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A Least Squares Method for Identification of Feedback Cascade Systems ⋆

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Abstract: The problem of identification of systems in dynamic networks is considered. Although the prediction error method (PEM) can be applied to the overall system, the non-standard model structure requires solving a non-convex optimization problem. Alternative methods have been proposed, such as instrumental variables and indirect PEM. In this paper, we first consider acyclic cascade systems, and argue that these methods have different ranges of applicability. Then, for a network with feedback connection, we propose an approach to deal with the fact that indirect PEM yields a non-convex problem in that case. A numerical simulation may indicate that this approach is competitive with other existing methods.

Keywords: System identification, Least-squares algorithm, Feedforward networks, Feedback loops, Closed-loop identification

1. INTRODUCTION

Due to the rising complexity of systems encountered in engineering problems, identification of systems that are embedded in a dynamic network has become an increasingly relevant problem. Thus, several contributions have recently been provided in this area, e.g., Dankers et al. (2014), Everitt et al. (2014), Van den Hof et al. (2013), Dankers et al. (2013), Everitt et al. (2013), Van den Hof et al. (2012), Hägg et al. (2011), Wahlberg and Sandberg (2008), Wahlberg et al. (2008).

A common particular case of such networks is the identification of acyclic cascade structures, e.g., the system in Fig. 1. It contains one external input, $u(t)$, and two outputs, $y_1(t)$ and $y_2(t)$, with measurement noises $e_1(t)$ and $e_2(t)$, respectively, which, for the purpose of this paper, are Gaussian, white, and uncorrelated to each other, with variances λ_1 and λ_2. A general discussion on identification and variance analysis of this type of cascade systems is taken in Wahlberg et al. (2008).

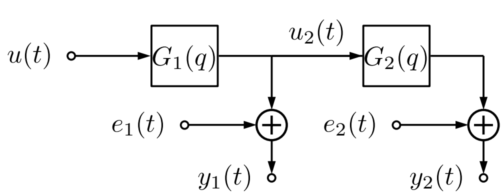

Fig. 1. Cascade system with two transfer functions.

The goal of system identification is to estimate the transfer functions $G_1(q)$ and $G_2(q)$, where q is the forward-shift operator. Mathematically, the system in Fig. 1 can be described by

$$y_1(t) = G_1(q)u(t) + e_1(t) \tag{1a}$$
$$y_2(t) = G_2(q)G_1(q)u(t) + e_2(t). \tag{1b}$$

First, notice that the transfer function $G_1(q)$ can be estimated with (1a) from the signals $u(t)$ and $y_1(t)$, using standard system identification techniques. Likewise, the product $G_2(q)G_1(q) =: G_{21}(q)$ can be estimated in a similar fashion from (1b), using $u(t)$ and $y_2(t)$ as data. However, the input to the transfer function $G_2(q)$, indicated in Fig. 1 as $u_2(t)$, is not known. Therefore, $G_2(q)$ cannot be estimated directly using a similar approach. A possible strategy to obtain $G_2(q)$ from the previously obtained estimates of $G_1(q)$ and $G_{21}(q)$ is to use the relation $G_2(q) = G_{21}(q)G_1^{-1}(q)$. However, that does not allow imposing a particular structure on $G_2(q)$. Furthermore, if $G_1(q)$ and $G_{21}(q)$ are estimated in the previously presented way, information that could be useful for the estimation is neglected. For example, using also $y_2(t)$ when estimating $G_1(q)$ can improve the variance of the estimates (see Everitt et al. (2013)).

Another possibility, which solves the problem of imposing structure, is to estimate $G_2(q)$ using $y_2(t)$ and an estimate of $u_2(t)$ as data. However, the presence of input noise makes this an errors-in-variables (EIV) problem (Söderström (2007)). When applied to this type of problem, standard system identification methods typically yield parameter estimates that are not consistent. Instrumental variable (IV) methods (Söderström and Stoica (2002)) can be used to solve this problem, since some choices of instruments provide consistent estimates (see, e.g., Söderström and Mahata (2002) and Thil et al. (2008)). A generalized IV approach for EIV identification in dynamic networks has been proposed in Dankers et al.

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(2014). However, using the system in Fig. 1 as an example, notice that $y_2(t)$ is still not used when estimating $G_1(q)$.

If the prediction error method (PEM) is applied to a model structure parametrized according to (1) and to the individual structures of $G_1(q)$ and $G_2(q)$, the obtained estimates are asymptotically efficient (see, e.g., Ljung (1999)). However, for such a model structure, PEM requires, in general, solving a non-convex optimization problem.

In Wahlberg et al. (2008), indirect PEM (Söderström et al. (1991)) is suggested as a suitable method for identification of cascade systems. In this method, PEM is first applied to a higher-order model. In a second step, this model is reduced to the model of interest in an optimal way, in the sense that the obtained estimate has the same asymptotic properties as if PEM had been applied to the smaller model directly. If the model in the first step is easier to estimate than the model of interest, the original problem is simplified.

