Available online at www.sciencedirect.com
ScienceDirect
IFAC PapersOnline CONFERENCE PAPER ARCHIVE
IFAC-PapersOnLine 48-28 (2015) 156–161

Control of a Constrained Flexible Rotor on Active Magnetic Bearings

Jie Zhao* Hai-Tao Zhang* Ming-Can Fan Yue Wu***
Huan Zhao***

* *School of Automation, Huazhong University of Science and Technology, Wuhan 430074, P.R. China (e-mail: zht@mail.hust.edu.cn)*
** *Department of Mathematics, Huizhou University, Huizhou 516007, Guangdong, P.R. China*
*** *School of Mechanical Science and Engineering, Huazhong University of Science and Technology, Wuhan 430074, P.R. China*

Abstract: This paper investigates the stability of a flexible rotor supported on Active Magnetic Bearings (AMBs) subject to input and output constraints. For a specified rotor-AMB system, we propose a discrete-time constrained model predictive control (MPC) algorithm based controller. Simulation results are provided to illustrate the effectiveness of the proposed controller in stabilizing the system. Moreover, it is found that when the input constraints are fixed, the vibration amplitudes of the high speed rotor can be suppressed by reducing the output constraints bound.

Keywords: Model Predictive Control (MPC), input and output constraints, Active Magnetic Bearings (AMBs), flexible rotor, vibration suppression

1. INTRODUCTION

These years have witnessed the emergence of applying Active Magnetic Bearings(AMBs) to a large variety of industrial rotating machineries Knospe [2007], e.g., AMB supported turbines, compressors, and machining spindles, etc. By contrast with traditional bearings, the most appealing advantages of AMBs lie in the higher rotor speed and the capability of generating non-contacting active magnetic forces to firmly hold the rotor so as to mitigate rotor vibrations.

However, rotor-AMB systems are open-loop unstable MIMO (multi-input, multi-output) systems. Therefore, to use AMB in industrial machineries systems, it is necessary to develop a closed-loop controller to guarantee its stability, which has recently become an active research topic in machining technology. The details of designing, establishing and modeling of a rotor-AMB system can be referred to Mushi et al. [2012], where a PID controller was implemented to an AMB to suspend a rotor. In Maslen and Sawicki [2007], a μ-synthesis control strategy was performed on a flexible rotor supported on AMBs. By using LQG methods and H_2 robust control approaches, Mushi *et al.* Mushi et al. [2008] stabilized a rotor-AMB plant with variational cross-coupling stiffness. To date, there are few works dealing with input and output constraints of rotor-AMB systems which ubiquitously exist in engineering applications. However, input constraints are often encountered in industrial process due to the physical limits of the actuators Cheng et al. [2015]. Moreover, the output constraints should be addressed as well since the rotor displacements have to be within the nominal air gap between rotor and stator of the AMBs. The difficulty in controller design lies in the nonlinear dynamics nature of the rotor with constraints and the severe coupling of the rotor and the bearing. Therefore, it is still an urgent and challenging task to develop a niche constrained active vibration controller for rotor-AMB systems.

Model Predictive Control(MPC) method has been widely applied in manufacturing industries due to its capability in dealing with constraints, system uncertainties, nonlinearities, and system couplings Cheng et al. [2015], Maciejowski [2002], Zhang et al. [2011], Zhang and Chen [2014]. At each sampling time, MPC produces the dynamics of the system by an internal model, and then solves online a receding horizon optimization problem to obtain an optimal input trajectory that minimize a given performance index Mayne et al. [2000]. By model prediction, receding horizon optimization and feedback rectification, MPC technique is promising to be borrowed to address the active control of rotor-AMB systems.

The main contribution of this paper is to develop a niche MPC scheme to stabilize the MIMO rotor-AMB system with input and output constraints. More specifically, we design an MPC approach to the stabilization problem, i.e., regulating the output displacement to zero. Accordingly, the control input voltage converges to zero as well since voltage settles to zero as the rotor vibration vanishes. Significantly, smaller displacement y implies better mit-

⋆ Corresponding Author is: Hai-Tao Zhang, zht@mail.hust.edu.cn.
This work is supported by National Natural Science Foundation of China (NNSFC) under Grants 61322304, 51328501, 51120155001, the National Science and Technology Major Projects of China under Grant No. 2012ZX04001-012-01-05.

2405-8963 © 2015, IFAC (International Federation of Automatic Control) Hosting by Elsevier Ltd. All rights reserved.
Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2015.12.117

igating capability on the rotor vibrations, which is highly preferable by industrial machining systems.

The rest of this paper is organized as follows. In Section 2, the rotor-AMB system model is briefly introduced. In Section 3, the dynamics internal model are derived and an MPC controller are accordingly developed. Afterwards, numerical simulations are conducted in Section 4 to show the effectiveness of the proposed MPC controller in stabilizing the vibrations of the high-speed rotor. Finally, conclusions are drawn in Section 5.

Throughout this paper, the following notations are used. A vector $\xi \in \mathbb{R}^m > 0$ if and only if all of its m entries are all positive. Matrices $\mathbf{I}$ and $\mathbf{0}$ represent identity and zero matrices, respectively. The matrix $\mathrm{diag}\{A_1, A_2, \ldots, A_n\}$ is a block diagonal matrix with diagonal entries being $A_1, A_2, \ldots, A_n$. The notation $\| \cdot \|_\infty$ denotes the infinity norm and $\| * \|_Q^2 := *^\mathsf{T} Q *$ where '$*$' is a column vector and Q is a positive definite matrix. The notation $\hat{x}(k+i|k)$ indicates the prediction value x at the $(k+i)$-th step on the basis of the currently available information at k-th step.

