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Constrained trajectory planning and actuator design for electromagnetic heating systems

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ABSTRACT

A trajectory planning approach for electromagnetic heating systems is presented capable to optimize the electrical excitation and spatial configuration of the corresponding actuator (e. g. inductor or electrode). In order to accurately incorporate the electromagnetic and thermal phenomena, a PDE constrained optimization problem is formulated. The optimality conditions are derived in the function space of the original problem formulation following a *first optimize then discretize* approach to ensure that not only the system dynamics but rather the whole optimality system can be solved by FEM-based simulation software. The major advantage lies in coping with different types of electromagnetic heating systems. Numerical results illustrate the applicability and accuracy of the trajectory planning.

1. Introduction

Electromagnetic heat-up techniques are used in many technical and medical applications to heat up specific regions. Essential degrees of freedom for the trajectory planning are the electrical excitation and the position and shape of the actuator. This allows one to tailor the intensity and spatial distribution of the electromagnetic heat source to the automation goals of a heat-up process. Typical applications are heat treatment processes in steel industry (Hömberg, 2004), hyperthermia therapy in oncology (Neufeld, 2008), or crystal growth processes (Kobayashi, 1993).

A rigorous mathematical modeling of electromagnetic heating systems results in a set of coupled partial differential equations (PDEs) comprising electromagnetic and thermal phenomena. Thus, the optimization of the actuator configuration and excitation constitute a multiphysics problem offering several methodological challenges (Rund, Chudej, Kerler, Pesch, & Sternberg, 2012). Moreover, the trajectory planning problem is complicated if input and state constraints have to be taken into account (Hinze, Pinnau, Ulbrich, & Ulbrich, 2009).

The trajectory planning of electromagnetic heating systems also involves numerical challenges. For instance, the distinctly different temporal and spatial scales of the electromagnetic and thermal phenomena can only be resolved numerically using sophisticated solvers (Hömberg & Sokolowski, 2003; Rodríguez, Fernandes, & Valli, 2003). The wide range of geometrical setups including complex spatial domains is another crucial point.

PDE constrained optimization is a promising means for optimal control and shape design because of its ability to cope with infinite-dimensional problems through a universal solution approach. The related theory is a well-developing field in mathematics and benefits from the continuous development of computing capacity (Attouch, Buttazzo, & Michaille, 2014; Hinze et al., 2009; Lions, 1971). In general, the approaches *first discretize then optimize* (FDTO) and *first optimize then discretize* (FOTD) are distinguished in the literature to deal with PDE constrained optimization problems (Gunzburger, 2003; Hinze et al., 2009). The major difference between the two approaches is the function space in which the optimality conditions are derived (Hinze et al., 2009).

An FDTO approach first discretizes the optimization problem in space, and optionally also in time, to obtain a finite-dimensional approximation (Hinze & Tröltzsch, 2010; Quarteroni & Valli, 1994). This allows one to apply well-known methods and algorithms from finite-dimensional optimization to derive and solve the optimality conditions (Bryson, 1999). Nevertheless, FDTO approaches rely on discretization schemes that either limit the applicability to problems with simple geometries or call for a time-consuming implementation. Another crucial point concerns changing spatial domains through shape optimization. Thus, elaborate discretization techniques or a repetitive discretization of the original problem are required.

FOTD approaches derive the optimality conditions in the function space of the infinite-dimensional problem before discretization techniques and algorithms are applied to solve the optimality conditions

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(Hinze et al., 2009; Tröltzsch, 2010). Here, the major challenge lies in the formulation of the optimality conditions. In the case of shape optimization, the FOTD approach is complicated since the system dynamics only implicitly depends on the spatial domain of the actuator but not on explicit optimization variables.

Recent works on the trajectory planning of electromagnetic heating systems apply advanced optimization techniques. Most contributions, however, either neglect the full three-dimensional character of the problem or optimize the actuator shape and excitation sequentially. The contributions (Favennec, Labbé, & Bay, 2003; Masserey, Rappaz, Rozsnyo, & Touzani, 2004) address the optimal control of induction heating systems for two-dimensional geometries. The more general case of a three-dimensional setup is considered in Druet, Klein, Sprekels, Tröltzsch, and Yousept (2011), Tröltzsch and Yousept (2012) and Yousept (2010). The optimality conditions are derived by so-called formal FOTD approaches, cf. Tröltzsch (2010), or by analyzing a suitable control-to-state operator (Hinze et al., 2009).

A method for shape optimization is presented in Bodart, Boureau, and Touzani (2001) for a two-dimensional induction heating problem. Based on the assumption of an indefinitely thin actuator, an optimization problem is formulated that explicitly comprises the optimization variables. Evolutionary algorithms, also referred to as zero order methods, are used in Di Barba, Savini, Dughiero, and Lupi (2003) to optimize the shape of actuators. The optimization problem is solved not on the basis of optimality conditions but by testing a sequence of suitable candidates of the optimization variables. However, evolutionary algorithms are restricted to few optimization variables and rather simple geometries since many evaluations of the infinite-dimensional system dynamics are required.

An alternative approach for shape optimization relies on shape sensitivities as shown in Hömberg and Sokolowski (2003). Shape gradients are used to determine a descent direction for the boundary of the actuator. Similarly, the method presented in Kim, Byun, and Park (2009) combines shape gradients with a level set method (Sethian, 1999). To this end, the spatial domain of the actuator and its surrounding is covered by an implicit function. This allows one to optimize not only the shape but also the structure and topology of the actuator. The proper derivation of the optimality conditions, however, depends on the definition of transformation operators which requires a deep mathematical insight into the problem.