In this contribution, we restrict ourselves to the case that each transfer function is a single-input single-output (SISO) finite impulse response (FIR) model. First, in Section 2, we revisit the application of IV methods to EIV problems. In Section 3, we review indirect PEM, and extend the discussion in Wahlberg et al. (2008) regarding the application to cascade structures. In Section 4, we point out that this method can be applied even if not all the transfer function outputs in a network are measured. Then, we consider a feedback cascade structure in Section 5, for which indirect PEM does not avoid non-convexity, and propose an intermediate step using the method in Galrinho et al. (2014). A numerical simulation is presented in Section 6, followed by a discussion in Section 7.

2. ERRORS-IN-VARIABLES METHODS

Consider the SISO system $G(q)$, and assume that data is generated according to

$$\begin{cases} y_o(t) = G(q)u(t) \\ y(t) = y_o(t) + \tilde{y}(t) \end{cases}, \tag{2}$$

where $y_o(t)$ is the true system output, $y(t)$ is the measured output, corrupted by noise $\tilde{y}(t)$, and the input $u(t)$ is assumed to be known. We introduce the assumption that $G(q)$ is FIR, and parametrize it accordingly as

$$G(q,\theta) = \theta_1 q^{-1} + \theta_2 q^{-1} + \cdots + \theta_n q^{-n},$$

and that $\tilde{y}(t)$ is Gaussian white noise.

The prediction error method (PEM) serves as benchmark in the field, since it is well known to provide asymptotically efficient estimates if the model orders are correct (Ljung (1999)). The essential idea of PEM is to minimize a cost function of the prediction errors. In this setting, PEM consists on minimizing the cost function

$$V(\theta) = \frac{1}{N}\sum_{t=1}^{N} \left(y(t) - G(q,\theta)u(t)\right)^2, \tag{3}$$

if a quadratic cost is used, and where N is the number of samples available. Then, the minimizer of (3) is an asymptotically efficient estimate of θ, if the model orders are correct. In general, PEM requires solving a non-convex optimization problem. However, for this particular model structure, the minimizer of (3) can be obtained by solving a least squares (LS) problem. Defining the regression vector as

$$\varphi^\top(t) := [u(t-1)\ u(t-2)\ \ldots\ u(t-n)],$$

it is possible to write

$$y(t) = \varphi^\top(t)\theta + \tilde{y}(t),$$

where

$$\theta = [\theta_1\ \theta_2\ \ldots\ \theta_n]^\top.$$

Further, if we define

$$y = [y(1)\ y(2)\ \ldots\ y(N)]^\top,$$

$$\Phi^\top = [\varphi(1)\ \varphi(2)\ \ldots\ \varphi(N)]^\top,$$

and $\tilde{y}$ analogously to y, we can write

$$y = \Phi^\top\theta + \tilde{y}.$$

An estimate of θ, which corresponds to the minimizer of (3), can be obtained by LS, computing

$$\hat{\theta} = \left(\Phi\Phi^\top\right)^{-1}\Phi y. \tag{4}$$

We consider now the case when the true input is not known, but it can be measured, and is corrupted by measurement noise. In this case, the data is generated according to

$$\begin{cases} y_o(t) = G(q)u_o(t) \\ u(t) = u_o(t) + \tilde{u}(t) \\ y(t) = y_o(t) + \tilde{y}(t) \end{cases},$$

where $u_o(t)$ is the true input, $\tilde{u}(t)$ the input measurement noise, and $u(t)$ the measured input. This setting corresponds to an errors-in-variables (EIV) problem (Söderström (2007)). In this scenario, we have that

$$\begin{cases} y(t) = \varphi^\top(t)\theta + v(t,\theta) \\ v(t,\theta) = \tilde{y}(t) - \tilde{\varphi}^\top(t)\theta \end{cases},$$

where, if φ_o is defined analogously to φ, but containing true input values u_o, then

$$\tilde{\varphi}(t) = \varphi(t) - \varphi_o(t).$$

Because $v(t,\theta)$ is not white, if the parameter vector θ is estimated according to (4), the obtained estimate is not consistent.

Instrumental variable (IV) methods are appropriate to deal with EIV problems. The basic idea of IV methods is to choose a vector of instruments $z(t)$ that is uncorrelated with the error $v(t,\theta)$, while being highly correlated with $\varphi(t)$. Then, for such an instrument vector, computing

$$\hat{\theta} = \left(Z\Phi^\top\right)^{-1} Zy,$$

where

$$Z = [z(1)\ z(2)\ \ldots\ z(N)],$$

yields a consistent estimate of θ under certain excitation conditions.

There is no unique way to define $z(t)$. One approach, proposed in Söderström and Mahata (2002), is to choose

$$z^\top(t) = [u(t-1-d_u)\ \ldots\ u(t-d_u-n_{z_u})],$$

where $d_u \geq n$. Another possibility, proposed in Thil et al. (2008), is to also include past outputs in the instrument vector, according to

$$\begin{aligned} z^\top(t) = [\ & -y(t-1-d_y)\ \ldots\ -y(t-d_y-n_{z_y}) \\ & u(t-1-d_u)\ \ldots\ u(t-d_u-n_{z_u})], \end{aligned} \tag{5}$$

where d_y is at least the order of the filter (it must be a moving average (MA) filter) applied to the noise. For the considered FIR case, $d_y \geq 0$.