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Constrained nonlinear filter for vehicle sideslip angle estimation with no a priori knowledge of tyre characteristics

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ABSTRACT

The vehicle sideslip angle is one of the most functional feedbacks for the actual control systems of vehicle dynamics. The measurement of the sideslip angle is expensive and unsuitable for common vehicles. Consequently, its estimation is nowadays an important task.

This paper focuses on the vehicle sideslip angle estimation adopting a constrained unscented Kalman filter (CUKF) that takes into account state constrains during the estimation process. State boundaries are useful in real-world applications to prevent unphysical results and to improve the estimator robustness. The proposed technique fully takes into account the measurement noise and nonlinearities. A vehicle model with single track has been adopted for the design of the estimator. Simulations have been carried out and comparisons with the unscented Kalman filter (UKF) are illustrated. Performance of the estimators have been checked through the application to experimental data. The results show the goodness of the CUKF, able to give an estimate fully in accordance with the measurement. Moreover, the results show that the CUKF, due to the presence of the boundaries, outperforms the UKF.

1. Introduction

In recent years considerable interest has been focusing on the estimation of the vehicle sideslip angle. Indeed, the directional behaviour and stability of the vehicle are strongly related to the sideslip angle, that is often requested in vehicle dynamics control systems (Melzi & Sabbioni, 2011; Rajamani, 2005; Russo, Strano, & Terzo, 2015; van Zanten, 2000; van Zanten, Erhardt, & Pfaff, 1996; Villagra, d'Andréa Novel, Fliess, & Mounier, 2011).

Generally, expensive instrumentation are adopted for measuring the vehicle sideslip angle. As a consequence, several methodologies for vehicle sideslip angle estimation have been proposed in literature. A sideslip angle estimation method based on a layered neural network has been proposed in Sasaki and Nishimaki (2000). In Solmaz and Baslamisli (2012) and Grip et al. (2008), nonlinear observers are developed in order to work for different adhesion characteristics. A particle filter technique is described in Cheng, Correa, and Charara (2011) where the relation between the tyre road forces and the sideslip angle has been described with a Dugoff model.

The extended Kalman filter (EKF) approach has been extensively adopted for state estimation in vehicle dynamics (Chen & Hsieh, 2001; Kim, 2009). This method could be applied to nonlinear systems by performing a local linearization. However, inevitably estimation errors occur due to the linearization of the vehicle mathematical model. In order to overcome this limit, a family of sigma-point Kalman filters has been recently proposed (Mariani & Ghisi, 2007; van der Merwe & Wan, 2003). These filters generate a population of so-called sigma-points on the basis of the current mean and covariance of the state vector, and allow them to propagate according to the actual nonlinear system dynamics. The unscented Kalman filter (UKF) (Julier & Uhlmann, 1996; Julier, Uhlmann, & Durrant-Whyte, 2000) belongs to this class of nonlinear filters, and one of the main advantages over the EKF is that the UKF does not require linearization phases and gradient computation of the state evolution equations. In Antonov, Fehn, and Kugi (2011) and Doumiati, Victorino, Charara, and Lechner (2009) , the UKF has been adopted for vehicle state estimation and the results have shown that the UKF outperforms the EKF, allowing the employment of larger sampling time.

Despite the fact that the UKF outperforms the EKF, some deficiencies still remain. One of the most important deficiencies of the UKF is that constraints on state variables cannot be taken into account and consequently the filter could fail in cases of inaccurate system modelling.

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Many approaches have been developed for UKF with constrained problems, also called constrained UKF (CUKF) (Kandepu, Foss, & Imsland, 2008; Kolas, Foss, & Schei, 2009; Mandela, Kuppuraj, Rengaswmy, & Narasimhan, 2012; Vachhani, Narasimhan, & Rengaswamy, 2006; Wu & Wang, 2014).

In this paper, a CUKF for vehicle sideslip angle estimation is presented, avoiding a local linearization of the vehicle mathematical model and taking into account the state boundaries.

Within this context, the interaction forces have been considered as states by using a random walk model approach. In this way, no reference model has to be employed to take into account tyre behaviour and, consequently, no tuning procedure is necessary for the tyres that are adopted on the vehicle. For comparison purpose, the UKF has been chosen as benchmark due to its demonstrated suitability in vehicle dynamics (Antonov et al., 2011). The results demonstrate the effectiveness of the CUKF for the estimate of the vehicle sideslip angle, due to the presence of the boundaries on the state variables that allow a better performance in terms of convergence and estimation error (Kandepu et al., 2008; Mandela et al., 2012).

The paper is organized as follows: Sections 2 and 3 focus on the UKF and the CUKF, respectively; the vehicle model is presented in Section 4; while Sections 5 and 6 illustrate simulation and experimental results, respectively.

