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Full-state autopilot-guidance design under a linear quadratic differential game formulation

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ABSTRACT

Full-state single-loop and full-state two-loop autopilot-guidance architectures are derived under a linear quadratic differential game formulation. In the full-state single-loop case, the guidance command is injected directly to the actuator, whereas in the full-state two-loop case, it is the input to the autopilot loop. To prevent impractical end-game scenarios, where the states diverge to unacceptable values, a cost function that includes appropriate running cost terms on some of the states is proposed. The conditions for obtaining an equivalence relation between the full-state single-loop and full-state two-loop architectures are derived under a linear quadratic differential game formulation and the proposed cost function. Under such a formulation, the two full-state architectures are identical if and only if the number of guidance commands matches the number of available controllers. The guidance laws performance is illustrated using an interceptor missile having forward and aft controls in linear and nonlinear settings, while considering two types of evasion strategies. The first strategy is a linear controller based on the linear quadratic differential game solution. The second strategy is a "bang–bang" controller based on the optimal evasion solution. It is shown that the linear evasion strategy may not be suitable to represent a realistic evading strategy. In addition, the conditions for the existence of a saddle point solution are analyzed for the two full-state guidance laws.

1. Introduction

The term full-state guidance law refers to an autopilot-guidance system that has a full-state feedback into the guidance loop, and thus the coupling between the guidance and flight-control (G&C) loops is taken into account. Such a design may enhance the interceptor's performance and has the potential to meet advanced design requirements, i.e. improved accuracy and extended kill envelope. Two types of full-state G&C architectures were considered in previous papers (Idan, Shima, & Golan, 2007; Levy, Shima, & Gutman, 2013, 2015, 2017; Menon & Ohlmeyer, 2001; Menon, Sweriduk, & Ohlmeyer, 2003; Palumbo, Reardon, & Blauwkamp, 2004; Rusnak & Levi, 1991; Shima, Idan, & Golan, 2006; Shkolnikov, Shtessel, & Lianos, 2001): full-state single-loop (FS-SL) and full-state two-loop (FS-TL). In the FS-TL case, the inner autopilot loop is designed independently of the outer guidance one, whereas in the FS-SL case, the guidance command is injected directly to the actuator, without a definite autopilot.

In the literature, the term "integrated guidance law" has been used to describe full-state G&C systems. In Palumbo et al. (2004), Shima et al. (2006), Idan et al. (2007), Menon and Ohlmeyer (2001) and Menon et al. (2003), the term "integrated" referred to FS-SL guidance systems, whereas in Shkolnikov et al. (2001) the term referred to an FS-TL autopilot-guidance system. The general solution of an optimal guidance law with a full-state feedback was derived for an arbitrary order autopilot model in Rusnak and Levi (1991). Theoretical results concerning the equivalence of the two full-state architectures were presented under one-sided optimal control formulation for linear quadratic optimization problems (Levy et al., 2013, 2015) and for nonlinear optimization problems with bounded controls (Levy et al., 2017).

In practice, the controller is bounded, which results in a nonlinear system during saturation. In fact, during saturation the G&C loop is opened and if in addition the open loop transfer function is unstable or close to instability, the attitude angle may diverge to unacceptable values (Gutman, Rubinsky, Shima, & Levy, 2013). In the nonlinear approach, the states can be kept at reasonable values by limiting the commands and using a carefully designed autopilot. In the linear quadratic approach, this can be done indirectly by adding running cost terms of each of the controllers to the cost function.

Perfect information of the target future maneuver is usually not available. Hence, an appropriate alternative to the optimal control formulation is the zero-sum pursuit-evasion game formulation (Isaacs,

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1965), where only the information on the target maneuver capability is required. The linear quadratic differential game (LQDG) formulation was presented in Ho, Bryson, and Baron (1965) while assuming ideal dynamics for both adversaries. In Ben-Asher and Yaesh (1998), this assumption was replaced by first-order dynamics for both the missile and the target. In Ben-Asher, Levinson, Shinar, and Weiss (2004) it was shown that an inclusion of a running cost on the state variables in LQDG guidance laws has a disturbance attenuation effect. In Turetsky and Shinar (2003), the bounded-control and LQDG formulations were compared by assuming first-order dynamics for both players.

In the present paper, FS-SL and FS-TL autopilot-guidance laws are derived under an LQDG formulation, by assuming: linear dynamics for both adversaries, perfect information of the states, and unbounded controls. The equivalence condition of the two full-state guidance laws is provided for a cost function that contains running cost terms of the states. According to the theorem, the two full-state optimization problems are identical if and only if the number of guidance commands is identical to the number of available controllers. This result encourages usage of the FS-TL architecture over the FS-SL one in practical autopilot-guidance systems. In these systems the autopilot is an important component since it ensures the inner stability of the airframe if the guidance loop is inactive. Thus, by following the equivalence condition, the FS-TL architecture can have the best of both worlds: inner stable autopilot loop, and at the same time, the same cost as the FS-SL architecture.

