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Model Validation Criteria for System Identification in Time Domain

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Abstract: The fit ratio is one of the most commonly used criteria to evaluate a result of system identification in the time domain. This criterion is given by the root mean squared error (RMSE) divided by the standard deviation of the measured signal. However, the fit ratio has some problems. For example, it can take negative values because it is not normalized, and it is easy to obtain a better value for a low-amplitude signal than for a high-amplitude signal. In this paper, we introduce some normalized criteria from the field of physical geography and consider criteria that resolve these problems. We evaluated these criteria through two case studies. We found that the correlation coefficient was effective to evaluate the phase, and a criterion obtained from the triangle inequality was effective for evaluating the gain and phase.

Keywords: System identification, Validation, Time domain, Error criteria, Correlation coefficients

1. INTRODUCTION

Recently, mathematical models have been used for the design of control systems even in the field of industrial applications. The mathematical models are often constructed by physical modeling or system identification. In this paper, we focus on system identification. Once the models are obtained, it is necessary to validate the some aspects of them. This step is called model validation. For example, Ljung (1998) noted that

- simulation in the time domain,
- comparison of some methods in the frequency domain, and
- pole and zero cancellation in the s (or z) domain

are often checked. Also, independence between the residuals and the past input is usually tested using the sample covariance. If the model is constructed by physical modeling, the feasibility of the physical parameters is often checked and is used to evaluate.

Simulation in the time domain is the most useful and intuitive validation method for evaluating the goodness of the model. Mean squared error (MSE) or root mean squared error (RMSE) are often used for the comparison criteria. Because these criteria are based on a physical quantity, their orders can be significantly different, and they do not seem to be suitable for discussion in general terms about comparison of two waveforms.

Therefore, a criterion based on a dimensionless quantity that does not depend on physical quantity appears to be required for simulation. Some criteria based on dimensionless quantities (called dimensionless criteria in this paper) have been proposed in a field of physical geography (especially hydrology and climatology) since the 1970s. First of all, Nash and Sutcliffe (1970), and Garrick et al. (1978) proposed the coefficient of efficiency E to evaluate the rainfall runoff model. The coefficient of efficiency is sometimes called the Nash-Sutcliffe model efficiency coefficient (NSE). This criterion is given by the MSE divided by the variance of the observed values, and it takes a maximum value when the predicted values are identical to the observed values. However, it takes negative values when the MSE is greater than the variance of the measured values. Therefore, Willmott (1981) and Willmott et al. (1985) proposed the index of agreement, which normalizes the MSE to a velue from 0 to 1.

In the field of system identification, the fit ratio (sometimes called the NRMSE fitness value), which is the RMSE divided by the standard deviation of the measured values, is often used for evaluation. This criterion is implemented in System Identification Toolbox for MATLAB® (Mathworks.co.jp, 2015). However, similarly to the coefficient of efficiency, this criterion takes values from $-\infty$ to 100, making it unsuitable for use as an intuitive criterion for evaluation.

In this paper, we introduce some normalized criteria from the field of physical geography, and consider criteria that resolve the fit ratio's problem. Then we evaluate the criteria using some case studies.

2. UNNORMALIZED DIMENSIONLESS CRITERIA

We introduce two unnormalized criteria, that is, the coefficient of efficiency and the fit ratio. Then, we show the problems of these criteria.

2.1 Coefficient of efficiency

The coefficient of efficiency (hereinafter denoted by E) is defined by

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$$E = 1 - \frac{\sum_{k=1}^{N} (y(k) - \hat{y}(k))^2}{\sum_{k=1}^{N} (y(k) - \overline{y})^2}, \tag{1}$$

where $y(k)$ and $\hat{y}(k)$ are the measured output and the simulated output at time k, respectively, $\overline{y}$ is the average value of y, and N is the number of samples. E takes a value of 1 when the MSEs of y and $\hat{y}$ are equal to 0, and E takes a negative value when the MSE is greater than the variance of the measured output. Furthermore, E takes a value of $-\infty$ when the simulated output diverges. In this way, E does not intuitively appear to be associated with the similarity between y and $\hat{y}$.

However, E was recommended as a criterion for evaluating the rainfall runoff model by the ASCE Task Committee (1993) and Legates and McCabe (1999) because it has a meaningful interpretation. One interpretation is that if $E \leq 0$, the mean measured value is a better predictor than the simulated value. Additionally, Moriasi et al. (2007) claimed that if $E > 0$, the model can be viewed as having acceptable performance. Thus, we can conclude that

(1) $E \leq 0$: model is not acceptable,
(2) $0 < E \leq 1$: model is acceptable.

In this paper, we call this Moriasi's rule.