Lagrange multiplier techniques offer an elegant way to cope with state constraints (Bergounioux & Kunisch, 2002a, b; Casas, 1997; Hintermüller & Kunisch, 2010). Another well established approach are inner and outer barrier methods (Masserey et al., 2004; Schiela, 2009). There are some works that systematically take state constraints into account when optimizing the control or shape of electromagnetic heating systems (Druet et al., 2011; Tröltzsch & Yousept, 2012; Yousept, 2010). The most contributions, however, use a barrier method, cf. Masserey et al. (2004), or consider no state constraints, cf., for instance, Bodart et al. (2001), Di Barba et al. (2003), Favennec et al. (2003), Hömberg and Sokolowski (2003), Kim et al. (2009) and Masserey et al. (2004).

This article presents an FOTD approach to simultaneously optimize the excitation and position or shape of electromagnetic actuators subject to input and state constraints defined on three-dimensional spatial domains. The specification of a cost functional allows one to adapt the optimization problem to typical applications such as surface hardening or hyperthermia therapy.

Special emphasis is paid to cope with the wide variety of geometrical setups that comes along with typical applications of electromagnetic heating. To this end, the optimization framework presented in Rhein and Graichen (2016) and Rhein, Utz, and Graichen (2015a) is extended to allow the optimization of actuator shapes. A further improvement of the trajectory planning compared to Rhein and Graichen (2016) and Rhein et al. (2015a) is an augmented Lagrangian method to cope with state constraints.

The article is structured as follows. Section 2 introduces a PDE system for mathematically describing the system dynamics of electromagnetic heating systems. The formulation of a saddle-point problem and the derivation of its optimality conditions is addressed in Section 3. The numerical solution of the optimality conditions considering a close interaction of optimization algorithms and state-of-the-art FEM software is presented in Section 4. Finally, Section 5 illustrates the applicability and accuracy of the optimization-based trajectory planning approach for various electromagnetic heat-up processes.

2. Electromagnetic heating systems

The trajectory planning as presented in this contribution predicts the system dynamics of electromagnetic heating systems by means of a highly accurate PDE system. It comprises the Maxwell equations and the heat equation to account for electromagnetic and thermal phenomena (Lienhard, 2004; Stratton, 1941). The following lines introduce the system dynamics and discuss the control tasks of typical electromagnetic heating systems.

2.1. Geometrical setup and basic principle

Fig. 1 shows the geometrical setup for which the electromagnetic and thermal phenomena are mathematically described. The region of interest $\Omega := \Omega_o \cup \Omega_c \cup \Omega_a$ with spatial coordinates $x := [r, \varphi, z]^\mathsf{T}$ comprises the workpiece Ω_o, the actuator Ω_c, and the ambient air Ω_a.

Electromagnetic heating systems rely on a sinusoidal excitation of the actuator to generate an electromagnetic field penetrating the workpiece, cf. Fig. 1. This induces Joule heat losses within the workpiece and increases the temperature (Rudnev, Loveless, Cook, & Black, 2002). However, the electromagnetic field is affected by various distortion effects leading to an unevenly distributed Joule heat source. This complicates the problem of heating up the workpiece to desired temperature profiles (Rodríguez & Valli, 2010; Rudnev et al., 2002).

The field distortion effects cause an uneven Joule heat source distribution and can be characterized by the skin, end, and edge effect, as shown in Fig. 1. The skin effect describes a concentration of the electromagnetic field to the surface layer of the workpiece. The end and edge effect represent field distortion effects at end and edge layers of the workpiece, since in these areas the magnetic field lines are contracted or stretched depending on the material parameters and severity of the skin effect (Rodríguez & Valli, 2010; Rudnev et al., 2002).

The position and shape of the actuator serve as design variables to manipulate the spatial distribution of the electromagnetic heat source. An optimal actuator configuration should minimize distortion effects of the electromagnetic field or rather adapt them to the desired heat-up behavior of the workpiece. The actuator excitation serves as a further design variable and can be used to adapt the intensity of the heat source.

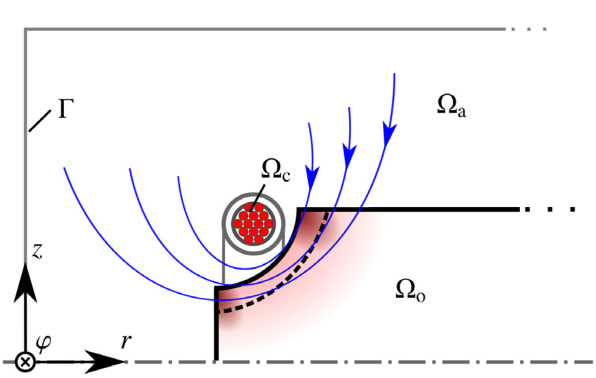

Fig. 1. Geometrical setup of an electromagnetic heating system (not to scale). The actuator Ω_c generates the magnetic field (blue) leading to Joule heating (red) in the workpiece Ω_o. The position, shape and electrical excitation of the actuator are the degrees of freedom to adapt the spatial and temporal distribution of the heat source. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)