira et al., 2014). $Y_0(t)$ describes the behavior of linear loads, whose currents have sinusoidal waveform and are in phase with the corresponding voltages. Thus, $Y_0(t)$ is represented by a linear, time-varying element defined by

$$Y_0(t) \in \Delta,\ \Delta := \{Y_0(t) \in \mathbb{R} : Y_{min} \le Y_0(t) \le Y_{max}\}, \tag{1}$$

where its lower and upper limits, denoted here by Y_{min} and Y_{max}, are defined by the nominal load (Y_{min}) and the lowest load (Y_{max}) of the UPS. On the other hand, the current source $i_d(t)$ represents a harmonic disturbance caused by nonlinear loads.

In a state-space framework, the state vector $\mathbf{x_p}(t) = [i(t)\ \ v_{out}(t)]'$ is composed of the inductor current $i(t)$ and the capacitor voltage $v_{out}(t)$. Based on this state vector, dynamic equations describing the behavior of the UPS can be stated as follows (Pereira et al., 2014):

$$\begin{cases} \dot{\mathbf{x}}_p(t) = \mathbf{A}_p\big(Y_0(t)\big)\,\mathbf{x}_p(t) + \mathbf{B}_p\,u(t) + \mathbf{B}_{d_p}\,i_d(t) \\ y_p(t) = \mathbf{C}_p\,\mathbf{x}_p(t) \\ e(t) = r(t) - y_p(t), \end{cases} \tag{2}$$

where $y_p(t)$ is the voltage to be controlled, $r(t)$ is the voltage reference to be tracked by $y_p(t)$, and $e(t)$ is the tracking error. Matrices $\mathbf{A}_p\big(Y_0(t)\big)$, $\mathbf{B}_p$, $\mathbf{B}_{d_p}$, and $\mathbf{C}_p$ are obtained by circuit analysis and given by the following expressions.

$$\mathbf{A}_p\big(Y_0(t)\big) = \begin{bmatrix} -\dfrac{R_{L_f}}{L_f} & -\dfrac{1}{L_f} \\ \dfrac{1}{C_f} & -\dfrac{Y_0(t)}{C_f} \end{bmatrix},\ \mathbf{B}_p = \begin{bmatrix} \dfrac{K_{\mathrm{PWM}}}{L_f} \\ 0 \end{bmatrix},$$

$$\mathbf{B}_{d_p} = \begin{bmatrix} 0 \\ -\dfrac{1}{C_f} \end{bmatrix},\quad \mathbf{C}_p = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

It should be observed that the matrix $\mathbf{A}_p\big(Y_0(t)\big)$ changes with the load admittance $Y_0(t)$ connected at the inverter output, thus changing the dynamic behavior of the system. This fact requires the use of robust control methods to ensure an adequate level of performance under parametric uncertainties.

3. Internal model controllers

In this section, it is detailed the formulation of resonant and repetitive controllers based on the IMP.

3.1. Resonant controller

According to the IMP, a closed loop stable system can track a sinusoidal reference with frequency ω_0 and also completely reject disturbances at this frequency if the following term is inserted into the controller transfer function:

$$G_c(s) = \frac{1}{s^2 + w_0^2}. \tag{3}$$

Hence, $G_c(s)$ presents a resonance peak with infinite magnitude at the frequency of the signal to be rejected or followed, which is the main characteristic of a resonant controller (Chen, 1995).

Since $G_c(s)$ has two marginally stable poles on the imaginary axis ($\pm j\omega_0$), it is usual to add two zeros to improve closed-loop stability (Pereira et al., 2014). Therefore, the resonant controller transfer function can be defined as follows, where k_2, k_3, and k_4 are constants to be determined.

$$G_{rs}(s) = \frac{k_2(s^2 + \omega_0^2) + k_4 s + k_3}{s^2 + w_0^2}, \tag{4}$$

One possible realization of (4) in the state space is

$$\begin{cases} \dot{\mathbf{x}}_{rs}(t) = \mathbf{A}_{rs}\,\mathbf{x}_{rs}(t) + \mathbf{B}_{rs}\,u_{rs}(t) \\ y_{rs}(t) = \mathbf{C}_{rs}\,\mathbf{x}_{rs}(t) + D_{rs}\,u_{rs}(t), \end{cases} \tag{5}$$

where $\mathbf{x}_{rs}(t) = [x_{rs1}(t)\ \ x_{rs2}(t)]' \in \mathbb{R}^2$ is the state vector, $u_{rs}(t)$ and $y_{rs}(t)$, respectively, are the input and output signals of the resonant controller. Remaining controller matrices are defined below for a given reference frequency ω_0.

$$\mathbf{A}_{rs} = \begin{bmatrix} 0 & 1 \\ -\omega_0^2 & 0 \end{bmatrix},\ \mathbf{B}_{rs} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$\mathbf{C}_{rs} = \begin{bmatrix} k_3 & k_4 \end{bmatrix},\ D_{rs} = k_2.$$