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An Algorithm for System Identification of a Discrete-Time Polynomial System without Inputs ⋆

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Abstract: A subalgebraic approximation algorithm is proposed to estimate from a set of time series the parameters of the observer representation of a discrete-time polynomial system without inputs which can generate an approximation of the observed time series. A major step of the algorithm is to construct a set of generators for the polynomial function from the past outputs to the future outputs. For this singular value decompositions and polynomial factorizations are used. A detailed example is described.

Keywords: System identification, algorithms, nonlinear systems, algebraic systems theory.

1. INTRODUCTION

The system identification of polynomial systems is motivated by the need for models of biochemical reaction systems in the life sciences. Also in the area of control engineering and of economics there is a need to determine parameter values of such control systems from data. As far as the authors have been able to determine there is no satisfactory algorithm for the general problem of determining the parameter values of these systems.

The problem of the paper is to determine a system in the class of discrete-time polynomial systems without inputs in the form of a polynomial observer realization. The polynomial observer system when supplied with a time series of outputs should produce a predicted time series which is a reasonable approximation of the supplied time series.

The relevant literature on polynomial systems and their system identification is briefly summarized. At the time this paper is written there are available results on the realization theory of polynomial and of rational systems see Sontag (1979), Bartosiewicz (1988), Němcová and van Schuppen (2009), Němcová and van Schuppen (2010). The problem of structural identifiability of polynomial and of rational systems was solved by J. Němcová in Němcová (2010). Earlier papers of the authors include Nemcova and van Schuppen (2009), Němcová et al. (2012). Various aspects of system identification of polynomial systems are also discussed in Boulier and Lemaire (2009), Gevers et al. (2013), Bazanella et al. (2014).

The algorithm proposed in this paper determines a polynomial system in the form of an observer, thus driven by the available output. It will be proven using system theory that an observer polynomial system is equivalent with the combined conditions: (1) the state of the observer at any time is a polynomial function of the past outputs; (2) the future outputs are a polynomial function only of the current state; and (3) the next state is a polynomial function only of the current state and the current output. The main step of the algorithm is to construct an approximate generator set for the polynomial algebra generated by the function from the past outputs to the future outputs. The subalgebra generated by this generator set is then an approximate subalgebra of the algebra of the function from past outputs to future outputs. These properties justify the term of a subalgebraic approximation algorithm and show the close analogy with the subspace identification algorithm.

At this time it is not yet possible to develop an indepth comparison of the proposed algorithm with existing algorithms. The plan of the authors is to develop such a comparison during the coming years. The main alternative methods are that of the *System Identification Toolbox* and that of the eigenfunction approach, which apply to arbitrary nonlinear systems, not to polynomial systems in particular. There are several reasons why such a comparison is difficult. First one must produce an observer for the polynomial system. Once one has an observer then the algorithm most used is the infimization of an approximation criterion between the time series and the predictions produced by a parametrized observer. But this nonconvex optimization problem is rather difficult as has been well reported in the literature. The alternative of the proposed subalgebraic approximation algorithm seems much more attractive based on the computational complexity criterion.

There is available, as a report, the first version of this paper with the detailed algorithm, Němcová et al. (2015). This conference paper provides an example and an exposition on the basis of the algorithm without technical details. Further publications are in preparation.

2. EXAMPLES

Only one example is provided with many details due to space limitations.

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Example 2.1. The computations of the proposed algorithm are shown for an elementary example. The example is simplified much so as to shorten the exposition. Simultaneously this example shows the steps of the algorithm. Consider the polynomial system,

$$x(t+1) = f(x(t)),\ x(0) = x_0, \tag{1}$$
$$y(t) = h(x(t)), \tag{2}$$
$$n = 2,\ d_y = 1,\ x(t) \in \mathbb{R}^n,\ y(t) \in \mathbb{R}^{d_y},$$
$$f(x) = \begin{pmatrix} a_{11}x_1 + a_{12}x_1x_2, \\ a_{21}x_1 + a_{22}x_2 \end{pmatrix} = \begin{pmatrix} 0.9x_1 - 0.3x_1x_2 \\ -0.2x_1 + 0.8x_2 \end{pmatrix},$$
$$h(x) = x_1 + x_2,$$
$$a = \begin{pmatrix} a_{11} & a_{12} & a_{21} & a_{22} \end{pmatrix}^T = \begin{pmatrix} 0.9 & -0.3 & -0.2 & 0.8 \end{pmatrix}^T,$$
$$\mathbb{Z}_+ = \{1, 2, \ldots\},\ \mathbb{N} = \{0, 1, 2, \ldots\},\ d_a = 4.$$

A set of time series was generated by starting the system at particular initial states whose values are not displayed here. The choice of the initial conditions of the system for each of the time series is rather critical so as to avoid instabilities of the system. The resulting times series are denoted by,

$$\left\{ \left\{ \overline{y}(k,s) \in \mathbb{R}^{d_y},\ s \in T \right\}, k \in \mathbb{Z}_{d_s} \right\},\ \text{where,} \tag{3}$$
$$T = \{1, 2, \ldots, t_1 \in \mathbb{Z}_+\},\ t_1 = 20,\ d_y = 1,\ d_s = 8 \in \mathbb{Z}_+.$$

Select a time $t \in T$ less or equal to $t_1/2 \in \mathbb{Z}_+$. Select the lengths of the future t^+ and of the past t^- of the time series at time t such that $t - t^-$ and $t + t^+ \in T$. The numbers chosen are $t = 10$, $t^- = 8$, $t^+ = 8$, $t - t^- = 2$, and $t + t^+ = 18$.