2. Unscented Kalman filter

The UKF has been proposed by Julier, Uhlrnann, and Durrant-Whyte (1995) and further improved Julier (2002). Since UKF does not require evaluating Jacobian and Hessian matrices, and has superior accuracy compared to EKF in terms of approximating the statistics of highly nonlinear systems, it is suitable for estimating fairly complex dynamical systems.

The statistical properties of a random variable in the unscented transformation are described with sigma points. As a consequence, the statistical behaviour of the transformed random variable is obtained by applying the nonlinear transformation to the sigma points. The UKF algorithm is briefly described in the following, since it has been employed for a comparative analysis. Consider the following continuous nonlinear state space description with discrete measurements sampled at regular intervals with sampling period Δt

$$\begin{aligned} &\mathbf{x}_{k+1} = \mathbf{x}_k + \int_{k\Delta t}^{(k+1)\Delta t} \mathbf{f}\left[\mathbf{x}(\tau), \mathbf{u}_k\right] d\tau + \mathbf{w}_k \quad \mathbf{x}_k \equiv \mathbf{x}(k\Delta t) \\ &\mathbf{y}_{k+1} = \mathbf{h}(\mathbf{x}_{k+1}, \mathbf{u}_{k+1}) + \mathbf{v}_{k+1} \end{aligned} \tag{1}$$

where $\mathbf{x} \in \mathbf{R}^n$ is the n-dimensional vector of system state, $\mathbf{f}$ and $\mathbf{h}$ are nonlinear functions, $\mathbf{u}$ is the input vector, $\mathbf{w}$ is the process noise characterized by the covariance $\mathbf{Q}$, $\mathbf{y} \in \mathbf{R}^m$ is the m-dimensional vector of measurement, $\mathbf{v}$ is the Gaussian white measurement noise with covariance $\mathbf{R}$ and k is the kth time step. The main task is to estimate the system state, *i.e.*, calculate the mean as well as the covariance of system state at the $(k+1)$th step, based on the state estimation at the kth step and the measurements at the current $(k+1)$th step.

Given the filtered state estimates $\hat{\mathbf{x}}_{k|k}$, which have been obtained using all the measurements made up to time t_k, and the input $\mathbf{u}_k$, the predicted state estimates $\hat{\mathbf{x}}_{k+1|k}$ can be obtained as

$$\hat{\mathbf{x}}_{k+1|k} = \hat{\mathbf{x}}_{k\,|\,k} + \int_{k\Delta t}^{(k+1)\Delta t} \mathbf{f}\left[\mathbf{x}(\tau), \mathbf{u}_k\right] d\tau; \quad \hat{\mathbf{x}}_{k\,|\,k} \equiv \hat{\mathbf{x}}(k\Delta t) \tag{2}$$

A set of $2n+1$ sigma points $\mathbf{X}_{k\,|\,k,i}$ with associated weights $\mathbf{W}_i$ are chosen symmetrically about $\hat{\mathbf{x}}_{k|k}$ as follows:

$$\begin{aligned} &\mathbf{X}_{k\,|\,k,0} = \hat{\mathbf{x}}_{k\,|\,k}, \quad \mathbf{W}_0 = \frac{\kappa}{n+\kappa} \\ &\mathbf{X}_{k\,|\,k,i} = \hat{\mathbf{x}}_{k\,|\,k} + \left(\sqrt{(n+\kappa)\mathbf{P}_{k\,|\,k}}\right)_i, \quad \mathbf{W}_i = \frac{1}{2(n+\kappa)} \\ &\text{and} \\ &\mathbf{X}_{k\,|\,k,i+n} = \hat{\mathbf{x}}_{k\,|\,k} - \left(\sqrt{(n+\kappa)\mathbf{P}_{k\,|\,k}}\right)_i, \quad \mathbf{W}_{i+n} = \frac{1}{2(n+\kappa)} \end{aligned} \tag{3}$$

where $\left(\sqrt{\mathbf{P}_{k\,|\,k}}\right)_i$ is the ith column of the matrix square root of the error covariance matrix $\mathbf{P}_{k\,|\,k}$, $\mathbf{W}_i$ is the weight associated with the corresponding point and κ is a tuning parameter.

