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Model predictive control for offset-free reference tracking of fractional order systems

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ABSTRACT

In this paper an offset-free model predictive control scheme is presented for fractional-order systems using the Grünwald–Letnikov derivative. The infinite-history fractional-order system is approximated by a finite-dimensional state-space system and the modeling error is cast as a bounded disturbance term. Using a state observer, it is shown that the unknown disturbance at steady state can be reconstructed and modeling errors and other persistent disturbances can be attenuated. The effectiveness of the proposed controller–observer ensemble is demonstrated in the optimal administration of an anti-arrhythmic medicine with fractional-order pharmacokinetics.

1. Introduction

1.1. Background

Fractional calculus is a mathematical extension of the classic calculus of integer-order derivatives and integrals. In fractional calculus, derivatives and integrals are extended to non-integer orders which possess fascinating properties. One of the most remarkable properties of fractional-order derivatives is that they are nonlocal operators, that is, unlike their integer-order counterparts, they cannot be evaluated at a certain point solely by knowing how the function behaves in a neighborhood of this point; instead, the whole history of the function needs to be taken into account (Podlubny, 1998).

Fractional dynamics have been used to model phenomena exhibiting hereditary properties and long or infinite memory transients. Such phenomena include semi-infinite transmission lines with losses (Clarke, Achar, & Hanneken, 2004), viscoelastic polymers (Hilfer, 2000), magnetic core coils (Schäfer & Krüger, 2006), ultra capacitors (Gabano, Poinot, & Kanoun, 2015), anomalous diffusion in semi-infinite transmission bodies (Guo, Li, & Wang, 2015) and several bio-medical applications (Magin, 2010; Magin, Ortigueira, Podlubny, & Trujillo, 2011; Sopasakis, Sarimveis, Macheras, & Dokoumetzidis, 2017). Fractional systems find also several applications in physics (Hilfer, 2000). Podlubny, Petráš, Skovranek, and Terpák (2016) offer a thorough overview of the wealth of available toolboxes and software that allow the simulation and controller design for fractional-order systems. Fractional-order systems and controllers have made their appearance in industrial applications by the extension of the classical PID controller to fractional-order $PI^{\lambda}D^{\mu}$ ones (Beschi, Padula, & Visioli, 2016; Feliu-Batlle & Rivas-Perez, 2016; Monje, Vinagre, Feliu, & Chen, 2008; Roy & Roy, 2016). In Feliu-Talegon, San-Millan, and Feliu-Batlle (2016) fractional order control is applied for the attenuation of vibrations in flexible structures.

During the last few years, a number of works appeared in the literature on the development of Model Predictive Control (MPC) methodologies for fractional order systems. MPC has gained great popularity in industry and academia due to its inherent capability to take into account state and input constraints, handle complex system dynamics and be resilient to external disturbances (Rawlings & Mayne, 2009). In Boudjehem and Boudjehem (2010, 2012) and Romero, de Madrid, Mañoso, Milanés, and Vinagre (2013), MPC formulations were presented, based on simple input–output fractional order models and using integer-order approximations of the transfer function of the system. In Romero et al. (2013), the proposed fractional order MPC was demonstrated on the low-speed control of gasoline-propelled cars. In Joshi, Vyawahare, and Patil (2015), both input–output and state space fractional order models were considered as predictive models in MPC. In Domek (2011) the use of fractional order Takagi–Sugeno fuzzy models was proposed in the synthesis of fractional MPC.

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A common limitation of all aforementioned works is that although they recognize the importance of MPC in handling input and output/state constraints, they do not take explicitly into account those constraints in the proposed MPC formulations. In Rhouma and Bouani (2014) input constraints were considered in the formulation of the MPC optimization problem, however, without constraints on the state variables and without theoretical stability guarantees.

In practical applications, integer-order derivatives are often used to approximate fractional-order systems (Vinagre, Podlubny, Hernandez, & Feliu, 2000). Unfortunately these methods come without guarantees of stability and satisfaction of constraints. In particular, when these approximations are based on frequency-domain procedures, no stability guarantees can be derived and this is a severe shortcoming in safety-critical applications such as drug administration. It should be noted again that the behavior of fractional-order systems depends on the whole history of their trajectories, therefore it is very difficult to provide cogent evidence based on simulations alone without theoretical backup.

An alternative approach has been proposed by Guermah, Djennoune, and Bettayeb (2010) where fractional-order systems are modeled as infinite-dimensional state space systems leading to theoretically interesting stability conditions which, nonetheless, are not tractable and cannot be used for controller design.

In our previous work, a controller design approach based on the Grünwald–Letnikov scheme (Sopasakis, Ntouskas, & Sarimveis, 2015; Sopasakis & Sarimveis, 2017; Sopasakis et al., 2017) was proposed. A finite-dimensional approximation was introduced to arrive at a linear time-invariant system and cast the discrepancy between the real and the approximate system as an additive bounded uncertainty term. A worst-case MPC formulation was presented which leads to *asymptotically stable* behavior towards the origin in presence of state and input constraints, even when an approximate finite-history model is employed, unlike alternative approaches (Guermah et al., 2010; Romero, Tejado, Suarez, Vinagre, & Angel, 2009).

1.2. Contributions

In this paper an MPC formulation is proposed that achieves offset-free reference tracking for fractional-order systems taking into account input/state constraints. Modeling error is cast as a disturbance term with which the state-space model of the nominal dynamical system is augmented. A state observer is then used to simultaneously estimate the system state and the disturbance using a simple disturbance model. As a result, the closed-loop system can reject disturbances associated with the aforementioned finite-memory approximation, but also other modeling errors due to inexact knowledge of the system parameters, while guaranteeing constraint satisfaction.

