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Iterative Pole–Zero model updating: A combined sensitivity approach

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A B S T R A C T

A crucial step in the control of a weakly damped high precision motion system is having an accurate dynamic model of the system from actuators to sensors and to the unmeasured performance variables. A (reduced) Finite Element (FE) model may be a good candidate apart from the fact that it often does not sufficiently match with the real system especially when it comes to machine-to-machine variation. To improve the dynamic properties of the FE model toward the dynamic properties of a specific machine, an Iterative Pole–Zero (IPZ) model updating procedure is used that updates numerical poles and zeros of Frequency Response Functions (FRFs) toward measured poles and zeros, which can be extracted from the measured FRFs. It is assumed that in a practical situation, the model (physical) parameters that cause discrepancy with the real structure are unknown. Therefore, the updating parameters will be the eigenvalues of the stiffness and/or damping (sub)matrix. In this paper, an IPZ model updating is introduced which combines the sensitivity functions of both poles and zeros (with respect to the corresponding updating parameters) together with the cross sensitivity functions between poles and zeros. The procedure is verified first using simulated experiments of a pinned-sliding beam structure and then using non-collocated FRF measurement results from a cantilever beam setup.

1. Introduction

High-precision motion stages are an important part of high-tech systems such as wafer scanners and microscopes (Devasia, Eleftheriou, & Moheimani, 2007; van de Wal, van Baars, Sperling, & Bosgra, 2002; Wassink, van de Wal, Scherer, & Bosgra, 2005). Motion stages are basically made of lightly damped flexible structures resulting in flexible dynamic behavior of the system. For accurate positioning of stages, knowledge of an accurate yet low-order parametric model of the system from actuators to the sensors and to the unmeasured performance variables at the Point-Of-Interest (POI) is unavoidable. In many situations, identification techniques are used for calculation of the parametric model from actuators to the sensors. However, in other situations where the performance variable is located in a different location than the sensor location, identification techniques cannot be used. In these situations, reduced-order FE models may be used to accurately predict the performance variables. However, FE models normally do not sufficiently match with the real structure due to simplification in FE modeling or due to manufacturing tolerances.

To improve the accuracy of a FE model in terms of matching the dynamic behavior with the real structure, model updating techniques are well-known tools. In principle there are two types of model updating techniques: direct methods and iterative methods. In Arora (2011), it is shown that iterative methods generally give better matching of FRFs with experimental data and that predictions based on iterative methods are better than those based on direct methods beyond the considered frequency range. Within the iterative methods, there are two categories of model updating techniques. The first category contains modal-based techniques and is concerned with updating modal properties such as eigenfrequencies, antiresonance frequencies, and mode shapes in an attempt to reduce the residuals between numerical and measured modal quantities, e.g. see Dorosti, Fey, Heertjes, van de Wal, and Nijmeijer (2014), Jaishi and Ren (2007), Mottershead, Link, and Friswell (2011). The second category contains FRF-based techniques and attempts at reducing the residuals between numerical and measured FRFs directly, see e.g. Abrahamsson, Bartholdsson, Hallqvist, Olsson, Olsson, and Sallstrom (2014), Arora, Adhikari, and Friswell (2015), Dorosti, Heck, Fey, Heertjes, van de Wal, and Nijmeijer (2011). In Jaishi and Ren (2005), a comparative study is given on the model updating approaches using either modal or FRF residuals.

One of the key issues in model updating is how to select appropriate design parameters. In some situations, it is clear which (physical)

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model parameter values are uncertain. In other situations, e.g. for geometrically complex structures with many mechanical connections, it may be far from trivial to identify which physical parameters are causing differences between the numerical and the experimental target quantities. Even if the uncertain physical parameter is known, derivation of the sensitivity of the model to that parameter may be computationally highly demanding. Moreover, the relation between the model and the physical parameters is often lost as soon as model reduction is applied. In those situations, generic parameters may be better candidates for model updating. Generic element parameterization is for example based on allowing changes to the eigenvalues and eigenvectors of the stiffness matrices of the structural elements or substructures (Titurus, Friswell, & Starek, 2003). Performance of FE model updating techniques under selection of different classes of parameters is compared in Ahmadian, Gladwell, and Ismail (1997). It is important to mention that having an (over)determined model updating problem is often desirable, i.e. having fewer or the same number of design parameters compared to the number of measured quantities (Mottershead et al., 2011). This is because having more design parameters than residuals increases the chance of creating an ill-conditioned problem and may also lead to physically unacceptable parameter updates.

