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Optimal Control of Droop Controlled Inverters in Islanded Microgrids

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Abstract: For an islanded microgrid modeled by a Kuramoto oscillators nonlinear model, we design the distributed optimal controller using the maximum principle optimization theory. We first quantify synchrony in terms of phases and droop coefficients at the inverters in the microgrid and then we maximize it. We prove that the solution of the distributed optimal control problem exists and we find it. We evaluate performance in a simulation case.

Keywords: Optimal control, islanded microgrids, Kuramoto model, renewable energy.

1. INTRODUCTION

A microgrid is an interconnected low voltage group of devices consisting of distributed energy resources (DERs) and loads. It is typically seen by the main grid as a single controllable entity and it connects or disconnects to the main grid on previously defined events and therefore works in grid connected or islanded mode respectively (Hatziargyriou et al., 2007). DERs can be either AC resources such as wind turbines or DC resources such as solar panels and, for both cases, AC/AC or DC/AC voltage source inverters are needed to ensure network synchronization. A microgrid may consist of a wind turbine, solar energy resource, storage device and loads connected in a logical bus (or ring) as shown in Figure 1. Microgrids facilitate distributed generation and high penetration of renewable energy sources and hence increase power quality and reliability of electric supply (Barker and Herman, 2002). Fault events within the connection with the main grid could lead to an islanded mode of operation (Katiraei et al., 2005).

Frequency control is needed in both grid connected and island modes. Control strategies for island modes are discussed in (Peas Lopes et al., 2006) where it was shown that the forced islanding of the microgrid can be performed safely under several different power importing and exporting conditions. They also showed that management of storage devices are essential to implement successful control strategies. In this paper we take advantage of the existence of storage devices to provide a desired feasible control flexibility to respond to renewable energy sources that do not have a well predicted power generation behavior. In a grid connected mode, the microgrid - depending on the amount of power generated and consumed - acts as either a load when the power consumption within the microgrid exceeds the supply, or as a generator when the supply exceeds the consumption. The latter case is called power penetration where the grid injects power to the main grid (Mathiesen et al., 2011). Although this penetration reduces the overall amount of power needed to be supplied by the main grid, the fluctuating and intermittent nature of this renewable generation causes variations of power flow that can significantly affect the operation of the electrical grid and causes frequency instabilities (Kroposki et al., 2008). Wind generation for instance is a growing renewable energy resource but a known challenge is to effectively integrate a significant amount of wind power into the power network (Georgilakis, 2008).

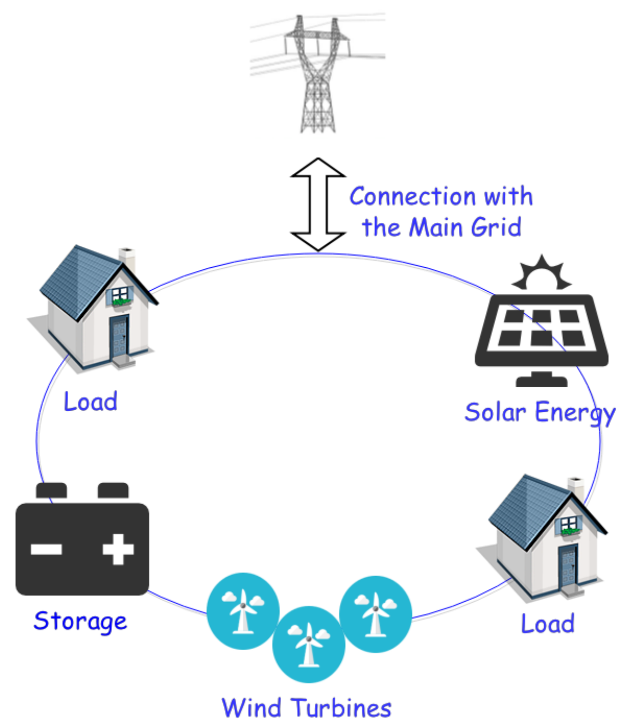

Fig. 1. Microgrid basic elements

Figures (2) and (3) show a 24 hours simulation of power consumption and supply for two households that use solar energy resources.The black curve is the amount of power supplied by the main grid, the blue curve is the household consumption while the red curve is the generated power from the solar energy resources. The simulation starts at 12:00 am at night where there is no solar energy generation so the power consumption equals exactly the power supplied by the grid. In the morning, the solar

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energy generator starts generating power and therefore the main grid supply decreases. The maximum solar energy generation occurs in the afternoon time where the grid supply becomes negative, which means that this household is now injecting power into the main grid and therefore isi seen as a generator.

So the problem of interest is how to use the droop values at the inverters in order to achieve frequency stability around the nominal value. In this paper we design the optimal controller for the islanded mode while the grid connected mode will be analyzed in future work. Many control strategies have been discussed in the literature, but they assume either linear models or linearized ones. In (Lasseter and Piagi, 2000) and (Chandorkar et al., 1993), a control scheme based on droop concepts to operate inverters feeding a standalone ac system is presented. Some droop control methods are proposed in (Li and Li, 2011), (Kim et al., 2011), and (Rokrok and Golshan, 2010). Here we develop an optimal control algorithm for the nonlinear model formulation.

