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Design, Analysis and Performance Evaluation of Fractional Order Proportional Integral for Three Interacting Tank Process in Frequency Domain considered as Third Order System

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Abstract: Conventional Integer order Proportional Integral Derivative (IOPID) are the workhorse for the control of almost 90% of the Industrial processes due to its structural simplicity. With the application of fractional calculus in the field of control engineering, Fractional Order (FO) PID controllers are gaining popularity since it requires a slight modification of the integer order PID controller. Tuning of the Integer Order PID controller parameters are by well known techniques like Ziegler Nichols method, Cohen Coon Method, etc mostly time domain based and few frequency techniques are also available. In the present research work, an attempt has been made to tune the FOPI controller using Frequency domain specifications. Frequency domain specifications considered for the design includes the Phase Margin specification, Gain crossover frequency specification and robustness to gain variations. The proposed control schemes are applied to three interacting tank process represented as third order system and verified under servo, regulatory, servo-regulatory response and robustness conditions. Comparison of the time domain performance indices are performed to guarantee the superiority of the proposed scheme.

Keywords: Fractional Order PI controller, Three Interacting Tank Process, Robustness to gain variations

1. INTRODUCTION

The control of liquid level in tanks and flow between tanks is a basic problem in the process industries. In many such cases the liquids are stored in one tank then sent to another tanks in a controlled manner. Often the tanks are so coupled together that the levels interact and this must be also controlled. It is well known fact that level and flow control in tanks are the heart of all chemical engineering systems. Thus the control in optimal way of these parameters in process industries also proves to be very beneficent from economic point of view. Petrochemical industries, pharmaceutical industries, food and beverages industries involves interacting tanks in series for their operations. Interacting tanks in series are higher order processes and mildly nonlinear in nature. Fractional calculus, the non-integer order involves arbitrary real or complex order integrals and derivatives. In recent past, fractional calculus has found widespread applications in the field of science and technology [1-5]. At present the second case IO (Integer order) plant with FO (fractional order) controller is realized for the three tank interacting system. Intuitively, with non-integer order controllers for integer order plants, there are more flexibility in adjusting the gain and phase characteristics than using IO controllers. These flexibilities make FO control a powerful tool in designing control system in both time and frequency domain. The $PI^{\lambda}D^{\square}$ controller is the extension of conventional Integer order PID controller involving two more parameters μ and λ.

Two special cases of $PI^{\lambda}D^{\square}$ controller are PI^{λ} controller [6] and $PD^{\square}$ controller [7,8]. Parameter tuning are by either analytical method, which involves calculation of the parameters considering frequency domain specifications, such as phase margin, gain cross over frequency and robustness to the gain variation [9] or graphical method which involves plotting the stabilizing boundary curves in the parameter space of the controller [10]. Fractional MIGO based tuning rule for FOPI controller which used reduced order models of higher order process to take a First Order Plus Dead Time (FOPDT) form only, which is not sufficient to describe the complex dynamic behaviour of the real world problem [11]. From specified phase margin ($\square_m$), gain cross-over frequency (ω_{gc}) and robustness criteria, a tuning methodology for FOPI/FOPD controllers for controlling integer order systems have been discussed[12, 13]. A set of tuning rules is presented [14] for integer-order PID and fractional-order PID controllers for FOPDT model for minimum integral absolute error with minimum sensitivity constraint. Two schemes of fractional order proportional integral controllers for a class of fractional order systems [15] is discussed. A method to design classical PID controllers (with proper derivative action) for a class of fractional order plants with time delays is developed [16]. To achieve the desired specifications of a gain and phase margins for plants with time-delay that stabilized with FO-PID controller a lead compensator is designed [17].

Section 2 discusses about the information regarding three interacting tank process, mathematical modeling, state space formulation and computation of transfer function. Section 3 elaborates the design of FOPI controller considering frequency domain specification. Section 4 deals with analysis of the proposed FOPI controller for three interacting tank process considered as third order system followed by the conclusion.

2. Three Interacting Tank Process Description

The hydraulic system considered consists of three identical cylindrical tanks with equal cross-sectional area (A). These three tanks are connected by two cylindrical pipes of the same cross sectional area (α). The process liquid is pumped to the first interacting tank from the sump by pump-1 through the control valve-1 and the input flow to the first

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interacting tank is F_{in1}. The levels in the three interacting tanks are measured using differential pressure transmitter. The three tanks are interconnected with manual valves. The objective is to control the level in the third tank by varying the flow F_{in1} of the first tank. The schematic diagram for a three interacting system is shown in Figure1.