In this paper the notation is used such that $y : T_1 \to Y = \mathbb{R}^{d_y}$ is a symbolic vector-valued function while a time series with actual numbers is denoted with an overbar as in equation (3).

Construct the future and the past at time t of the time series $k \in \mathbb{Z}_{d_s}$ and denote these by

$$\begin{aligned} &(\overline{y}^+(k,t), \overline{y}^-(k,t)) \\ &= \left(\begin{pmatrix} \overline{y}(k,t) \\ \vdots \\ \overline{y}(k,t+t^+-1) \end{pmatrix}, \begin{pmatrix} \overline{y}(k,t-1) \\ , \vdots \\ \overline{y}(k,t-t^-) \end{pmatrix} \right) \\ &\in \mathbb{R}^{d_{y^+}} \times \mathbb{R}^{d_{y^-}}. \end{aligned}$$

To simplify the exposition, below the index of the time series is omitted except when needed.

Below a polynomial in the elements of the vector $\overline{y}^-(t)$ is used which, for the single-output system of this example, is denoted by,

$$L(\overline{y}^-(t))^K \in \mathbb{R}[\overline{y}(t-1), \ldots, \overline{y}(t-t^-)], \tag{4}$$
$$([\overline{y}^-(t)]^K)_i = \prod_{j=1}^{t^-} \overline{y}(t-j)^{K(i,j)} \in \mathbb{R}, \tag{5}$$
$$d_y \in \mathbb{Z}_+,\ d_{y^-} = t^- d_y,\ d_{y^+} = t^+ d_y, \tag{6}$$
$$d_{v^-} = \prod_{i=1}^{d_{y^-}} (k_{\max,y^-}(i) + 1), \tag{7}$$
$$k_{\max} \in \mathbb{N}^{d_{y^-}},\ L \in \mathbb{R}^{d_{v^+} \times d_{v^-}},$$
$$K \in \mathbb{N}^{d_{v^-} \times d_{y^-}},\ 0 \le K_{i,j} \le k_{\max,y^-}(j),\ \forall\,(i,j).$$

For this example,

$$d_y = 1,\ d_{y^-} = t^- d_y = 8,\ d_{y^+} = t^+ d_y = 8,$$
$$d_{v^-} = 2^{d_{y^-}} = 2^8 = 256,$$
$$\overline{y}^-(t) = \begin{pmatrix} \overline{y}(t-1) \\ \overline{y}(t-2) \\ \vdots \\ \overline{y}(t-t^-) \end{pmatrix} \in \mathbb{R}^8,\ k_{\max,y^-} = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} \in \mathbb{N}^8,$$
$$L \in \mathbb{R}^{8 \times 256},\ \text{is arbitrary,}$$
$$K = \begin{pmatrix} 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1 \\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 0 \\ \vdots \qquad\qquad\quad \vdots \\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \end{pmatrix} \in \mathbb{R}^{256 \times 8}.$$

The algorithm consists of two major steps.
Step 1. Compute a polynomial factorization according to the formulas,

$$\overline{y}^+(t) \approx h_o(g_o(\overline{y}^-(t))), \tag{8}$$
$$\overline{x}(t) = g_o(\overline{y}^-(t)) \in \mathbb{R}^{d_{x_o}}, \tag{9}$$
$$\overline{y}^+(t) \approx h_o(\overline{x}(t)), \tag{10}$$
$$g_o(\overline{y}^-(t)) = L_1(\overline{y}^-(t))^{K_1},\ h_o(\overline{x}(t)) = L_2 x(t)^{K_2}. \tag{11}$$

The algorithm for the approximate polynomial factorization involves a singular value decomposition of a matrix for which the following values for the cummulatives of the singular values were obtained,

$$D = \begin{pmatrix} D_{11} & & \ldots & 0 \\ 0 & D_{22} & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & & \ldots & D_{n_1,n_1} \end{pmatrix},\ D_{\text{cum}} = \begin{pmatrix} 0.8164 \\ 0.9575 \\ 0.9929 \\ \vdots \\ 1.0000 \end{pmatrix},$$
$$D_{\text{cum},i} = \sum_{j=1}^{i} D_{jj} / \sum_{k=1}^{n_1} D_{kk},\ i = 1, 2, \ldots, n_1. \tag{12}$$

It is of interest to choose the state-space dimension of the observer system, which equals the algebraic dimension of the subalgebra defined below, as small as possible. However, one also wants to achieve a reasonable approximation which leads one to a higher state-space dimension. In the computations one must make a choice for the state-space dimension.

The choice of the state-space dimension can be influenced by a bound on the approximation for the cummulative singular values. For the times series used the state space dimension of the observer was chosen to be $d_{x_o} = 3$. It is a property of polynomial systems that the dimension of the observer system may be higher than that of the system generating the output as will be described in a future publication on observers. The computation is such that $\text{rank}(L_1) = d_{x_o}$. The matrices of the polynomials are not displayed to save space. The quality of the approximation of the state vector is summarized in the following table for time series 1. Here the output refers to the future outputs at time $t \in T$ of time series 1.