This paper is organized as follows: In section 2 the microgrid model based on Kuramoto model is discussed. In section 2 we formulate the control problem and prove existence of solution. In section 4 we solve the optimization problem and show simulation results.

2. MICROGRID MODEL

In a microgrid like the one shown in figure (5) that includes N inverters, the electrical active power injected into the network at the ith inverter is given by (Kundur, 1994)

$$P_{e,i} = \sum_{j=1}^{N} E_i E_j [B_{ij} \sin(\delta_i - \delta_j) + G_{ij}(\delta_i - \delta_j)], \quad (1)$$

where E_i, E_j are the nodal voltage magnitudes at inverters i and j respectively, δ_i, δ_j are the nodal voltage phases at inverters i and j respectively, B_{ij}, G_{ij} are the real and imaginary parts of $y_{ij} = B_{ij} + jG_{ij}$ respectively, where y_{ij} is the ijth entry in the nodal admittance matrix Y.

For a pure inductive admittance matrix Y, (1) becomes:

$$P_{e,i} = \sum_{j=1}^{N} E_i E_j B_{ij} \sin(\delta_i - \delta_j) \quad (2)$$

In frequency droop control, the power demand changes the frequency ω_i at inverter i by

$$\omega_i = \omega^* - (d_i P_{e,i} - P_i^*), \quad (3)$$

where ω^* is the nominal frequency, P_i^* is the nominal active power injection at inverter i and d_i is the ith droop coefficient. The frequency droop controller (3) can be written as

$$\dot{\delta}_i = P_i^* - d_i P_{e,i}, \quad (4)$$

where $\dot{\delta}_i = \omega_i - \omega^*$ is the frequency deviation from the nominal frequency ω^* at inverter i (Simpson-Porco et al., 2013).
Substituting (2) in (4) gives the dynamics:

$$\dot{\delta}_i = P_i^* - d_i \sum_{j=1}^{N} E_i E_j B_{ij} \sin(\delta_i - \delta_j), \quad (5)$$

$$\delta_i(0) = \delta_i^0,$$

It has been shown by Simpson-Porco et al. (2013) that for a microgrid model whose elements are connected in parallel as shown in figure (5) and described by (5) is equivalent to a network of n Kuramoto phase coupled oscillators model given by (Kuramoto, 1975):

$$\dot{\delta}_i = \Omega_i - d_i \sum_{j=1}^{N} a_{ij} \sin(\delta_i - \delta_j), \quad (6)$$

where $\delta_i \in S^1$ (the unit circle) is the phase of oscillator i, Ω_i is the natural frequency, a_{ij} is the coupling strength between oscillators i and j, and d_i is a the ith oscillator coefficient. A Kuramoto oscillator network is shown in figure (4).

In the next section we show the existence of the optimal control problem.

3. THE OPTIMAL CONTROL PROBLEM FORMULATION AND EXISTENCE OF SOLUTION

Let $r(t) = \left| \frac{1}{N} \sum_{i=1}^{N} e^{j\delta_i(t)} \right|$ quantify the synchrony in the network such that $r(t) = 1$ refers to a perfectly synchronized network while $r(t) = 0$ means there is no synchronization (Strogatz, 2000).
So the optimization problem is:

$$\sup_{D \in U} J(D) = \int_0^T r^2(\delta, D) dt \quad (7)$$
$$= \int_0^T \frac{1}{N^2} \left[\sum_{j=1}^{N} \sin(\delta_j)^2 + \sum_{j=1}^{N} \cos(\delta_j)^2 \right] dt,$$

subject to the constraints:
$\dot{\delta}_i = P_i^* - d_i \sum_{j=1}^{N} E_i E_j B_{ij} \sin(\delta_i - \delta_j),$
$\delta_i(0) = \delta_i^0,$

where $\delta = [\delta_1, \cdots, \delta_N]^T$ is the phase vector at nodes $1, \cdots, N$ and is also called the states vector, $D = [d_1, \cdots, d_N]^T$ is the vector of droop coefficients at nodes $1, \cdots, N$ and is also called the controls vector. A practical constraint for D is that it should be bounded, i.e. $\|D\| \le C$ for some constant C where $\|\cdot\|$ is the Euclidean norm, so U in (7) is the set of feasible controls defined as:

$$U := \{D \in R^N : \|D\| \le C\} \quad (8)$$

Theorem 1. A solution exists for the optimization problem (7).

Proof. Let D^* denotes the optimal control vector when they exist and δ^* denotes the corresponding optimal states vector. Choose a sequence $\{D^n\}$ in U such that

$$J(D^n) = \max_{D \in U} J(D) \quad (9)$$