2. MODELING OF THE ROTOR-AMB SYSTEM

In our MPC scheme, we will use the AMB model designed by Mushi *et al.* Mushi et al. [2012] as the internal model for model prediction. More precisely, a finite element method(FEM) is first used to approximate the dynamics of a flexible rotor. It is assumed that the axial motion of the rotor is independent of the radial motion, then the rotor dynamics model is reduced to a 20-ordered model with two rigid modes and three bending modes. Then, a magnetic reluctance circuit method is adopted to seek a suitable bias current to perform the bias flux linearization. As the displacement is within the nominal air gap between the stator and the rotor, the force generated by the AMB is approximated by a linear function of the rotor-stator air gap and the current flowing through the stator windings. The residual dynamics of the PWM power amplifiers, position sensors, additional filters and sampling components are regarded by a simple gain model with time-delay, which is afterwards approached by a Padé series. Finally, the approximated Padé series are appended to the model of the rotor and AMB system.

The cross coupled forces are ignored to focus on the input and output constraints. Hence we consider a simple rotor-AMB model as below, which considers the dynamics of the rotor, AMBs, sensors, amplifiers, filters and sampling components.

$$\begin{aligned} \dot{\eta} &= A_1 \eta + B_1 u, \\ y_f &= C_f \eta, \end{aligned} \tag{1}$$

with $\eta := [\, x_m \ x_s \ x_a \ x_f \,]^\mathsf{T}$, $B_1 := [\mathbf{0}^\mathsf{T}, \mathbf{0}^\mathsf{T}, B_a^\mathsf{T}, \mathbf{0}^\mathsf{T}]^\mathsf{T}$,

$$A_1 := \begin{bmatrix} \hat{A}_m & \mathbf{0} & B_{1m} K_i C_a & \mathbf{0} \\ B_s C_m & A_s & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & A_a & \mathbf{0} \\ \mathbf{0} & B_f C_s & \mathbf{0} & A_f \end{bmatrix},$$

u is the input voltages acting on the amplifiers, x_m, x_s, x_a, x_f denote the states of the rotor-AMB, sensors, amplifiers and filters, respectively, and y_f represents the x- and y-directional displacements of the two support AMBs, respectively.

Note the order of the overall model (1) is very high that brings some control design difficulties. Therefore, a reduced model of (1) is considered as below, which is called a nominal model denoted by (A_c, B_c, C_c).

$$\begin{aligned} \dot{x}_c &= A_c x_c + B_c u_c, \\ y_c &= C_c x_c, \end{aligned} \tag{2}$$

with $x_c \in \mathbb{R}^{36}$ and $u_c, y_c \in \mathbb{R}^4$.

3. MODEL ANALYSIS AND CONTROLLER DESIGN

3.1 Model analysis

With the A_c, B_c and C_c matrices given in Mushi et al. [2012], the eigenvalues of A_c (or the system model (2)) have real parts at $\{317.7208, 190.7556, 317.7208, 190.7556\}$, and hence the open-loop system (2) is unstable. Further more, it is neither a controllable nor observable system. Hence, pole placement methods are not feasible to stabilize the model. However, by using Hautus-test Hautus [1969], we find that the model (2) is stabilizable as well as detectable. This is critical for implementation of MPC scheme for tackling the instability of the rotor-AMB system in this paper.

3.2 Controller design

Accordingly, we develop a discrete-time MPC controller to deal with the input and output constraints. Recall that the output constraints are considered to avoid contacts between the high-speed rotor and the bearings. Thus, in our controller design, we are more concerned about output displacement constraints rather than the input voltage due to the fact that smaller vibrations implies a more stable rotor. The closed-loop system structure of the MPC controller is shown in Fig. 1, where the receding horizon optimization is implemented to calculate MPC law to address input/ouput constraints, the plant is the rotor-AMB system described below, and $\Gamma(k)$ denotes a future output reference trajectory (specifically, $\Gamma(k) := 0$ for stabilization problem), Υ and Ψ are feedback gain matrices of $u(k)$) and $x(k)$, respectively.

After discretization by MATLAB command *c2d* with sampling time $T_s = 0.01$ second, the linear continuous model in Mushi et al. [2012] is converted into a linear discrete-time model (A, B, C) as below,

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k), \quad x \in \mathbb{R}^m, \quad u \in \mathbb{R}^p, \\ y(k+1) &= Cx(k+1), \quad y \in \mathbb{R}^q, \end{aligned} \tag{3}$$

where $m := 36$, $p := 4$ and $q := 4$.

Assume that the state vector is measurable, i.e., $\hat{x}(k|k) = x(k)$, and any disturbances or measurement noise is unknown Maciejowski [2002]. In the future dynamics iteration, we have used $\hat{u}(k|k)$ rather than $u(k)$ at k-th step as when we need to compute its predictions we do not yet know what $u(k)$ will be. Assume that the input will be only updated at times $k, k+1, \ldots, k+H_u-1$, and will remain constant after that, i.e., $\hat{u}(k+i|k) = \hat{u}(k+H_u-1|k)$ for