Most of the modal-based model updating techniques are concerned with updating the eigenfrequencies, see e.g. Jaishi and Ren (2007), Mottershead et al. (2011). Recently, some effort has been dedicated to include the effect of antiresonances in the model updating, see Arora (2014), D'Ambrogio and Fregolent (2003), Hanson, Waters, Thompson, Randall, and Ford (2007), Jones and Turcotte (2002), Mottershead (1998). However, all of these model updating techniques are based on updating physical design parameters. Contrary, in IPZ model updating, the eigenvalues of the stiffness and/or damping matrix of the (sub)structure are introduced as the generic parameters. This is done because errors in stiffness and/or damping modeling are more likely to occur than errors in mass modeling. The choice for these generic parameters comes with the advantages that the number of design parameters will be limited, and that the exact location of the model error does not need to be known.

IPZ model updating in general tries to reduce the pole and zero residuals between numerical and measured values, which are obtained from a few FRF measurements from the existing actuator/sensor configuration. In this paper, an IPZ model updating technique is introduced which uses a standard gradient-based technique to update generic parameters such that the poles and zeros of a reduced numerical model iteratively converge to their experimentally estimated counterparts. To do so, equations are derived to calculate increments for the pole and zero generic parameters simultaneously. This is done by using combined sensitivities. Not only the pole sensitivities w.r.t. the pole generic parameters and the zero sensitivities w.r.t. the zero generic parameters are used, but also the cross sensitivities between the poles and the zero generic parameters (and vice versa) are incorporated. Subsequently, first the stiffness (or damping) structure matrix and subsequently the substructure (or damping) stiffness matrices are updated sequentially.

In a nutshell, contributions of IPZ model updating can be summarized in the following areas. First, updating of complex-valued poles and zeros is done instead of (mostly used in the literature) real-valued resonance and antiresonance frequencies. In other words, the damping matrix is updated as well as the stiffness matrix. The majority of the existing updating procedures are dedicated to undamped structures. Second, model updating is carried out using generic parameters instead of physical parameters which is suited for geometrically complex structures where the erroneous physical parameters are hardly known. Third, model updating is performed on the reduced-order model.

The remainder of the paper is organized as follows. Section 2 is dedicated to a short recap on model reduction including residual flexibility, which will be used in IPZ model updating. In Section 3, the theoretical framework of IPZ model updating using combined sensitivities is discussed. Simulation is a powerful tool for verification of techniques, since we have access to the expected results. Therefore, in Section 4, a pinned-sliding beam structure is introduced as a case study to verify the IPZ model updating technique with combined sensitivities. The IPZ model updating technique with combined sensitivities is experimentally validated through non-collocated FRF measurement results from a cantilever beam setup in Section 5. Finally, some conclusions are drawn in Section 6.

2. Model reduction

Updating a FE model of a complex system with many Degrees-Of-Freedom (DOFs), typically in the order of 10^6, is generally computationally expensive. Moreover, from a system and control point of view we are often interested in a specific frequency range which may include rigid body modes as well as a limited number of flexible modes that have relevant contributions to the input–output behavior of the system. Therefore, the original FE model will be reduced. It is assumed that the linear dynamic behavior of a mechanical structure can be described by

$$\mathbf{M}_n\ddot{\mathbf{q}}_n + \mathbf{B}_n\dot{\mathbf{q}}_n + \mathbf{K}_n\mathbf{q}_n = \mathbf{f}_n, \tag{1}$$

with $\mathbf{q}_n \in \mathbb{R}^{n\times 1}$ the vector with n DOFs, $\mathbf{f}_n \in \mathbb{R}^{n\times 1}$ the external load vector, $\mathbf{M}_n = \mathbf{M}_n^T \succ 0$ the positive-definite mass matrix, $\mathbf{B}_n = \mathbf{B}_n^T \geq 0$ the positive semi-definite damping matrix, $\mathbf{K}_n = \mathbf{K}_n^T \geq 0$ the positive semi-definite stiffness matrix, and $\mathbf{M}_n, \mathbf{B}_n, \mathbf{K}_n \in \mathbb{R}^{n\times n}$. Using a model reduction technique based on eigenmodes and residual flexibility modes, the following reduced-order dynamic equation is derived