Preliminary results related to the present work appeared in Levy, Shima, and Gutman (2014). The present paper expands on Levy et al. (2014) in several directions:

1. The conditions for the existence of a saddle point solution are analyzed for both FS-SL and FS-TL architectures.
2. Derivation of the optimal evasion strategy in a practical bounded control setting against an LQDG maneuvering missile.
3. A thorough analysis of the FS-SL and FS-TL performance is done using linear and nonlinear test scenarios, while considering two types of target acceleration commands. The first evasion strategy is linear and is based on the LQDG formulation. The second evasion strategy is of "bang–bang" type and is based on the optimal evasion solution against an LQDG guided missile.

The remainder of this paper is organized as follows. The linearized model derivation and autopilot-guidance design are given in Section 2 and Section 3, respectively. The equivalence of the two full-state guidance laws is presented in Section 4. The test case and the corresponding guidance laws formulations are given in Section 5 and Section 6, respectively. Simulation results were made for two types of target strategies: an LQDG one and the optimal evasion strategy against an LQDG guided missile. These strategies are presented in Section 7, followed by the simulation results in Section 8, and the concluding remarks. The nonlinear models of the relative kinematics and missile lateral dynamics are presented in Appendix.

2. Linearized model derivation

This section provides the design assumptions and describes the linearized end-game geometry used for the synthesis of the guidance laws and their analysis.

2.1. Design assumptions

The derivation of the guidance laws will be performed based on the following assumptions:

1. A skid-to-turn roll-stabilized missile is considered. The motion of such a missile can be separated into two perpendicular channels, thus allowing to treat only a planar motion.
2. Linear dynamics for the target and the missile.

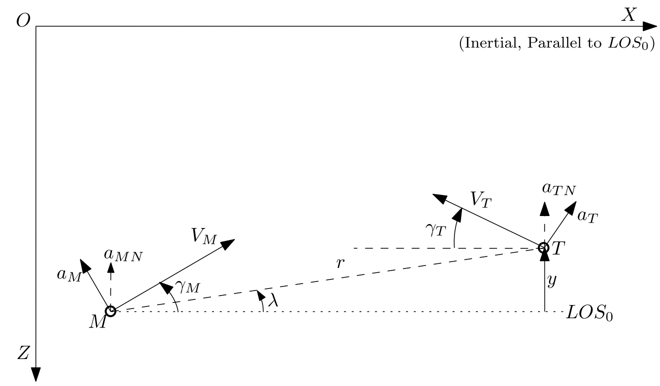

Fig. 1. Planar engagement geometry.

3. The missile and target deviations from the collision triangle are small during the end-game, consequently the relative end-game trajectory can be linearized about the nominal line of sight (LOS).
4. Constant speeds are assumed for both the missile and the target.

2.2. End-game scenario description

Fig. 1 presents a schematic view of the planar end-game geometry, where X axis is aligned with the initial LOS (LOS_0) and Z axis is perpendicular to it. The subscripts M and T denote the missile and the target, respectively. V, a, and γ denote the speed, normal acceleration, and path angle. a_{MN}, and a_{TN} are respectively the missile and target accelerations normal to LOS_0. r is the range between the adversaries and λ is the angle between the LOS and the X axis. y is the relative displacement between the target and the missile normal to the X axis.

The missile and the target accelerations normal to the initial LOS are

$$a_{MN} \approx a_M \cos(\gamma_{M0}), \quad a_{TN} \approx a_T \cos(\gamma_{T0}) \tag{1}$$

Then, the corresponding kinematic equation is

$$\ddot{y} = a_{TN} - a_{MN} \tag{2}$$

2.3. Linear equations of motion

The general set of equations can be classified into three categories: kinematics equations, dynamics equations, and servo model equations. Thus, the general state vector is given by

$$\mathbf{x} = \begin{bmatrix} \mathbf{x_K} \\ \mathbf{x_D} \\ \mathbf{x_S} \end{bmatrix} \tag{3}$$

where $\mathbf{x_K} \in \mathbb{R}^{n_K \times 1}$ denotes the kinematics states, e.g. the missile-target separation; $\mathbf{x_D} \in \mathbb{R}^{n_D \times 1}$ denotes the dynamics states, e.g the missile's angular rates; $\mathbf{x_S} \in \mathbb{R}^{n_S \times 1}$ denotes the servo model states.[1]

The target dynamics is assumed to be ideal to consider a realistic scenario where there is no information on its dynamics. In this manner, by assuming ideal target dynamics, the worst-case scenario is taken into account. Let $v \in \mathbb{R}^1$ denote the target controller, then

$$a_{TN} = v \tag{4}$$

The dynamics and servo equations of the missile are given as follows

$$\begin{bmatrix} \dot{\mathbf{x}}_\mathrm{D} \\ - \\ \dot{\mathbf{x}}_\mathrm{S} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_\mathrm{D} & \\ \hline [\mathbf{0}] & | \;\mathbf{A}_\mathrm{S} \end{bmatrix} \begin{bmatrix} \mathbf{x}_\mathrm{D} \\ - \\ \mathbf{x}_\mathrm{S} \end{bmatrix} + \begin{bmatrix} [\mathbf{0}] \\ - \\ \mathbf{B}_\mathrm{S} \end{bmatrix} \mathbf{u}_\mathrm{S} \tag{5}$$

[1] The equations of motions take into account the servo dynamics.