2.2 Fit ratio

The fit ratio (hereinafter denoted by FIT) is defined by

$$\mathrm{FIT} = \left\{ 1 - \frac{\sqrt{\sum_{k=1}^{N} (y(k) - \hat{y}(k))^2}}{\sqrt{\sum_{k=1}^{N} (y(k) - \overline{y})^2}} \right\} \times 100\ \% \ . \tag{2}$$

When $y(k)$ is identical to $\hat{y}(k)$ for all $k = 1, ..., N$, FIT becomes 100. Since in the second term the RMSE is normalized by the standard deviation of y, FIT can take negative values when the RMSE is greater than the standard deviation of y, similarly to E, i.e.,

$$\sqrt{\sum_{k=1}^{N} (y(k) - \hat{y}(k))^2} > \sqrt{\sum_{k=1}^{N} (y(k) - \overline{y})^2}.$$

In particular, if the RMSE tends to ∞, FIT tends to $-\infty$. This property of FIT appears to make it unsuitable for the evaluation of system identification.

2.3 Problems of these criteria

In this section we show through an example that E, FIT, and Moriasi's rule are not appropriate for evaluating the result of system identification. As an illustrative example, let us consider the Linear Time-Invariant (LTI) system:

$$y(k) = G(q)u(k), \tag{3}$$

where $u(k)$ and $y(k)$ are the input and output at time k, respectively, $G(q)$ denotes a discrete time transfer function, and q denotes a shift operator. As the identification input

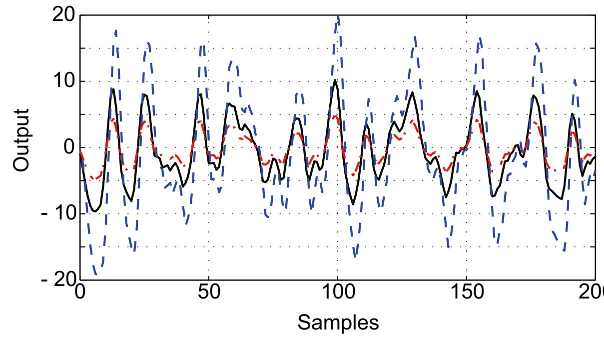

Fig. 1. Waveforms in the example: gain shift case (black solid line: $y(k)$, Blue dashed line: $\hat{y}_+(k)$, Red dashed line: $\hat{y}_-(k)$)

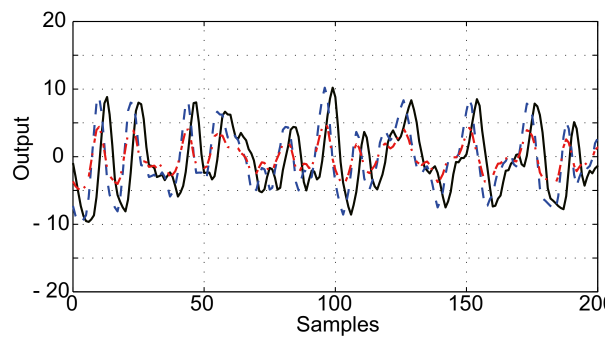

Fig. 2. Waveforms in the example: phase shift case (black solid line: $y(k)$, blue dashed line: $y_p(k)$, Red dashed line: $y_{p-}(k)$)

u, two periods of a pseudo-random binary signal (PRBS) with a period of 1023 are used.

We assume that true system is given by

$$G(q) = \frac{q^{-1} + 0.5q^{-2}}{1 - 1.5q^{-1} + 0.7q^{-2}}.$$

Now we assume that following four models are estimated:

$$\hat{G}_+(q) = 2.01G(q), \quad \hat{G}_-(q) = 0.49G(q),$$
$$\hat{G}_p(q) = G(q)q^{-3}, \quad \hat{G}_{p-}(q) = 0.49G(q)q^{-3},$$

where the first two models $\hat{G}_+$ and $\hat{G}_-$ are the same except for their gains, and the latter two models $\hat{G}_p$ and $\hat{G}_{p-}$ are the same except for their gains. We apply u to the models, and we obtain the following four simulated outputs:

$$\hat{y}_+(k) = \hat{G}_+(q)u(k), \quad \hat{y}_-(k) = \hat{G}_-(q)u(k),$$
$$\hat{y}_p(k) = \hat{G}_p(q)u(k), \quad \hat{y}_{p-}(k) = \hat{G}_{p-}(q)u(k).$$

Moreover, if $\hat{G} = 0$, $\hat{y}$ takes a value of 0 all the time. We call this output "zero output". We calculate E and FIT for four simulated outputs and the zero output.

Figure 1 shows the true output y (black solid line) and the simulated outputs $\hat{y}_+$ and $\hat{y}_-$ (blue and red dashed lines, respectively) for 0 to 200 samples. In the same way, Fig.2 shows the true output y and the simulated outputs $\hat{y}_p$ and $\hat{y}_{p-}$ for 0 to 200 samples. Table 1 shows the values of E and FIT and the acceptance results according to Moriasi's rule. E and FIT take negative values for $\hat{y}_+$ having about twice the amplitude of y, but they take positive values for $\hat{y}_-$ having about half the amplitude of y. Similarly, they take negative values for $\hat{y}_p$ having three taps delay and