Unlike the controller design approaches we discussed in Section 1.1, which use frequency-domain approximations (i.e., integer-order approximations of the transfer function), the approach we propose in this paper uses a time-domain approximation based on the Grünwald–Letnikov derivative.

The proposed MPC strategy is demonstrated in a case study emerging from pharmacokinetics and pharmacodynamics, where fractional-order systems are becoming increasingly popular over the last years. The work of Kytariolos, Dokoumetzidis, and Macheras (2010) introduced fractional-order dynamics in pharmacokinetics, highlighting why the classical *in-vitro–in-vivo correlations* theory fails. Certain nonlinearities, anomalous diffusion, deep tissue trapping, diffusion across fractal manifolds such as systems of capillaries, synergistic and competitive actions are cases that can hardly be modeled by integer-order systems (Dokoumetzidis & Macheras, 2008). Fractional-order pharmacokinetic dynamics can be cast as physiologically based pharmacokinetic (PBPK) or compartmental models by properly re-writing the mass balance equations using fractional-order derivatives in a way that mass balances are not violated (Dokoumetzidis, Magin, & Macheras, 2010). The controlled drug administration for drugs with fractional dynamics is a key enabler of an effective and realistic therapy and a valuable tool for the clinical practice, especially in presence of constraints (Sopasakis & Sarimveis, 2014).

This paper is organized as follows: In Section 2 we describe fractional-order dynamical systems in terms of the Grünwald–Letnikov derivative and derive control-oriented approximations of bounded error. In Section 3 we propose an MPC scheme for offset-free control using a state observer for an augmented system which is able to attenuate modeling errors. Lastly, In Section 4 we present such an offset-free MPC for the control of amiodarone administration to patients and show that the controlled system is resilient to inexact knowledge of the pharmacokinetic parameters of the patients (which are, typically, not known).

1.3. Notation

Hereafter, $\mathbb{R}$ and $\mathbb{N}$ denote the sets of real and nonnegative integers respectively. We denote by $\mathbb{N}_{[k_1,k_2]}$ the set of all integers in the closed interval $[k_1, k_2]$. The set of real n-dimensional vectors is denoted by $\mathbb{R}^n$ and the set of m-by-n matrices by $\mathbb{R}^{m\times n}$. All sets are denoted by calligraphic uppercase letters and all matrices are denoted by uppercase letters. Vectors and scalars are denoted by lowercase letters. The transpose of a matrix A is denoted by A'.

2. Fractional-order systems

2.1. Discrete-time fractional operators

In this section a fractional-order differential operator, the Grünwald–Letnikov derivative is introduced. Let $f : \mathbb{R} \to \mathbb{R}^n$ be a bounded function. Let us first introduce the *Grünwald–Letnikov difference* of f at t of order $\alpha > 0$ and step size $h > 0$, which is defined as

$$\Delta_h^\alpha f(t) = \sum_{j=0}^{\infty} (-1)^j \binom{\alpha}{j} f(t - jh) \tag{1}$$

where $\binom{\alpha}{0} = 1$ and for $j \in \mathbb{N}, j > 0$, $\binom{\alpha}{j} = \prod_{i=0}^{j-1} \frac{\alpha-i}{i+1}$. Furthermore, let us define $c_j^\alpha = (-1)^j \binom{\alpha}{j}$ and note that $|c_j^\alpha| \leq \alpha^j/j!$ for all $j \in \mathbb{N}$, therefore, the sequence c_j^α is absolutely summable and Δ_h^α is well defined. It is now clear that in order to estimate $\Delta_h^\alpha f(t)$ for non-integer orders α the whole history of $f(t)$ is needed.

The Grünwald–Letnikov operator leads to the definition of the Grünwald–Letnikov fractional derivative of order α as

$$\mathrm{D}^\alpha f(t) = \lim_{h\to 0} \frac{\Delta_h^\alpha f(t)}{h^\alpha} \tag{2}$$

provided that the limit exists. It can be verified that D^k, for $k \in \mathbb{N}$, are the ordinary integer-order derivatives with respect to t and, by convention, $\mathrm{D}^0 f(t) = f(t)$.

Using the definition above, it is easy to describe fractional-order dynamical systems with state $x : \mathbb{R} \to \mathbb{R}^n$ and input $u : \mathbb{R} \to \mathbb{R}^m$ as

$$\sum_{i=0}^{l} A_i \mathrm{D}^{\alpha_i} x(t) = \sum_{i=0}^{r} B_i \mathrm{D}^{\beta_i} u(t) \tag{3}$$

where $l, r \in \mathbb{N}$, A_i and B_i are matrices of appropriate dimensions and all powers α_i and β_i are nonnegative.

For the discretization of a fractional system, an Euler-type method is used to approximate D^α in (3), for a fixed time step size h, using $h^{-\alpha}\Delta_h^\alpha$. Using the forward operator for the derivatives of the states, and the backward operator for the input variables, the discretization of Eq. (3) becomes

$$\sum_{i=0}^{l} \bar{A}_i\, \Delta_h^{\alpha_i} x_{k+1} = \sum_{i=0}^{r} \bar{B}_i\, \Delta_h^{\beta_i} u_k. \tag{4}$$

For convenience in (4) it is: $x_k = x(kh), u_k = u(kh)$ for $k \in \mathbb{Z}$, and $\bar{A}_i = h^{-\alpha_i} A_i$, $\bar{B}_i = h^{-\beta_i} B_i$.