Process Parameters

$A_1 = A_2 = A_3 = 615.75 cm^2$; $\alpha_1 = \alpha_2 = \alpha_3 = 5.0671\ cm^2$;
$\beta_{12} = 0.9$, $\beta_{23} = 0.8$, $\beta_3 = 0.3$, $K_1 = K_2 = 75\ cm^3/vs$

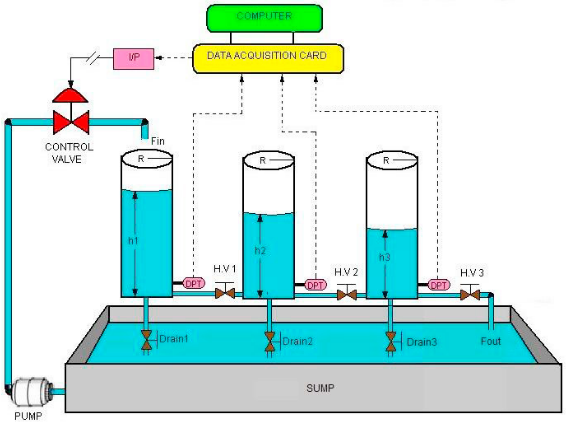

Figure 1: Three Tank Interacting System

The transfer function between F_{in1} and h_3 is considered in the present work which is a third order single input single output system. To obtain the interacting third order process, the inflow to tank 2 is shut down and input to the process is considered to be the inflow to tank 1 and the output from the process is the height of the third tank. Table 1 shows the transfer function obtained for the different regions from the process data.

Table 1. Third order transfer function relating F_{in1} and h_3

Region	*Third Order Transfer Function for the three interacting tank process*
I	$\frac{0.012}{s^3+1.295s^2+0.3237s+0.0028}$
II	$\frac{0.003}{s^3+0.65s^2+0.0822s+0.0004}$
III	$\frac{0.0012}{s^3+0.4063s^2+0.0323s+0.0001}$

3. Design of FOPI and IOPID controller considering frequency domain specification

It is obvious that the Fractional Order PI (FOPI) controller is capable of meeting the three frequency domain specifications, whereas Integer Order PI (IOPI) controller can meet only two specification at a time. The reason behind this is that FOPI controllers gives the freedom of having three controller parameters K_p, K_i and λ being in PI domain, but in IOPI controller there are only two controller parameters K_p and K_i. Therefore in integer order one has to go for PID controller to meet the given design specifications in frequency domain. So the Integer Order PID (IOPID) controller has been designed meeting the same criteria as FOPI controller is satisfying. An attempt has been made to do a comparative study between the two controllers.

3.1. Design of Fractional order PI (FOPI) for three interacting tank process considered as Third Order System

For the three tank interacting process, considering input flow to the first tank as input and output as height of the third tank, the transfer function in general is given by

$$P(s)=\frac{K}{d_1 s^3+d_2 s^2+d_3 s+d_4} \quad (1)$$

where K = gain of the process and d_1, d_2, d_3, d_4 = coefficients of the denominator polynomial.

Putting s=jω, $$P(j\omega)=\frac{K}{d_1(j\omega)^3+d_2(j\omega)^2+d_3(j\omega)+d_4} \quad (2)$$

Therefore the magnitude of the above $P(j\omega)$, will be given by

$$|P(j\omega)|=\frac{K}{\sqrt{(d_4-d_2\omega^2)^2+\left(d_3\omega-d_1\omega^3\right)^2}} \quad (3)$$

And the phase angle is given by:

$$Arg(P(j\omega))=-\tan^{-1}\left(\frac{d_3\omega-d_1\omega^3}{d_4-d_2\omega^2}\right) \quad (4)$$

The transfer function of the fractional order PI controller is given by

$$C(s)=K_p+\frac{K_pK_i}{s^\lambda}=K_p\left(1+\frac{K_i}{s^\lambda}\right) \quad (5)$$

Putting s=jω, in the above equation :

$$C(j\omega)=K_p\left(1+K_i(j\omega)^{-\lambda}\right) \quad (6)$$

Since, it is known that:

$$j^{-\lambda}=\cos\frac{\lambda\pi}{2}-j\sin\frac{\lambda\pi}{2} \quad (7)$$

Substituting this value j^{μ} in eqn(6), following is obtained:

$$C(j\omega)=K_p(1+K_i\omega^{-\lambda}(j)^{-\lambda})$$
$$=K_p\left(1+K_i\omega^{-\lambda}\left(\cos\frac{\lambda\pi}{2}-j\sin\frac{\lambda\pi}{2}\right)\right) \quad (8)$$

Thus the magnitude of $C(j\omega)$ is given by:

$$|C(j\omega)|=K_p\sqrt{1+K_i^2\omega^{-2\lambda}+2K_i\omega^{-\lambda}\cos\frac{\lambda\pi}{2}} \quad (9)$$

And the phase angle of $C(j\omega)$ is given by:

$$Arg(C(j\omega))=\tan^{-1}\left(\frac{-\not{K_p}K_i\omega^{-\lambda}\sin\frac{\lambda\pi}{2}}{\not{K_p}+\not{K_p}K_i\omega^{-\lambda}\cos\frac{\lambda\pi}{2}}\right) \quad (10)$$

Since in Fractional Order PI controller there are three controller parameters, so three following specification is taken into account:-

1) Phase Margin Specification

$$Arg[G(j\omega_{gc})]=Arg[C(j\omega_{gc})P(j\omega_{gc})]=-\pi+\phi_m \quad (11)$$

2) Gain Crossover Frequency Specification

$$|G(j\omega_{gc})|=|C(j\omega_{gc})P(j\omega_{gc})|=1 \quad (12)$$