The set $\mathbf{X}$ and $\hat{\mathbf{x}}_{k\,|\,k}$ have the same weighted mean due the symmetric placement of the sigma points and since the weights $\mathbf{W}_i$ sum is one. Therefore, the weighted covariance matrix of the sample $\mathbf{X}$ is equal to $\mathbf{P}_{k\,|\,k}$:

$$\mathbf{P}_{k|\,k} = \sum_{i=0}^{2n} \mathbf{W}_{k,i}(\mathbf{X}_{k|\,k,i} - \hat{\mathbf{x}}_{k|\,k})(\mathbf{X}_{k|\,k,i} - \hat{\mathbf{x}}_{k|\,k})^{\mathrm{T}} \tag{4}$$

The predicted set of sigma points are obtained by applying the nonlinear state space equation to the sigma points:

$$\mathbf{X}_{k+1|k,i} = \hat{\mathbf{x}}_{k\,|\,k,i} + \int_{k\Delta t}^{(k+1)\Delta t} \mathbf{f}\left[\mathbf{X}_i(\tau), \mathbf{u}_k\right] d\tau; \quad \mathbf{X}_{k\,|\,k,i} \equiv \mathbf{X}_i(k\Delta t), \tag{5}$$

and the predicted state estimate and the error covariance matrix are

$$\begin{aligned} &\hat{\mathbf{x}}_{k+1\,|\,k} = \sum_{i=0}^{2n} \mathbf{W}_{k,i}\mathbf{X}_{k+1\,|\,k,i} \\ &\mathbf{P}_{k+1\,|\,k} = \sum_{i=0}^{2n} \mathbf{W}_{k,i}(\mathbf{X}_{k+1\,|\,k,i} - \hat{\mathbf{x}}_{k+1\,|\,k})(\mathbf{X}_{k+1\,|\,k,i} - \hat{\mathbf{x}}_{k+1\,|\,k})^{\mathrm{T}} + \mathbf{Q}_k. \end{aligned} \tag{6}$$

Propagation of the sigma points through the nonlinear measurement equation provides the predicted measurements:

$$\boldsymbol{\Upsilon}_{k+1\,|\,k,i} = \mathbf{h}(\mathbf{X}_{k+1\,|\,k,i}, \mathbf{u}_{k+1}), \quad i = 0, 1, \ldots, 2n, \tag{7}$$

and the covariance matrix of innovations and the cross covariance matrix between predicted state estimate errors and innovations are computed as

$$\begin{aligned} &\mathbf{P}_{yy,k+1\,|\,k} = \sum_{i=0}^{2n} \mathbf{W}_{k,i}(\boldsymbol{\Upsilon}_{k+1\,|\,k,i} - \hat{\mathbf{y}}_{k+1\,|\,k})(\boldsymbol{\Upsilon}_{k+1\,|\,k,i} - \hat{\mathbf{y}}_{k+1\,|\,k})^{\mathrm{T}} + \mathbf{R}_{k+1} \\ &\mathbf{P}_{xy,k+1\,|\,k} = \sum_{i=0}^{2n} \mathbf{W}_{k,i}(\mathbf{X}_{k+1\,|\,k,i} - \hat{\mathbf{x}}_{k+1\,|\,k})(\boldsymbol{\Upsilon}_{k+1\,|\,k,i} - \hat{\mathbf{y}}_{k+1\,|\,k})^{\mathrm{T}} \end{aligned} \tag{8}$$

where

$$\hat{\mathbf{y}}_{k+1\,|\,k} = \sum_{i=0}^{2n} \mathbf{W}_{k,i}\boldsymbol{\Upsilon}_{k+1\,|\,k,i}. \tag{9}$$

Finally, the updated state estimates and the error covariance matrix of updated state estimates are

$$\hat{\mathbf{x}}_{k+1\,|\,k+1} = \hat{\mathbf{x}}_{k+1\,|\,k} + \mathbf{K}_{k+1}(\mathbf{y}_{k+1} - \hat{\mathbf{y}}_{k+1\,|\,k}) \tag{10}$$

$$\mathbf{P}_{k+1\,|\,k+1} = \mathbf{P}_{k+1\,|\,k} - \mathbf{K}_{k+1}\mathbf{P}_{yy,k+1\,|\,k}\mathbf{K}_{k+1}^{\mathrm{T}} \tag{11}$$

where

$$\mathbf{K}_{k+1} = \mathbf{P}_{xy,k+1\,|\,k}(\mathbf{P}_{yy,k+1\,|\,k})^{-1} \tag{12}$$

is the Kalman gain matrix.

3. Constrained unscented Kalman filter

An improved version of the UKF is the CUKF, introduced in Section 1, that can take into account constraints of the state variables.

The CUKF approach considered in this paper consists of two main aspects (Wu & Wang, 2014): in the prediction step, sigma points that violate bound constraints are moved onto the bounds, and the relevant sigma points within the boundary are moved correspondingly in order to obtain the symmetry of the new set of sigma points; (ii) in correction step, the state updating equation is used to generate transformed sigma points, and those transformed sigma points that violate bound constraints are projected to constraints boundary only when the updated state estimate exceeds the boundary (Wu & Wang, 2014). The details of above proposals are described as follows.