$$\mathbf{M}_r\ddot{\mathbf{q}}_r + \mathbf{B}_r\dot{\mathbf{q}}_r + \mathbf{K}_r\mathbf{q}_\mathbf{r} = \mathbf{f}_r, \tag{2}$$

with the reduced order mass matrix $\mathbf{M}_r = \mathbf{M}_r^T = {\mathbf{\Phi}_s^{-1}}^T \mathbf{\Phi}_r^T \mathbf{M}_n \mathbf{\Phi}_r \mathbf{\Phi}_s^{-1} \succ 0$, damping matrix $\mathbf{B}_r = \mathbf{B}_r^T = {\mathbf{\Phi}_s^{-1}}^T \mathbf{\Phi}_r^T \mathbf{B}_n \mathbf{\Phi}_r \mathbf{\Phi}_s^{-1} \geq 0$, and stiffness matrix $\mathbf{K}_r = \mathbf{K}_r^T = {\mathbf{\Phi}_s^{-1}}^T \mathbf{\Phi}_r^T \mathbf{K}_n \mathbf{\Phi}_r \mathbf{\Phi}_s^{-1} \geq 0$, $\mathbf{M}_r, \mathbf{B}_r, \mathbf{K}_r \in \mathbb{R}^{r\times r}$, and $\mathbf{f}_r = {\mathbf{\Phi}_s^{-1}}^T \mathbf{\Phi}_r^T \mathbf{f}_n \in \mathbb{R}^{r\times 1}$ the reduced external load column. $\mathbf{\Phi}_r \in \mathbb{R}^{n\times r}$ is a subset of the mode shape matrix, which column-wise consists of (a) rigid-body modes (if present), (b) a selected number of low-frequency modes, and (c) residual flexibility modes defined for externally loaded DOFs. Row-wise $\mathbf{\Phi}_r$ consists of all DOFs. The square matrix $\mathbf{\Phi}_s \in \mathbb{R}^{r\times r}$ is a subset of $\mathbf{\Phi}_r$, which column-wise consists of the same modes as in $\mathbf{\Phi}_r$, but row-wise consists of only desired physical DOFs including actuator, sensor, and unmeasured performance variable DOFs. This has been explained in more detail in Dorosti et al. (2014).

3. Iterative pole–zero model updating using combined sensitivities

Imagine that in a motion system, an accurate prediction of a performance variable (for a location different than the sensor location) is needed. Assume that an FRF measurement from an actuator to a sensor is available. Also assume that the FE model of the system is available but the generated numerical FRF between the actuator and the sensor shows discrepancy with the measured FRF. A solution to this problem is to first reduce the FE model of the system, and then update the reduced model using poles and zeros derived from the measured FRF. Subsequently, the updated reduced FE model can be used for accurate prediction of the performance variable.

Using the reduced-order dynamic equation in (2), the FRF corresponding to a sensor at DOF i and an actuator at DOF j can be described as

$$\mathbf{G}_{ij}(\omega) = \frac{\det(-\omega^2\mathbf{M}_s + j\omega\mathbf{B}_s + \mathbf{K}_s)}{\det(-\omega^2\mathbf{M}_r + j\omega\mathbf{B}_r + \mathbf{K}_r)}, \tag{3}$$

where $\mathbf{M}_s, \mathbf{B}_s, \mathbf{K}_s$ are the so-called substructure matrices which are constructed from the reduced-order matrices $\mathbf{M}_r, \mathbf{B}_r, \mathbf{K}_r$ respectively, by eliminating the ith column and the jth row corresponding to the sensor and actuator DOFs (Mottershead, 1998). Note that if $i \neq j$, the substructure matrices will generally be non-symmetric. Now assume that m_p experimental poles ($\lambda_{p,e} \in \mathbb{C}^{m_p\times 1}$) and m_z experimental zeros